

# Maths

By a group of supervisors



Interactive Application  
For Distance Learning

THE MAIN BOOK



**3<sup>rd</sup>** PREP.  
2021  
FIRST TERM



EL-MOASSER

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**First**

# **Algebra and Statistics**



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# UNIT 1

## Relations and functions



### ▶ Lessons of the unit :

1. Cartesian product.
2. Relation - Function (mapping).
3. The symbolic representation of the function - Polynomial functions.
4. The study of some polynomial functions.

▶ Use your smart phone or tablet to scan the QR Code and enjoy watching videos.





## LESSON

# 1

## Cartesian product

In this lesson, we shall know the concept of the Cartesian product and how to find it and how to represent it graphically.

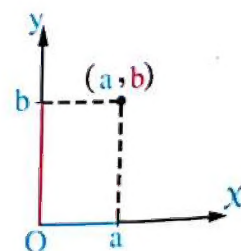
Before dealing with this subject, we shall remember together what we had studied about the ordered pair.

### The ordered pair

$(a, b)$  is called an ordered pair

- $a$  is called the first projection
- $b$  is called the second projection

and the ordered pair  $(a, b)$  could be represented by a point as shown in the opposite figure.



### Remarks

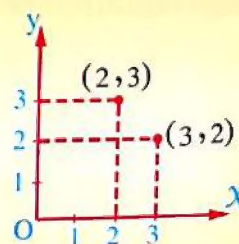
- If  $a \neq b$ , then  $(a, b) \neq (b, a)$

For example:  $(2, 3) \neq (3, 2)$

and when representing them graphically as shown in the opposite figure, we find that they are represented by two different points.

- The ordered pair is not a set.

i.e.  $(a, b) \neq \{a, b\}$



- $(a, a)$  is an ordered pair, while in the sets, we don't write  $\{a, a\}$ , but we write  $\{a\}$  without repeating the element  $a$
- There is an empty set of elements and denoted by the symbol  $\emptyset$ , but there is not an empty ordered pair.

### The equality of two ordered pairs

If  $(a, b) = (x, y)$ , then  $a = x$ ,  $b = y$

For example:

- If  $(a, b) = (3, -4)$ , then  $a = 3$ ,  $b = -4$
- If  $(x, 2) = (-5, y)$ , then  $x = -5$ ,  $y = 2$

**Example 1** Find the values of  $x$  and  $y$  in each of the following :

1  $(x^2 - 1, 8) = (48, \sqrt[3]{y})$       2  $(32, x + y) = (y^5, 2)$

**Solution**

1  $\therefore (x^2 - 1, 8) = (48, \sqrt[3]{y})$

$\therefore x^2 - 1 = 48$

$\therefore \sqrt[3]{y} = 8$

$\therefore x^2 = 49$

$\therefore y = 8^3$

$\therefore x = \pm 7$

$\therefore y = 512$

2  $\therefore (32, x + y) = (y^5, 2)$

$\therefore y^5 = 32$

$\therefore y = 2$  «for  $2^5 = 32$ »

$\therefore x + y = 2$  and by substituting by the value of  $y$  :

$\therefore x + 2 = 2$

$\therefore x = 0$

**TRY 1**  
by yourself

Find the values of  $x$  and  $y$  in each of the following :

1  $(x + 1, y^2) = (3, 9)$

2  $(x^3 - 5, 8) = (3, 3y - 7)$

3  $(x^2 - 2, 2y) = (y, \sqrt[3]{64})$

**Final answers**

of try by yourself  
questions are at the  
end of each lesson  
to check your answer.

## The Cartesian product of two finite sets

For any two finite and non empty sets  $X$  and  $Y$ , we get :

The Cartesian product of the set  $X$  by the set  $Y$  and it is denoted by  $X \times Y$  is the set of all ordered pairs whose first projection of each of them belongs to  $X$  and the second projection of each of them belongs to  $Y$

i.e.  $X \times Y = \{(a, b) : a \in X, b \in Y\}$

For example:

1 If  $X = \{1, 2\}$ ,  $Y = \{5, 7, 8\}$ , then :

$$\begin{aligned}
 X \times Y &= \{1, 2\} \times \{5, 7, 8\} \\
 &= \{(1, 5), (1, 7), (1, 8), (2, 5), (2, 7), (2, 8)\}
 \end{aligned}$$

		Second projection		
		5	7	8
First projection	1	(1, 5)	(1, 7)	(1, 8)
	2	(2, 5)	(2, 7)	(2, 8)

2 If  $X = \{1, 2\}$ ,  $Y = \{5, 7, 8\}$ , then :

$$\begin{aligned}
 Y \times X &= \{5, 7, 8\} \times \{1, 2\} \\
 &= \{(5, 1), (5, 2), (7, 1), (7, 2), (8, 1), (8, 2)\}
 \end{aligned}$$

		Second projection	
		1	2
First projection	5	(5, 1)	(5, 2)
	7	(7, 1)	(7, 2)
	8	(8, 1)	(8, 2)

From the previous, we notice that :

$$X \times Y \neq Y \times X \text{ where } X \neq Y$$

### Remark

The Cartesian product of the set  $X$  by itself and we denote it by  $X \times X$  or by  $X^2$  (it is read  $X$  two) is the set of all ordered pairs whose first projections and second projections belong both to  $X$

$$\text{i.e. } X \times X = \{(a, b) : a \in X, b \in X\}$$

For example: If  $X = \{1, 2\}$ , then :

$$\begin{aligned} X \times X &= \{1, 2\} \times \{1, 2\} \\ &= \{(1, 1), (1, 2), (2, 1), (2, 2)\} \end{aligned}$$

		Second projection	
		1	2
First projection	1	(1, 1)	(1, 2)
	2	(2, 1)	(2, 2)

### Remarks

- For any set  $X$  :  $X \times \emptyset = \emptyset \times X = \emptyset$  where  $\emptyset$  is the empty set.
- If  $(a, b) \in X \times Y$ , then  $a \in X$ ,  $b \in Y$

For example: If  $(3, 5) \in X \times Y$ , then  $3 \in X$ ,  $5 \in Y$

### Representing the Cartesian product of two finite sets

We can represent the Cartesian product of two finite sets in two methods :

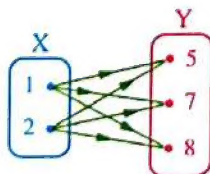
- 1<sup>st</sup> method : The arrow diagram.
- 2<sup>nd</sup> method : The graphical (Cartesian) diagram.

For example: Let  $X = \{1, 2\}$ ,  $Y = \{5, 7, 8\}$

We can represent the Cartesian product  $X \times Y$  where :

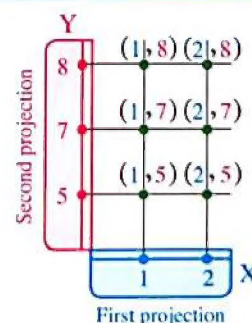
$X \times Y = \{(1, 5), (1, 7), (1, 8), (2, 5), (2, 7), (2, 8)\}$  as follows :

#### First : The arrow diagram



Where we draw an arrow going from each element representing the first projection (the elements of the set  $X$ ) to each element representing the second projection (the elements of the set  $Y$ )

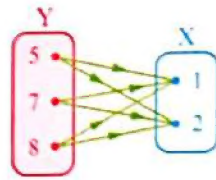
#### Second : The graphical (Cartesian) diagram



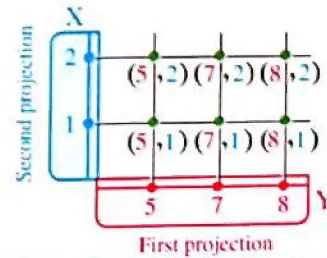
Where the elements of the set  $X$  are represented horizontally and the elements of the set  $Y$  are represented vertically and the points of intersection of the horizontal and vertical lines represent the Cartesian product of  $X \times Y$

- Similarly, we can represent  $Y \times X$  where :

$Y \times X = \{(5, 1), (5, 2), (7, 1), (7, 2), (8, 1), (8, 2)\}$  as follows :

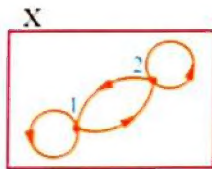


The arrow diagram

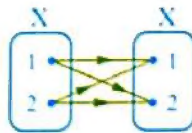


The Cartesian diagram

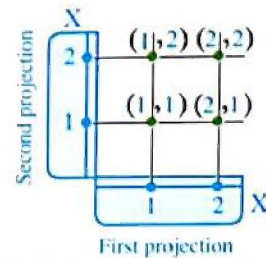
- Also, we can represent  $X \times X$  where :  $X \times X = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$  as follows :




or



The arrow diagram



The Cartesian diagram

**Notice that :** The figure  is called a loop to show that the arrow goes from the point and returns to the same point.

**Example 2** If  $X = \{2, 3, 4\}$  and  $Y = \{a, b\}$  find each of :

1  $X \times Y$

2  $Y \times X$

3  $X \times X$

, then find the number of elements of each of them.

**Solution**

1  $X \times Y = \{(2, a), (2, b), (3, a), (3, b), (4, a), (4, b)\}$   
the number of elements of  $X \times Y = 6$  ordered pairs.

2  $Y \times X = \{(a, 2), (a, 3), (a, 4), (b, 2), (b, 3), (b, 4)\}$   
the number of elements of  $Y \times X = 6$  ordered pairs.

3  $X \times X = \{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)\}$ , the number of elements of  $X \times X = 9$  ordered pairs.

### Remarks

If we denote the number of elements of any set by «n», then from the previous example, we find that :

$$\bullet n(X) = 3, n(Y) = 2 \quad \bullet n(X \times Y) = 6, n(Y \times X) = 6, n(X \times X) = 9$$

i.e. ①  $n(X \times Y) = n(Y \times X) = n(X) \times n(Y)$       ②  $n(X \times X) = (n(X))^2$  ”

### TRY 2

by yourself

If  $X = \{3, 4, 5\}$ ,  $Y = \{5, 6\}$ ,  $Z = \{1, 4\}$  Find each of the following :

①  $Y \times X$  and represent it by an arrow diagram

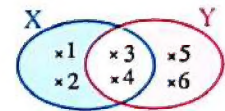
②  $X^2$  and represent it by a Cartesian diagram

③  $n(X \times Z)$

④  $n(Y^2)$

### Remember the operations on sets

If  $X = \{1, 2, 3, 4\}$ ,  $Y = \{3, 4, 5, 6\}$ , then :



- $X \cap Y$  = the set of elements which are common in X and  $Y = \{3, 4\}$
- $X \cup Y$  = the set of all elements in X or Y without repeating =  $\{1, 2, 3, 4, 5, 6\}$
- $X - Y$  = the set of elements which are in X and not in  $Y = \{1, 2\}$
- $Y - X$  = the set of elements which are in Y and not in  $X = \{5, 6\}$

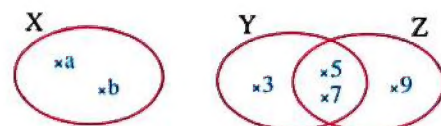
**Example 3** If  $X = \{a, b\}$ ,  $Y = \{3, 5, 7\}$ ,  $Z = \{5, 7, 9\}$

Represent the sets X, Y and Z by Venn diagram, then find :

①  $X \times (Y \cup Z), (X \times Y) \cup (X \times Z)$

②  $X \times (Y \cap Z), (X \times Y) \cap (X \times Z)$

③  $X \times (Z - Y), (X \times Z) - (X \times Y)$



### Solution

①  $\because Y \cup Z = \{3, 5, 7, 9\}$

$$\therefore X \times (Y \cup Z) = \{a, b\} \times \{3, 5, 7, 9\}$$

$$= \{(a, 3), (a, 5), (a, 7), (a, 9), (b, 3), (b, 5), (b, 7), (b, 9)\}$$

$$\begin{aligned} X \times Y &= \{a, b\} \times \{3, 5, 7\} \\ &= \{(a, 3), (a, 5), (a, 7), (b, 3), (b, 5), (b, 7)\} \end{aligned} \quad (1)$$

$$\begin{aligned} X \times Z &= \{a, b\} \times \{5, 7, 9\} \\ &= \{(a, 5), (a, 7), (a, 9), (b, 5), (b, 7), (b, 9)\} \end{aligned} \quad (2)$$

From (1) and (2) :

$$\begin{aligned} \therefore (X \times Y) \cup (X \times Z) &= \\ \{(a, 3), (a, 5), (a, 7), (a, 9), (b, 3), (b, 5), (b, 7), (b, 9)\} \end{aligned}$$

$$2 \quad \therefore Y \cap Z = \{5, 7\}$$

$$\therefore X \times (Y \cap Z) = \{a, b\} \times \{5, 7\} = \{(a, 5), (a, 7), (b, 5), (b, 7)\}$$

From (1) and (2) :

$$\therefore (X \times Y) \cap (X \times Z) = \{(a, 5), (a, 7), (b, 5), (b, 7)\}$$

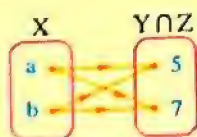
$$3 \quad \therefore Z - Y = \{9\}$$

$$\therefore X \times (Z - Y) = \{a, b\} \times \{9\} = \{(a, 9), (b, 9)\}$$

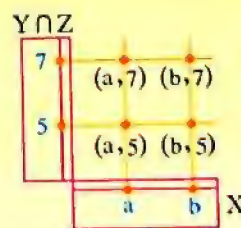
$$\text{From (1) and (2)} : \therefore (X \times Z) - (X \times Y) = \{(a, 9), (b, 9)\}$$

### Remark

In the previous example, we can represent  $X \times (Y \cap Z)$  by an arrow diagram and a Cartesian diagram as follows :



The arrow diagram



The Cartesian diagram

### TRY 3

by yourself

If  $X = \{2, 3\}$ ,  $Y = \{1, 3, 5\}$ ,  $Z = \{2\}$

Represent each of  $X$ ,  $Y$  and  $Z$  by Venn diagram, then find :

1  $Z \times (X \cap Y)$

2  $(Z \times X) \cup (Z \times Y)$

### The Cartesian product of two infinite sets

- We know that if  $X$  is a finite set (having  $n$  elements), then the Cartesian product  $X \times X$  is also a finite set (having  $n^2$  elements).

For example: If  $n(X) = 3$ , then  $n(X \times X) = 9$

- But if  $X$  is an infinite set, then  $X \times X$  is an infinite set also

As examples for that :

$$\mathbb{N} \times \mathbb{N} = \{(x, y) : x \in \mathbb{N}, y \in \mathbb{N}\}, \quad \mathbb{Z} \times \mathbb{Z} = \{(x, y) : x \in \mathbb{Z}, y \in \mathbb{Z}\},$$

$$\mathbb{Q} \times \mathbb{Q} = \{(x, y) : x \in \mathbb{Q}, y \in \mathbb{Q}\}, \quad \mathbb{R} \times \mathbb{R} = \{(x, y) : x \in \mathbb{R}, y \in \mathbb{R}\}$$

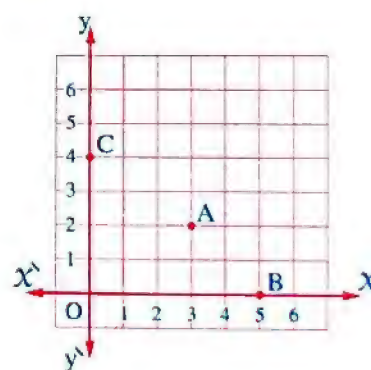
### Representing the Cartesian product of two infinite sets

- We know that if  $X$  is a finite set, we represent the Cartesian product  $X \times X$  graphically by a finite number of points.
- But if  $X$  is an infinite set, then the Cartesian product  $X \times X$  represented graphically by an infinite number of points.

The following is the graphical representation of each of:  $\mathbb{N} \times \mathbb{N}$ ,  $\mathbb{Z} \times \mathbb{Z}$ ,  $\mathbb{R} \times \mathbb{R}$ :

#### First Representing the Cartesian product $\mathbb{N} \times \mathbb{N}$ ( $\mathbb{N}^2$ )

- Represent the natural numbers on two perpendicular straight lines, one of them  $\overleftrightarrow{XX}$  is horizontal and the other  $\overleftrightarrow{yy}$  is vertical, where they intersect at the point which represents the number zero on each of them i.e.  $O = (0, 0)$
- The opposite figure shows a small part of the perpendicular graphical net of the Cartesian product  $\mathbb{N} \times \mathbb{N}$  which consists of the vertical and the horizontal straight lines that pass through the points which represent the natural numbers on each of  $\overleftrightarrow{XX}$  and  $\overleftrightarrow{yy}$
- And each point of the points of this net represents an ordered pair of the Cartesian product  $\mathbb{N} \times \mathbb{N}$

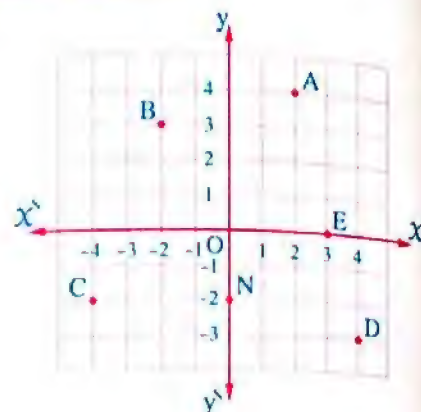


For example:

- The point A represents the ordered pair (3, 2)
- The point B represents the ordered pair (5, 0)
- The point C represents the ordered pair (0, 4)
- The point O represents the ordered pair (0, 0)

## Second Representing the Cartesian product $\mathbb{Z} \times \mathbb{Z}$ ( $\mathbb{Z}^2$ )

- Represent the integers on each of  $\overleftrightarrow{XX}$  and  $\overleftrightarrow{yy}$  which are intersecting at O (0, 0)
- The opposite figure shows a small part of the perpendicular graphical net of the Cartesian product  $\mathbb{Z} \times \mathbb{Z}$
- And each point of its points represents an ordered pair of the Cartesian product  $\mathbb{Z} \times \mathbb{Z}$



For example:

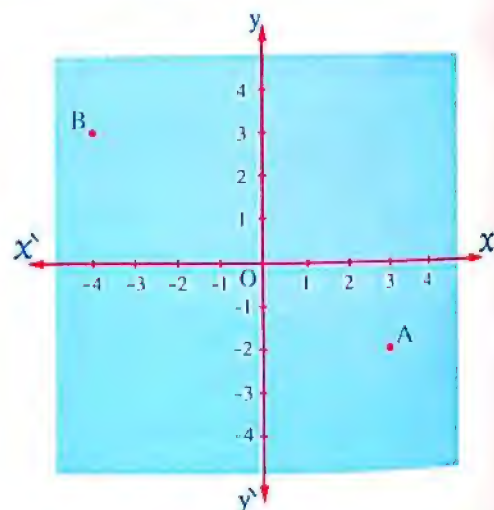
- The point A represents the ordered pair (2, 4)
- The point B represents the ordered pair (-2, 3)
- The point C represents the ordered pair (-4, -2)
- The point D represents the ordered pair (4, -3)
- The point E represents the ordered pair (3, 0)
- The point N represents the ordered pair (0, -2)

## Third Representing the Cartesian product $\mathbb{R} \times \mathbb{R}$ ( $\mathbb{R}^2$ )

- The perpendicular graphical net of the Cartesian product  $\mathbb{R} \times \mathbb{R}$  is an infinite extended surface from all sides and the opposite figure shows a small part of this region.
- Each point of this region represents an ordered pair of the Cartesian product  $\mathbb{R} \times \mathbb{R}$

For example:

- The point A represents the ordered pair (3, -2)
- The point B represents the ordered pair (-4, 3)



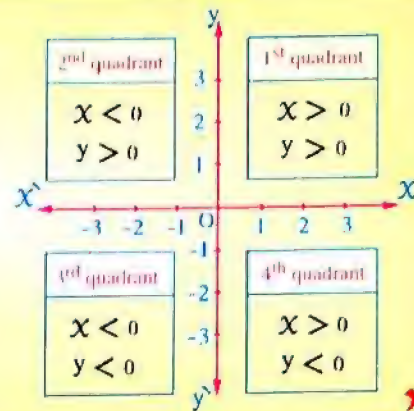
### Remarks

- 1 The horizontal straight line  $\overleftrightarrow{XX}$  is called X-axis or the horizontal axis and the vertical straight line  $\overleftrightarrow{yy}$  is called y-axis or the vertical axis.
- 2 The point of intersection of the two axes  $\overleftrightarrow{XX}$  and  $\overleftrightarrow{yy}$  is called the origin point.
- 3 If the point A represents the ordered pair (X, y) in the Cartesian product  $\mathbb{R} \times \mathbb{R}$ , then :
  - The first projection X is called the X-coordinate of the point A
  - The second projection y is called the y-coordinate of the point A

- 4 The two axes  $\overleftrightarrow{XX}$  and  $\overleftrightarrow{YY}$  divide the plane into four quadrants as shown in the opposite figure and we can determine the quadrant in which any point lies by knowing the signs of its two coordinates.

- 5 If the  $X$ -coordinate of the point  $= 0$   
 , then the point lies on  $y$ -axis

- 6 If the  $y$ -coordinate of the point  $= 0$   
 , then the point lies on  $X$ -axis

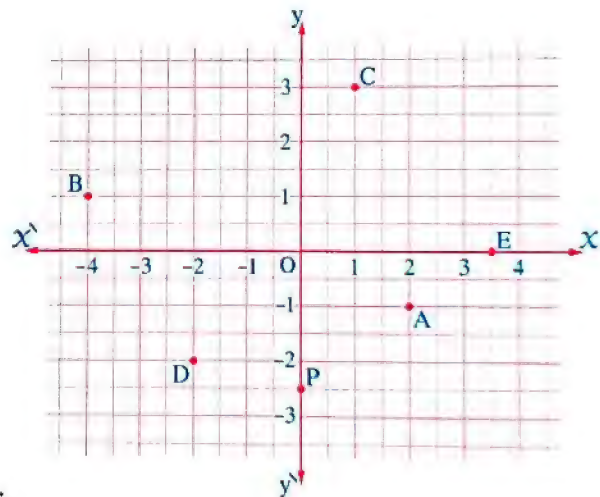


**Example 4** Mention the quadrant or the axis on which each of the following points lies on the perpendicular square net of the Cartesian product  $\mathbb{R} \times \mathbb{R}$  , then locate it on the net :

A (2 , -1) , B (-4 , 1) , C (1 , 3) , D (-2 , -2) , E  $(3\frac{1}{2} , 0)$   
 , P  $(0 , -2\frac{1}{2})$

**Solution**

- A (2 , -1) lies on the 4<sup>th</sup> quadrant
- B (-4 , 1) lies on the 2<sup>nd</sup> quadrant
- C (1 , 3) lies on the 1<sup>st</sup> quadrant
- D (-2 , -2) lies on the 3<sup>rd</sup> quadrant
- E  $(3\frac{1}{2} , 0)$  lies on the  $X$ -axis
- P  $(0 , -2\frac{1}{2})$  lies on the  $y$ -axis



**TRY 4**

by yourself

Mention the quadrant or the axis on which each point lie :

The point	(-3 , 5)	(2 , -1)	$(5 , \frac{1}{2})$	$(-\frac{1}{3} , -2\frac{1}{4})$	(0 , 5)	$(-1\frac{1}{5} , 0)$
The quadrant or the axis						

## The Cartesian product of two intervals

We studied that interval is subset of the set of the real numbers ( $\mathbb{R}$ ) and then the Cartesian product of two intervals is a subset of the Cartesian product  $\mathbb{R} \times \mathbb{R}$  and we can explain that in the following example.

**Example 5** If  $X = [0, 3]$  ,  $Y = [1, 3]$

Represent graphically using the perpendicular graphical net of the Cartesian product  $\mathbb{R} \times \mathbb{R}$  the region which represents each of :

1  $X \times Y$

2  $X \times X$

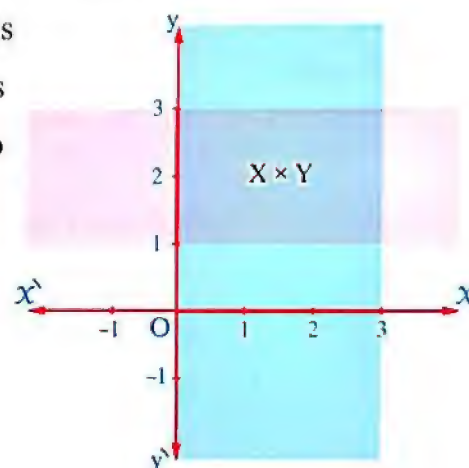
3  $Y \times Y$

, then show , in each case , which of the following points belongs to the previous Cartesian products :  $(2, 2)$  ,  $(1, 0)$  ,  $(0, 3)$

### Solution

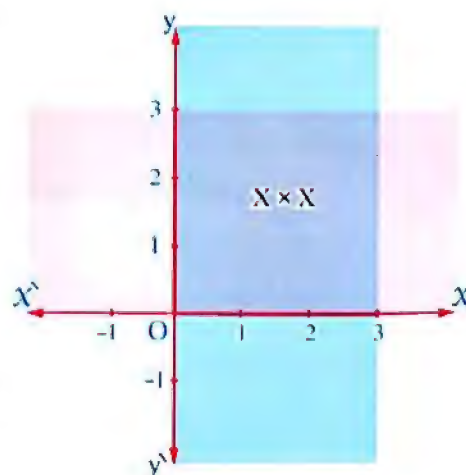
1 To represent  $X \times Y$  , we follow the following steps :

- Represent the interval  $X$  on  $X$ -axis
- Represent the interval  $Y$  on  $y$ -axis
- The intersection region of the two colours represents  $X \times Y$
- $(2, 2) \in X \times Y$  because it belongs to the region which represents  $X \times Y$
- $(1, 0) \notin X \times Y$  because it lies outside the region which represents  $X \times Y$
- $(0, 3) \in X \times Y$



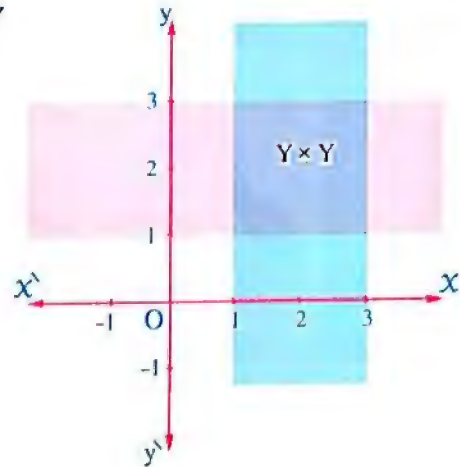
2 To represent  $X \times X$  graphically , do as follows :

- Represent the interval  $X$  one time on  $X$ -axis and another time on  $y$ -axis.
- The intersection region of the two colours represents  $X \times X$
- $(2, 2) \in X \times X$  ,  $(1, 0) \in X \times X$  and  $(0, 3) \in X \times X$



3 Similarly, we can represent  $Y \times Y$  as shown in the opposite figure :

- $(2, 2) \in Y \times Y$
- $(1, 0) \notin Y \times Y$
- and  $(0, 3) \notin Y \times Y$



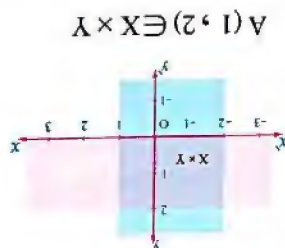
### TRY 5

in yourself

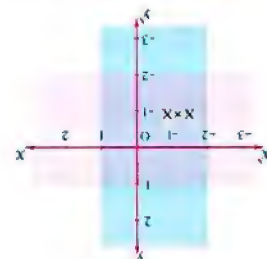
If  $X = [-2, 1]$  ,  $Y = [0, 2]$

Find the region which expresses each of the following using the perpendicular square net of the Cartesian product  $\mathbb{R} \times \mathbb{R}$

- 1  $X \times X$
- 2  $X \times Y$  and show which of the following points belongs to  $X \times Y$   
A (1, 2) , B (0, -2) , C (3, -1) , D (-2, -2)



A (1, 2)  $\in X \times Y$



5 1

2

4 The second, the fourth, the first, the third, y-axis, X-axis.

3 1  $\{(2, 3)\}$

2  $\{(2, 2), (2, 3), (2, 1), (2, 5)\}$

3 6

4 4

2  $\{(3, 3), (3, 4), (3, 5), (4, 3), (4, 4), (4, 5), (5, 3), (5, 4), (5, 5)\}$ , represent by yourself.

2 1  $\{(5, 3), (5, 4), (5, 5), (6, 3), (6, 4), (6, 5)\}$ , represent by yourself.

1 1  $X = 2$  ,  $y = \pm 3$

2  $X = 2$  ,  $y = 5$

3  $X = \pm 2$  ,  $y = 2$

At the end of each lesson, you will find the final answers of try by yourself questions in the same form.

Answers of try by yourself



## LESSON

# 2

## Relation - Function (mapping)

### First

### The relation

The relation from set  $X$  to set  $Y$  is a connection that connects some or all the elements of set  $X$  with some or all the elements of set  $Y$  and it is denoted by " $R$ ".

- The relation  $R$  from  $X$  to  $Y$  is the set of ordered pairs whose first projection belongs to  $X$  and its second projection belongs to  $Y$  and the first projection is connected with the second projection by this relation.

If  $(a, b) \in R$  where  $a \in X, b \in Y$

So, we express this as " $a R b$ "

- The relation  $R$  is a subset of the Cartesian product  $X \times Y$

i.e.  $R \subset X \times Y$

- This relation can be expressed by an arrow diagram or a Cartesian diagram (graphical).

**Example 1** If  $X = \{2, 5\}$ ,  $Y = \{1, 4, 7\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a < b$ " for every  $a \in X$ ,  $b \in Y$ . State the relation  $R$  and represent it by an arrow diagram and by a Cartesian diagram.

**Solution**

$\because 2$  is not less than 1  $\therefore (2, 1) \notin R$

$\because 2 < 4$   $\therefore (2, 4) \in R$

$\because 2 < 7$   $\therefore (2, 7) \in R$

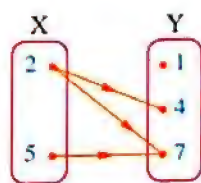
$\because 5$  is not less than 1  $\therefore (5, 1) \notin R$

$\because 5$  is not less than 4  $\therefore (5, 4) \notin R$

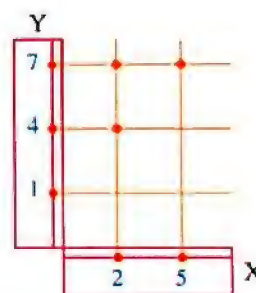
$\because 5 < 7$   $\therefore (5, 7) \in R$

$\therefore$  The relation  $R = \{(2, 4), (2, 7), (5, 7)\}$

The following figures represent the arrow diagram and the Cartesian diagram of this relation :



The arrow diagram



The Cartesian diagram

**TRY 1**  
by yourself

If  $X = \{1, 2, 3\}$ ,  $Y = \{3, 4, 5, 6\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + b = 6$ " for every  $a \in X$  and  $b \in Y$ .

**State the set of  $R$  and represent it by an arrow diagram.**

**Remark**

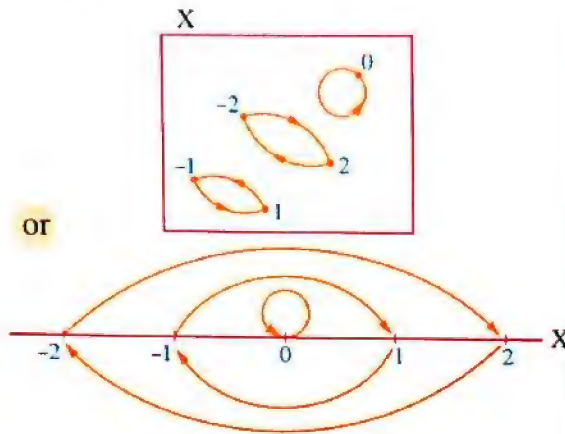
If  $R$  is a relation from  $X$  to  $X$ , then :  $R$  is a relation on  $X$  and the relation  $R \subset X \times X$

**Example 2** If  $X = \{-2, -1, 0, 1, 2\}$  and  $R$  is a relation on  $X$  where " $a R b$ " means " $a$  is the additive inverse of the number  $b$ " for every  $a \in X$  and  $b \in X$ . State  $R$ , then represent it by an arrow diagram and a Cartesian diagram.

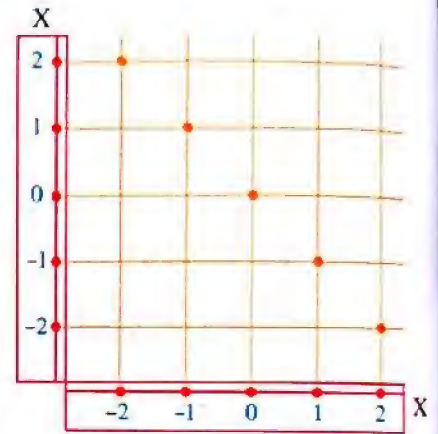
**Solution**

$$R = \{(-2, 2), (-1, 1), (0, 0), (1, -1), (2, -2)\}$$

• The arrow diagram :



• The Cartesian diagram :

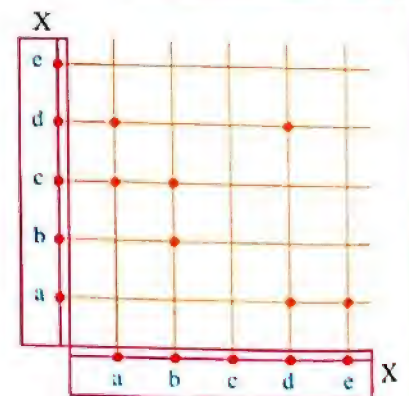
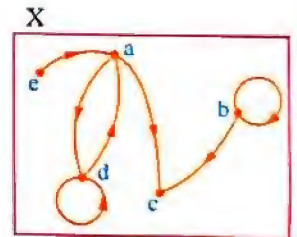


**Example 3** If the opposite arrow diagram represents the relation  $R$  on  $X$

State  $R$ , then represent it by a Cartesian diagram.

**Solution**

$$R = \{(a, c), (a, d), (b, b), (b, c), (d, d), (d, a), (e, a)\}$$



**TRY 2**  
by yourself

If  $X = \{1, 2, 4\}$  and  $R$  is a relation on  $X$  where " $a R b$ " means " $a$  is twice  $b$ " for every  $a \in X$  and  $b \in X$

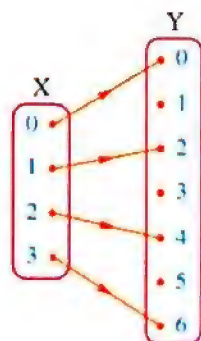
State  $R$  and represent it by a Cartesian diagram.

## Second Functions (Mapping)

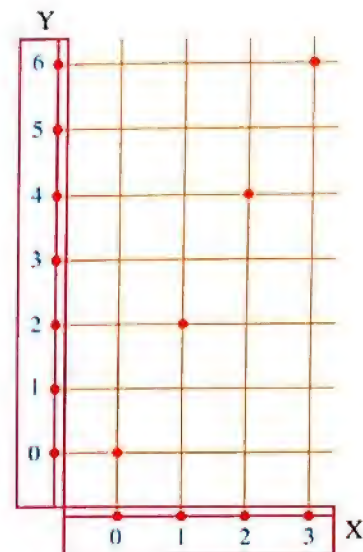
**Introductory example** If  $X = \{0, 1, 2, 3\}$ ,  $Y = \{0, 1, 2, 3, 4, 5, 6\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a = \frac{1}{2} b$ " for each  $a \in X, b \in Y$ . Write  $R$  and represent it by an arrow diagram and a Cartesian diagram.

**Solution**

$$R = \{(0, 0), (1, 2), (2, 4), (3, 6)\}$$



The arrow diagram



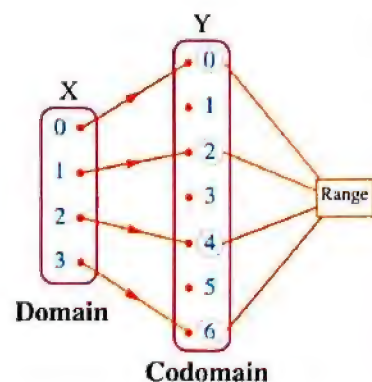
The Cartesian diagram

In the previous relation, we notice that :

Each element of the set  $X$  has been connected with one and only one element of the elements of the set  $Y$

Such as, this relation is called a function or (mapping).

- The set of  $X = \{0, 1, 2, 3\}$  is called "the domain of the function".
- The set of  $Y = \{0, 1, 2, 3, 4, 5, 6\}$  is called "the codomain of the function".
- The set  $\{0, 2, 4, 6\}$  is called "the range of the function" and it is a subset from the codomain of the function.



**Generally**

A relation from  $X$  to  $Y$  is said to be a function if :

- 1 In the relation , **each element** of the set  $X$  appears **only once** as a first projection in one of the ordered pairs of **the relation**.
- 2 In the arrow diagram which represents the relation , **each element** of  $X$  has **one and only one arrow** going out of it to one element of  $Y$
- 3 In the Cartesian diagram which represents the relation , **each vertical line** has **one and only one point** lying on it of the points which represent the relation.

**Example 4** If  $X = \{1, 2, 3, 4\}$  ,  $Y = \{1, 3, 5, 7\}$

Show which of the following relations represents a function from  $X$  to  $Y$  and if it is a function , mention its range :

$$R_1 = \{(2, 3), (1, 1), (3, 5), (3, 7), (4, 3)\}$$

$$R_2 = \{(1, 7), (2, 5), (4, 1)\}$$

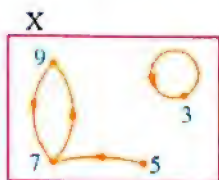
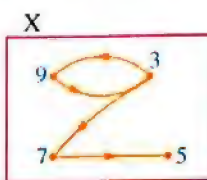
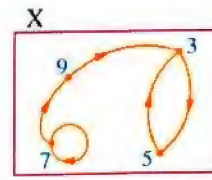
$$R_3 = \{(2, 3), (3, 3), (1, 5), (4, 7)\}$$

**Solution**

- $R_1$  is not a function because the element  $3 \in X$  appears as a first projection twice in two ordered pairs of the relation  $(3, 5)$  and  $(3, 7)$
- $R_2$  is not a function because the element  $3 \in X$  does not appear as a first projection in any ordered pair of the relation.
- $R_3$  is a function because each element of  $X$  appeared only once as a first projection in an ordered pair of the relation , the range of  $R_3$  is  $\{3, 5, 7\}$

**Example 5** If  $X = \{3, 5, 7, 9\}$

Show which of the following arrow diagrams represents a function on  $X$  (i.e. from  $X$  to  $X$ ) and if it is a function, mention its range :

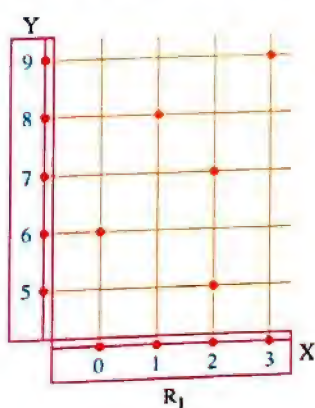
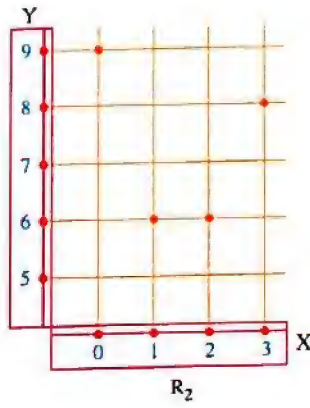
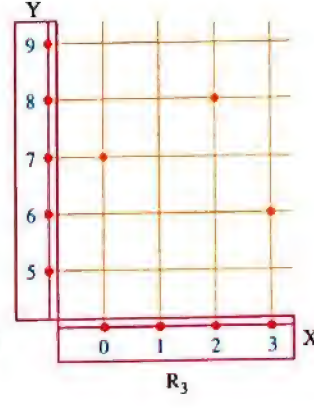
 $F_1$  $F_2$  $F_3$ 

**Solution**

- $F_1$  is a function because each element of  $X$  has only one arrow going out of it to one element of  $X$ , the range of the function  $F_1$  is  $\{3, 7, 9\}$
- $F_2$  is not a function because for the element  $5 \in X$  there are no arrows going out of it or because the element  $3 \in X$  has two arrows going out of it.
- $F_3$  is not a function because the element  $7 \in X$  has two arrows going out of it.

**Example 6** If  $X = \{0, 1, 2, 3\}$ ,  $Y = \{5, 6, 7, 8, 9\}$

Show which of the following Cartesian diagrams represents a function from  $X$  to  $Y$  and if it is a function, mention its range :

 $R_1$  $R_2$  $R_3$ 

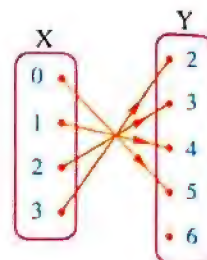
**Solution**

- $R_1$  is not a function because there are two points lying on the vertical line which passes through the element  $2 \in X$
- $R_2$  is a function because each vertical line has only one point lying on it, the range of the function  $R_2$  is  $\{6, 8, 9\}$
- $R_3$  is not a function because there is no point on the vertical line which passes through the element  $1 \in X$

**Example 7** If  $X = \{0, 1, 2, 3\}$ ,  $Y = \{2, 3, 4, 5, 6\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + b = 5$ " for each  $a \in X, b \in Y$ . Write the relation  $R$  and represent it by an arrow diagram. Mention giving reasons if  $R$  is a function from  $X$  to  $Y$  or not? And if it is a function, find its range.

**Solution**

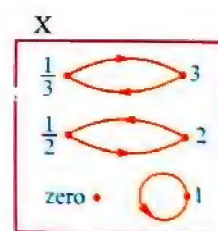
- $R = \{(0, 5), (1, 4), (2, 3), (3, 2)\}$
- $R$  represents a function from  $X$  to  $Y$  because each element of  $X$  connects with only one element of  $Y$
- The range of the function =  $\{5, 4, 3, 2\}$



**Example 8** If  $X = \{3, 2, 1, \text{zero}, \frac{1}{2}, \frac{1}{3}\}$  and  $R$  is a relation on  $X$  where " $a R b$ " means " $a$  is the multiplicative inverse of  $b$ " for each  $a \in X, b \in X$ . Write  $R$  and represent it by an arrow diagram and mention giving reasons if  $R$  represents a function or not.

**Solution**

- $R = \{(3, \frac{1}{3}), (2, \frac{1}{2}), (1, 1), (\frac{1}{2}, 2), (\frac{1}{3}, 3)\}$
- $R$  does not represent a function because the element  $\text{zero} \in X$  does not connect with any element in  $X$  (There is no arrow going out from zero in the arrow diagram which represents the relation)



**TRY 3**  
yourself

If  $X = \{1, 2, 3\}$ ,  $Y = \{1, 4, 6, 9\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a = \sqrt{b}$ " for each  $a \in X, b \in Y$ .

Write the relation  $R$  and represent it by an arrow diagram. Mention giving reasons if  $R$  is a function from  $X$  to  $Y$  or not? and if it is a function, mention its range.

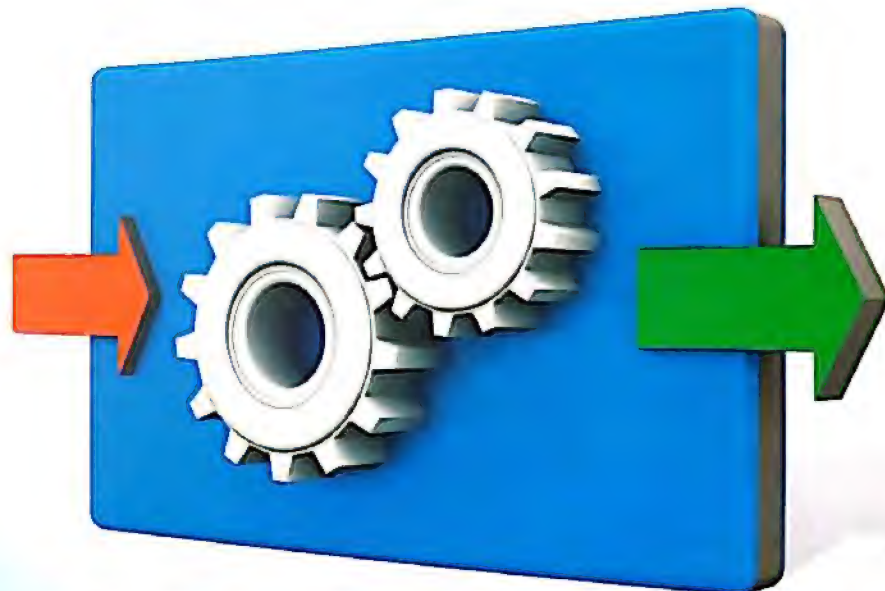
Yes,  $R$  is a function from  $X$  to  $Y$  because each element of  $X$  has one image in  $Y$ , its range =  $\{1, 4, 9\}$

**3**  $R = \{(1, 1), (2, 4), (3, 9)\}$ , represent by yourself.

**2**  $R = \{(2, 1), (4, 2)\}$ , represent by yourself.

**1**  $R = \{(1, 5), (2, 4), (3, 3)\}$ , represent by yourself.

**Answers**  
of try by yourself



## LESSON

# 3

## The symbolic representation of the function - Polynomial functions

### The symbolic representation of the function



- The function is usually denoted by one of the following letters as  $f$  or  $m$  or  $q$  or ... and the function  $f$  from the set  $X$  to the set  $Y$  is written mathematically as :
- If the ordered pair  $(x, y)$  belongs to the function, then the element  $y$  is called the image of the element  $x$  by the function  $f$  and we express it by one of the following two forms :

$f : X \longrightarrow Y$  and is read as  $f$  is a function from  $X$  to  $Y$

or  $m : X \longrightarrow Y$  and is read as  $m$  is a function from  $X$  to  $Y$  and so on ...

$f : x \longmapsto y$  it is read as  $f$  maps  $x$  to  $y$

or  $f : f(x) = y$  it is read as  $f$  is a function where  $f(x) = y$

**For example:**

If  $f : X \longrightarrow Y$  where  $f : x \longmapsto x^2$ , then  $f : 3 \longmapsto 9$

, also can be written in the form :  $f(x) = x^2$ , hence  $f(3) = 9$

### Remark

The mathematical form  $f(x) = x^2$  is called the rule of the function  $f$ , and it is used to find the image of any element of the domain by the function  $f$



### Remember that

If  $f$  is a function from the set  $X$  to the set  $Y$  i.e.  $f : X \longrightarrow Y$ , then :

- 1  $X$  is called the **domain** of the function  $f$
- 2  $Y$  is called the **codomain** of the function  $f$
- 3 The set of images of the elements of the set  $X$  by the function  $f$  is called the **range** of the function  $f$  which is a subset of the codomain  $Y$

**Example 1** If  $X = \{-1, 0, 1\}$ ,  $Y = \{0, -1, -2\}$  and the function  $f : X \longrightarrow Y$  where  $f(x) = x^2 - 1$ , find the set of the function  $f$  and represent it by an arrow diagram, then write its range.

### Solution

$$\therefore f(x) = x^2 - 1$$

$$\therefore f(-1) = (-1)^2 - 1 = 0$$

$$\therefore (-1, 0) \in \text{the set of the function } f$$

$$, f(0) = (0)^2 - 1 = -1$$

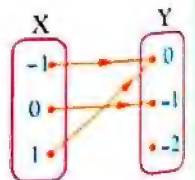
$$\therefore (0, -1) \in \text{the set of the function } f$$

$$, f(1) = (1)^2 - 1 = 0$$

$$\therefore (1, 0) \in \text{the set of the function } f$$

$$\therefore \text{The set of the function } f = \{(-1, 0), (0, -1), (1, 0)\}$$

The range of the function  $f = \{0, -1\}$



### Remark

If  $f$  is a function from the set  $X$  to itself i.e.  $f : X \longrightarrow X$ , then we say « $f$  is a function on  $X$ »

**Example 2** If  $f : \mathbb{N} \longrightarrow \mathbb{N}$  where  $\mathbb{N}$  is the set of natural numbers and  $f(x) = x + 1$  find  $f(0)$ ,  $f(1)$ ,  $f(2)$ ,  $f(3)$  and  $f(4)$ , then graph a part of the square net of the Cartesian product  $\mathbb{N} \times \mathbb{N}$  and represent on it 5 elements of this function. What is the range of the function?

**Solution**

$f(x) = x + 1$  for each  $x \in \mathbb{N}$

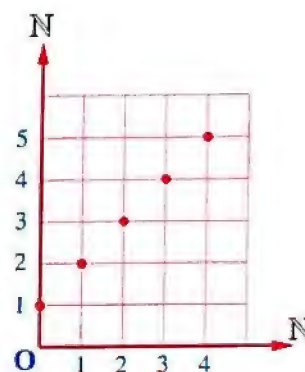
means that the image of any natural number by the function  $f$  is "the number + 1"

$$\therefore f(0) = 0 + 1 = 1, \quad f(1) = 2$$

$$, f(2) = 3, \quad f(3) = 4, \quad f(4) = 5$$

$$\therefore (0, 1), (1, 2), (2, 3), (3, 4), (4, 5)$$

are 5 elements of  $f$



- The range of  $f$  is all the natural numbers except zero. (because there is no natural number added 1 gives zero)

i.e. The range of  $f = \mathbb{N} - \{0\}$

**TRY 1**

by yourself

If  $X = \{2, 4, 6, 8\}$ ,  $Y = \{1, 2, 3, 4, 5, 6\}$  and the function  $f : X \longrightarrow Y$  where  $f(x) = \frac{1}{2}x$

**Write the set of the function  $f$  and represent it by a Cartesian diagram, then find its range.**

## Polynomial functions

### Definition

The function  $f : \mathbb{R} \longrightarrow \mathbb{R}$ ,  $f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$  where  $a_0, a_1, a_2, \dots, a_n \in \mathbb{R}$ ,  $n \in \mathbb{N}$  is called a polynomial function.

i.e.

The polynomial function is a function whose rule is a term or an algebraic expression in condition that the following should be identified :

- 1 Each of the domain and the codomain of the function is the set of real numbers.
- 2 The power (the index) of the variable  $x$  in any of its terms is a natural number.

**For example:** The following functions are all polynomial functions :

- $f : f(x) = 2x + 5$
- $g : g(x) = x^2 - 2x + 1$
- $k : k(x) = 8$
- $n : n(x) = 1 + \sqrt{2}x - 9x^3$

### Remark

If the domain or the codomain of a function is not the set of real numbers, then that function is not a polynomial function.

**For example:**

- $f : f(x) = \sqrt{x}$  is not a polynomial function because  $f(x)$  doesn't exist in  $\mathbb{R}$  if  $x$  equals a negative number.

For example :  $f(-1) \notin \mathbb{R}$  because  $\sqrt{-1} \notin \mathbb{R}$

, so the domain of the function  $f$  is not the set of real numbers.

- $h : h(x) = \frac{1}{x}$  is not a polynomial function

because  $h(x)$  doesn't exist in  $\mathbb{R}$  if  $x$  equals zero. i.e.  $h(0) \notin \mathbb{R}$

, so the domain of the function  $h$  is not the set of real numbers.

### Remark

When we search if the function is a polynomial or not, we do not simplify its rule.

For example:

The function  $f_1 : f_1(x) = x\left(x + \frac{1}{x}\right)$  doesn't represent a polynomial function

because  $f_1(0) \notin \mathbb{R}$  while the function  $f_2 : f_2(x) = x^2 + 1$  represents a polynomial function.

And notice that:  $x\left(x + \frac{1}{x}\right) = x^2 + 1$  for all real numbers except 0

### The degree of the polynomial function

The degree of the polynomial function is the highest power of the variable in the function rule.

For example:

- The function  $f_1 : f_1(x) = 3x - \frac{1}{2}$  is of the first degree (a linear function)
- The function  $f_2 : f_2(x) = \sqrt{5}x^2 - 3x + 4$  is of the second degree (quadratic function)
- The function  $f_3 : f_3(x) = x^3 - 5x^2 + 4$  is of the third degree (cubic function)

### Remark

The function  $f : f(x) = a$  where  $a \in \mathbb{R} - \{0\}$

is a polynomial function of zero degree. (a constant function) as  $f(x) = 3$

In the case of  $a = 0$  i.e. when  $f(x) = 0$ , then the function has no degree.

**Example 3** If  $f : \mathbb{R} \longrightarrow \mathbb{R}$ , mention the degree of  $f$  in each of the following :

1  $f(x) = 5 - 3x$

3  $f(x) = 5x - 3x^2 + x^3$

5  $f(x) = x(x^3 + x^2) - x^2(x^2 - 2x)$

2  $f(x) = 3x - x^2$

4  $f(x) = x^2(2 + x)^2$

### Solution

1  $f$  is of the first degree.

2  $f$  is of the second degree.

3  $f$  is of the third degree.

4  $\because f(x) = x^2(4 + 4x + x^2)$   
 $= 4x^2 + 4x^3 + x^4$

$\therefore f$  is a function of the fourth degree.

5  $\because f(x) = x^4 + x^3 - x^4 + 2x^3 = 3x^3$   
 $\therefore f$  is a function of the third degree.

### Notice that :

When you want to determine the degree of the function you should simplify its rule to the simplest form before telling its degree.

**TRY 2**  
by yourself

Which of the functions defined by the following rules represents a polynomial function, then determine its degree if it is a polynomial:

①  $f_1(x) = x(x^2 - 3)$

②  $f_2(x) = x\left(\frac{2}{x} + 5\right)$

③  $f_3(x) = x^2 - \sqrt{x} + 1$

④  $f_4(x) = x^2 - (x^2 - 4)$

**Example 4** If  $f : f(x) = x^2 - 2x + 5$

1 Find :  $f(1)$ ,  $f(0)$ ,  $f(-2)$ ,  $f\left(\frac{1}{2}\right)$  and  $f(\sqrt{5})$

2 Prove that :  $f(2\sqrt{2} + 1) = 2f(1 - \sqrt{2})$

**Solution**

1  $f(1) = (1)^2 - 2 \times (1) + 5 = 1 - 2 + 5 = 4$

Similarly,  $f(0) = 5$ ,  $f(-2) = 13$

,  $f\left(\frac{1}{2}\right) = 4\frac{1}{4}$  and  $f(\sqrt{5}) = 10 - 2\sqrt{5}$

2  $\therefore f(2\sqrt{2} + 1) = (2\sqrt{2} + 1)^2 - 2(2\sqrt{2} + 1) + 5$   
 $= 8 + 1 + 4\sqrt{2} - 4\sqrt{2} - 2 + 5 = 12$  (1)

,  $f(1 - \sqrt{2}) = (1 - \sqrt{2})^2 - 2(1 - \sqrt{2}) + 5$   
 $= 1 + 2 - 2\sqrt{2} - 2 + 2\sqrt{2} + 5 = 6$  (2)

From (1) and (2) :  $\therefore f(2\sqrt{2} + 1) = 2f(1 - \sqrt{2})$

**Example 5** If  $f(x) = 2x + b$  and  $g(x) = x^2 + b$  and if  $f(2) + g(-4) = 30$ , then find :  $f(-2) - g(2)$

**Solution**

$\therefore f(2) = 2 \times 2 + b = 4 + b$ ,  $g(-4) = (-4)^2 + b = 16 + b$

,  $\therefore f(2) + g(-4) = 30$

$\therefore 4 + b + 16 + b = 30$

$$\therefore 20 + 2b = 30$$

$$\therefore 2b = 30 - 20 = 10$$

$$\therefore b = \frac{10}{2} = 5$$

$$\therefore f(x) = 2x + 5, \quad g(x) = x^2 + 5$$

$$\therefore f(-2) = 2 \times (-2) + 5 = 1, \quad g(2) = 2^2 + 5 = 9$$

$$\therefore f(-2) - g(2) = 1 - 9 = -8$$



If  $f(x) = 2x + 5$  and  $g(x) = x - 6$ , then prove that:  $f(2) + 3g(3) = 0$

- 1 R = {(2, 1), (4, 2), (6, 3), (8, 4)}, represent by yourself.
- 2 The range of the function  $f = \{1, 2, 3, 4\}$   
The polynomial functions are:
- 3  $f_1$  is of the third degree,  $f_4$  is of the zero degree.  
Prove by yourself.

Answers of try by yourself

## LESSON

# 4

## The study of some polynomial functions

### First

### The linear function

#### Definition

The function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = ax + b$ ,  $a \in \mathbb{R} - \{0\}$ ,  $b \in \mathbb{R}$  is called a linear function (it is a polynomial function of the first degree).

#### Examples of linear function :

- $f : \mathbb{R} \longrightarrow \mathbb{R}$ ,  $f(x) = x - 1$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$ ,  $f(x) = 2x + 1$
- $f : \mathbb{R} \longrightarrow \mathbb{R}$ ,  $f(x) = 3x$

#### Notice that :

In each of the shown functions , the index of  $x$  is 1 , therefore each of them is a function of the first degree.

### The graphical representation of the linear function

- The linear function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = ax + b$ ,  $a \in \mathbb{R} - \{0\}$ ,  $b \in \mathbb{R}$  is represented graphically by a straight line intersecting :
  - The  $y$ -axis at the point  $(0, b)$
  - The  $x$ -axis at the point  $(-\frac{b}{a}, 0)$
- To represent a linear function , it is enough to find two ordered pairs belonging to the function.
- You can find a third ordered pair to check that the three points are on the same straight line.

#### Example 1 Graph each of the following linear functions :

1  $f : f(x) = 2x - 3$

2  $r : r(x) = -\frac{1}{2}x$

**Solution**

- 1 Determine 3 ordered pairs belonging to the function.

$$\because f(x) = 2x - 3$$

$$\therefore f(-1) = 2(-1) - 3 = -5$$

$$\therefore f(1) = 2 \times 1 - 3 = -1$$

$$\text{and } f(2) = 2 \times 2 - 3 = 1$$

$$\therefore (-1, -5) \in f$$

$$\therefore (1, -1) \in f$$

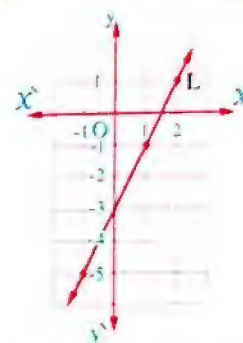
$$\therefore (2, 1) \in f$$

You can arrange these ordered pairs in the opposite table :

$x$	-1	1	2
$y = f(x)$	-5	-1	1

Locate these three points which represents the three ordered pairs in the Cartesian plane and draw the straight line L which passes through any two points of them. Then check that the third point lies on the same straight line.

Then this straight line is the graphical representation of this function.

**Notice that :**

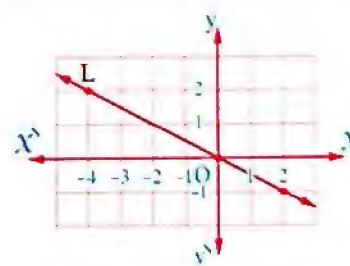
- The point of intersection with y-axis =  $(0, b) = (0, -3)$
- The point of intersection with x-axis =  $(-\frac{b}{a}, 0) = (\frac{3}{2}, 0)$

2  $\because r(x) = -\frac{1}{2}x$

$$\therefore$$

$x$	0	2	-4
$y = r(x)$	0	-1	2

From the opposite graph notice that , the straight line L passes through the origin point O  $(0, 0)$

**Generally**

The function  $f: \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = ax$ ,  $a \in \mathbb{R}^*$  is represented graphically by a straight line passing through the origin point  $(0, 0)$

**TRY 1**  
by yourself

Represent graphically each of the following linear functions :

1  $f: f(x) = 3x - 3$

2  $f: f(x) = 2x$

## Second The constant function

### Definition

The function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = b$ ,  $b \in \mathbb{R}$  is called a constant function.

For example:

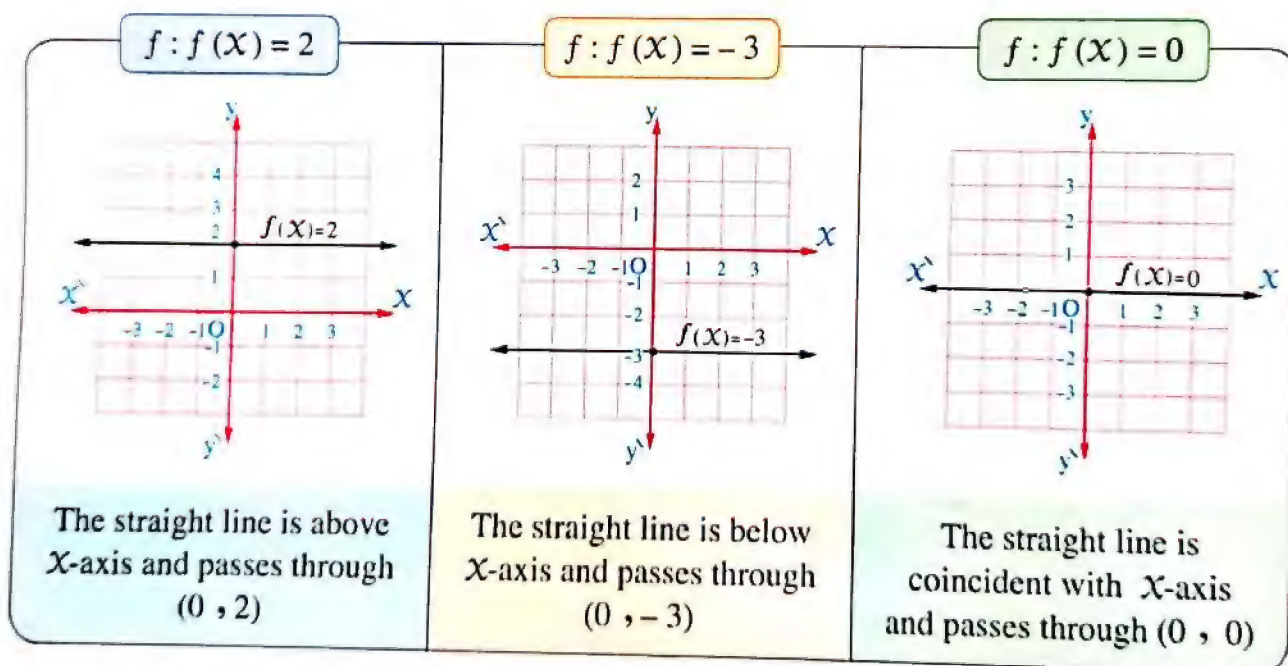
$f : f(x) = 5$  is a constant function where  $f(1) = 5$ ,  $f(0) = 5$ ,  $f(-2) = 5$ , ... and so on.

### Graphical representation of the constant function

The constant function  $f : f(x) = b$  (where  $b \in \mathbb{R}$ ) is represented by a straight line parallel to  $X$ -axis and passes through the point  $(0, b)$  this line is :

- **above**  $X$ -axis if  $b > 0$
- **below**  $X$ -axis if  $b < 0$
- **coincident** with  $X$ -axis if  $b = 0$

The following examples illustrate that :



### TRY 2 yourself

Represent graphically  $f : f(x) = -1$ , then find the following :

- 1 The degree of the function  $f$
- 2  $f(5)$
- 3  $f(2) + f(-2)$
- 4  $f(-x)$

### Third The quadratic function

#### Definition

The function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(X) = aX^2 + bX + c$  where  $a, b$  and  $c$  are real numbers,  $a \neq 0$  is called a quadratic function (it is a polynomial function of the second degree).

#### Examples of quadratic functions :

- $f : \mathbb{R} \longrightarrow \mathbb{R}, f(X) = X^2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}, f(X) = X^2 - 2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}, f(X) = 3X^2 - 7X + 2$
- $f : \mathbb{R} \longrightarrow \mathbb{R}, f(X) = 6 - X^2 + X$

#### Notice that :

In each of the shown functions, the highest index of  $X$  is 2 therefore each of them is a function of the 2<sup>nd</sup> degree.

### The graphical representation of the quadratic function

We know that the domain of the quadratic function is the set of real numbers  $\mathbb{R}$  which is an infinite set. So, to represent this function graphically, we should represent it on a certain interval by determining some of ordered pairs which belong to the function. Then we draw the curve (paved curve) passing through the points which represent these ordered pairs.

The following examples illustrate that :

#### Example 2 Graph each of the following quadratic functions :

- 1  $f : f(X) = X^2$  taking  $X \in [-3, 3]$
- 2  $f : f(X) = -X^2$  taking  $X \in [-3, 3]$

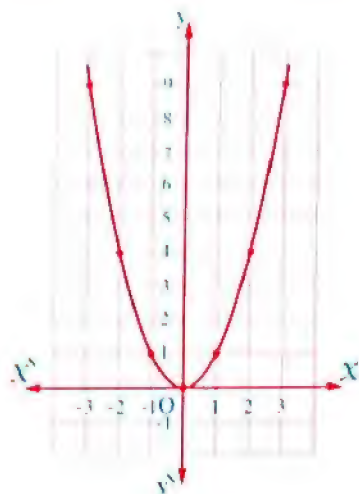
#### Solution

1  $f(X) = X^2$

$X$	-3	-2	-1	0	1	2	3
$f(X)$	9	4	1	0	1	4	9

2  $f(X) = -X^2$

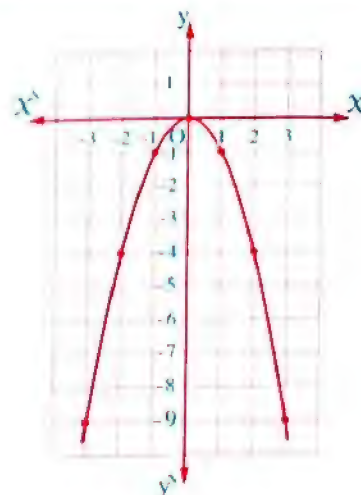
$X$	-3	-2	-1	0	1	2	3
$f(X)$	-9	-4	-1	0	-1	-4	-9



Notice that :

The coefficient of  $X^2 > 0$

- The point (0 , 0) is the point of the vertex of the curve , it is considered as a **minimum value** point of the curve because the whole curve **lies up on it**.
- The **minimum** value of the function is zero it equals the y-coordinate of the vertex of the curve.
- The curve is symmetric with respect to y-axis  
**i.e.** the y-axis is the line of symmetry of the curve and its equation is  $X = 0$



Notice that :

The coefficient of  $X^2 < 0$

- The point (0 , 0) is the point of the vertex of the curve , it is considered as a **maximum value** point of the curve because the whole curve **lies below it**.
- The **maximum** value of the function is zero it equals the y-coordinate of the vertex of the curve.
- The curve is symmetric with respect to y-axis  
**i.e.** the y-axis is the line of symmetry of the curve and its equation is  $X = 0$

### Generally

The quadratic function  $f : f(X) = aX^2 + bX + c$  where  $a$  ,  $b$  and  $c$  are real numbers ,  $a \neq 0$  has the following properties :

- 1 The vertex of the curve =  $\left( \frac{-b}{2a} , f\left(\frac{-b}{2a}\right) \right)$
- 2 If  $a$  (the coefficient of  $X^2$ ) is positive , then the curve is open upwards and the function has a minimum value equals  $f\left(\frac{-b}{2a}\right)$

- 3 If  $a$  (the coefficient of  $x^2$ ) is negative, then the curve is open downwards and the function has a maximum value equals  $f\left(\frac{-b}{2a}\right)$
- 4 The curve of the function is symmetric about the vertical line which passes through the vertex of the curve and the equation of that line :  $x = \frac{-b}{2a}$  and it is called the axis of symmetry of the curve.

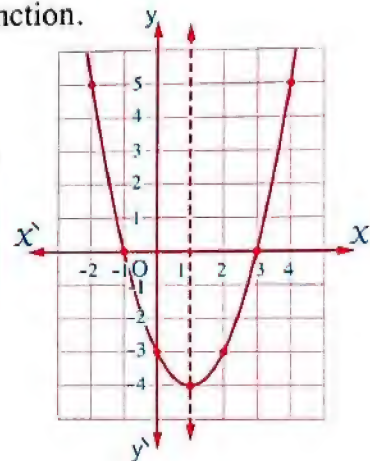
**Example 3** Graph the function  $f : f(x) = x^2 - 2x - 3$  taking  $x \in [-2, 4]$  from the graph, find :

- 1 The point of the vertex of the curve.
- 2 The equation of the line of symmetry.
- 3 The maximum or minimum value of the function.

**Solution**

$$f(x) = x^2 - 2x - 3$$

$x$	-2	-1	0	1	2	3	4
$f(x)$	5	0	-3	-4	-3	0	5




From the graph, we deduce that :


- 1 The vertex of the curve is  $(1, -4)$
- 2 The equation of the line of symmetry is  $x = 1$ , it is a straight line parallel to  $y$ -axis and passes through the vertex of the curve.
- 3 The minimum value of the function  $= -4$

### Remarks

We can form the table used in graphing the function  $f : f(x) = x^2 - 2x - 3$  where  $x \in [-2, 4]$  by using the scientific calculator which supports (Table) as follows :




- 1 Turn the calculator on (Table) as follows : Press **MODE** **SETUP**, then choose **TABLE**
- 2 Input data : Write the rule of the previous function, press successively the following buttons : **Start** **ALPHA** **1** **x<sup>2</sup>** **-** **2** **ALPHA** **1** **-** **3**
- 3 Press the button **=**, then at the beginning of the interval **START?** write **(-)** **2**, then press **=**

④ At the end of the interval END? write the number **4** , then press 

⑤ To determine the length of the interval STEP? write **1** , then press 

The table is formed in the display, you can move by using

button  up or down.

• To exit the program, press successively the buttons :    **1**

	X	F(X)
1	-2	5
2	-1	0
3	0	-3
4	1	-4
5	2	-3
6	3	0
7	4	5

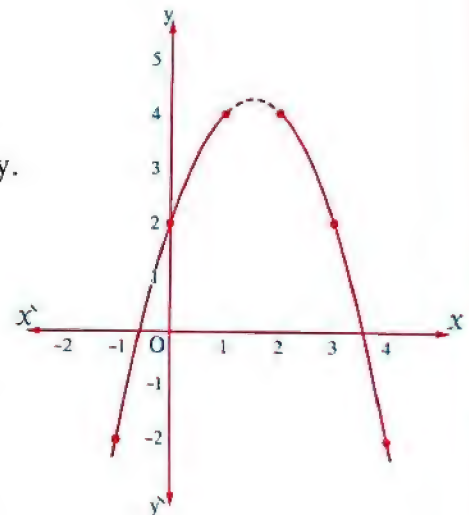
**Example 4** Graph the function  $f : f(x) = -x^2 + 3x + 2$  taking  $x \in [-1, 4]$ , then find :

- 1 The maximum value or minimum value of the function.
- 2 The equation of the line of symmetry.

**Solution**

x	-1	0	1	2	3	4
f(x)	-2	2	4	4	2	-2

When we represent these ordered pairs, we notice that the point of the vertex of the curve is not among these points which makes the drawing of the dotted part in the opposite figure is inaccurate, so the studying of the curve will be difficult, then we should find the vertex point of the curve algebraically as the following :



At the point of the vertex of the curve of the quadratic function, it will be :

- The X-coordinate  $= \frac{-b}{2a}$
  - The y-coordinate  $= f\left(\frac{-b}{2a}\right)$
- where b is the coefficient of  $x$ , a is the coefficient of  $x^2$

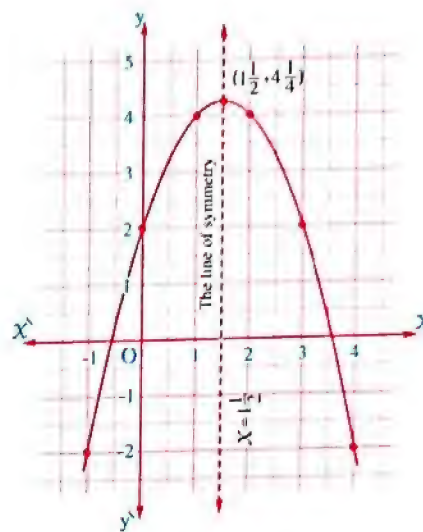
$$\therefore X \text{ at the vertex of the curve} = \frac{-3}{2 \times -1} = \frac{-3}{-2} = 1 \frac{1}{2}$$

$$\therefore f\left(1 \frac{1}{2}\right) = -\frac{9}{4} + \frac{9}{2} + 2 = 4 \frac{1}{4}$$

$$\therefore \text{The vertex of the curve is } \left(1 \frac{1}{2}, 4 \frac{1}{4}\right)$$

From the vertex of the curve ,  
we find that :

- 1 The maximum value =  $4 \frac{1}{4}$
- 2 The equation of the line of symmetry is  $X = 1 \frac{1}{2}$



### TRY 3

Graph the curve of the function  $f : f(X) = X^2 + 2X - 3$  on the interval  $[-4, 2]$  :  
From the graph, find:

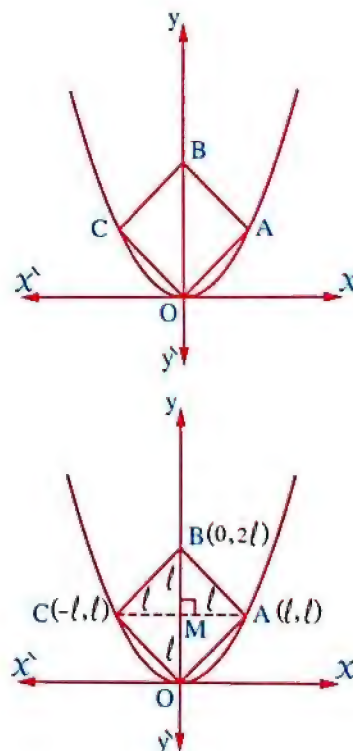
- 1 The maximum or minimum value of the function.
- 2 The equation of the line of symmetry.

### Example 5 In the opposite figure :

ABCO is a square and the curve represents  
the function  $f : f(X) = X^2$   
Find the coordinates of the points :  
A , B and C

#### Solution

Draw the square diagonal  $\overline{AC}$  to intersect  
the another diagonal  $\overline{BO}$  at the point M  
 $\therefore$  The two diagonals of the square are equal  
in length and bisect each other.  
 $\therefore MA = MB = MC = MO$  and let :  $MA = l$   
 $\therefore MA = MB = MC = MO = l$   
 $\therefore A(l, l), C(-l, l), B(0, 2l)$   
 $\therefore A(l, l) \in R$  where  $f(X) = X^2$

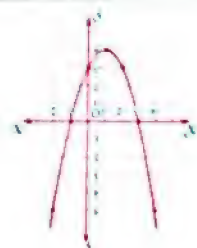


By substituting in the rule of the function

$$\therefore l = l^2 \qquad \qquad \qquad \therefore l^2 - l = 0 \qquad \qquad \therefore l(l - 1) = 0$$

$$\therefore l = 0 \text{ (refused)} \qquad \qquad \text{or } l - 1 = 0 \qquad \therefore l = 1$$

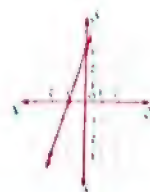
$\therefore A(1, 1)$ ,  $B(0, 2)$  and  $C(-1, 1)$



3



2



1 1

Answers of try by yourself

2  $x = -1$

1 The minimum value of the function = -4

3 -2

4 -1

1 The zero degree

2 -1



2

# UNIT 2

## Ratio, proportion, direct variation and inverse variation



### ▶ Lessons of the unit :

1. Ratio and proportion.
2. Follow properties of proportion.
3. Continued proportion.
4. Direct variation and inverse variation.

▶ Use your smart phone or tablet to scan the QR Code and enjoy watching videos.



## ► Unit Objectives :

**By the end of this unit, student should be able to :**

- recognize the concept of the ratio.
- recognize the properties of the ratio.
- recognize the concept of the proportion.
- recognize the properties of the proportion.
- recognize the concept of the continued proportion.
- use the properties of the ratio and the proportion for solving a lot of problems.
- recognize the concept of the direct variation.
- recognize the concept of the inverse variation.
- solve real life problems on the direct variation and the inverse variation.
- appreciate the role of mathematics in solving a lot of real life problems.

## LESSON

# 1

## Ratio and proportion

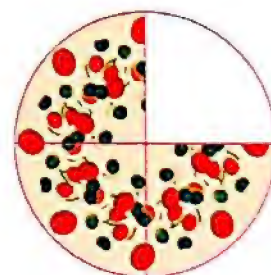
### First Ratio

We have studied in the primary stage that the ratio is one of methods of comparison between two quantities.

For example:

If a pie is divided into four equal parts and Hany ate one part only of it , then :

- The ratio of what Hany ate to the whole pie is 1 : 4  
and it is written as  $\frac{1}{4}$
- The ratio of what was left of the pie to the whole pie is 3 : 4  
and it is written as  $\frac{3}{4}$
- The ratio of what Hany ate to which was left of the pie is 1 : 3  
and it is written as  $\frac{1}{3}$



### Generally

If a and b are two real numbers , then :

The ratio between a and b is written  $a : b$  or  $\frac{a}{b}$

and is read a to b where :

a is called the antecedent of the ratio , b is called the consequent of the ratio , a and b are called together the two terms of the ratio.

## Properties of the ratio



### Property 1

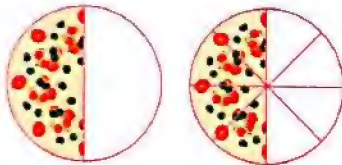
The value of the ratio does not change if each of its terms is multiplied or divided by the same non-zero real number.

$$a : b = ak : bk, k \in \mathbb{R}^*$$

For example:

$$\bullet 1 : 2 = 1 \times (4) : 2 \times (4)$$

$$\text{i.e. } 1 : 2 = 4 : 8$$



1 : 2

4 : 8

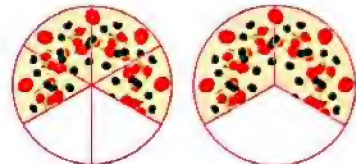
i.e.

$$a : b = \frac{a}{n} : \frac{b}{n}, n \in \mathbb{R}^*$$

For example:

$$\bullet 4 : 6 = \frac{4}{2} : \frac{6}{2}$$

$$\text{i.e. } 4 : 6 = 2 : 3$$



4 : 6

2 : 3

### Property 2

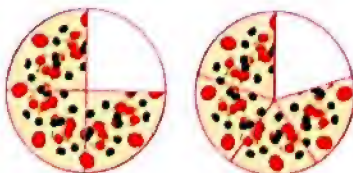
The value of the ratio ( $\neq 1$ ) changes if we add or subtract (to or from) each of its two terms a non-zero real number.

$$a : b \neq a + k : b + k, k \in \mathbb{R}^* \\ \text{where } a \neq b$$

For example:

$$\bullet 3 : 4 \neq 3 + (1) : 4 + (1)$$

$$\text{i.e. } 3 : 4 \neq 4 : 5$$



3 : 4

4 : 5

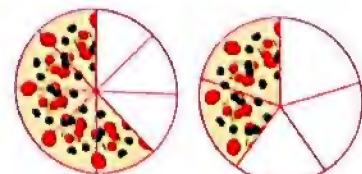
i.e.

$$a : b \neq a - k : b - k, k \in \mathbb{R}^* \\ \text{where } a \neq b$$

For example:

$$\bullet 5 : 8 \neq 5 - (3) : 8 - (3)$$

$$\text{i.e. } 5 : 8 \neq 2 : 5$$



5 : 8

2 : 5



## Second Proportion

The opposite table shows two sets of numbers.

If we look at these sets, we can notice that :

$$\frac{2}{8} = \frac{4}{16} = \frac{7}{28} = \frac{3}{12} = \frac{6}{24} \text{ each of them equals } \frac{1}{4}$$

The set (A)	2	4	7	3	6
The set (B)	8	16	28	12	24

In this case, we say that the numbers of set (A) are proportional to the corresponding numbers in the set (B)

The previous form which expresses the equality of two ratios or more is called proportion.

### Definition of proportion

It is the equality of two ratios or more.

i.e.

If  $\frac{a}{b} = \frac{c}{d}$ , then the quantities a, b, c and d are proportional.

And vice versa : If a, b, c and d are proportional, then :  $\frac{a}{b} = \frac{c}{d}$

- **a** is called the **first** proportional.
- **b** is called the **second** proportional.
- **c** is called the **third** proportional.
- **d** is called the **fourth** proportional.

**a** and **d** are called **extremes** and **b** and **c** are called **means**.

**For example:** The numbers 1, 4, 7 and 28 are proportional numbers, because  $\frac{1}{4} = \frac{7}{28}$

And : **1** is the first proportional, **4** is the second proportional, **7** is the third proportional, **28** is the fourth proportional, **1 and 28** are the extremes of this proportion and **4 and 7** are the means.

### Properties of proportion



#### Property 1

If  $\frac{a}{b} = \frac{c}{d}$ , then :  $a \times d = b \times c$  (the product of the extremes = the product of the means)

**The reason :** If we multiply each ratio by  $b \times d$ , we get :  $\frac{a}{b} \times b \times d = \frac{c}{d} \times b \times d$

i.e.  $a \times d = b \times c$

#### Example 1

- 1 Find the third proportion of the quantities : 3, 4, .... and 20
- 2 Find the fourth proportion of the quantities :  $18a^2b$ ,  $12ab^2$ ,  $21ab$  and ....

**Solution**

1 Let the third proportion be  $x$

$\therefore$  The quantities 3, 4,  $x$ , 20 are proportional.  $\therefore \frac{3}{4} = \frac{x}{20}$

From the previous property  $\therefore 3 \times 20 = 4x$

$\therefore 4x = 60 \quad \therefore x$  (The third proportion)  $= 15$

2 Let the fourth proportion be  $x$

$\therefore$  The quantities  $18a^2b$ ,  $12ab^2$ ,  $21ab$ ,  $x$  are proportional.

$\therefore \frac{18a^2b}{12ab^2} = \frac{21ab}{x} \quad \therefore \frac{3a}{2b} = \frac{21ab}{x} \quad \therefore 3a \times x = 21ab \times 2b$

$\therefore 3ax = 42ab^2 \quad \therefore x = \frac{42ab^2}{3a} \quad \therefore x = 14b^2$

$\therefore$  The fourth proportion  $= 14b^2$

**TRY 1**  
by yourself

If the quantities  $x$ , 23, 15 and 69 are proportional, **find the value of :  $x$**

**Example 2**

Find the number that will be added to each of the numbers : 1, 13, 7 and 31 to get proportional numbers.

**Solution**

Let the number be  $x$   $\therefore 1+x, 13+x, 7+x, 31+x$  are proportional.

$\therefore \frac{1+x}{13+x} = \frac{7+x}{31+x} \quad \therefore (x+1)(x+31) = (x+7)(x+13)$

$\therefore x^2 + 32x + 31 = x^2 + 20x + 91 \quad \therefore 32x - 20x = 91 - 31$

$\therefore 12x = 60 \quad \therefore x = 5 \quad \therefore$  The required number  $= 5$

**Example 3**

If  $(2x+5) : (3x-3) = 5 : 4$  Find the value of :  $x$

**Solution**

$\therefore \frac{2x+5}{3x-3} = \frac{5}{4}$

$\therefore 4(2x+5) = 5(3x-3)$

$\therefore 8x+20 = 15x-15$

$\therefore 20+15 = 15x-8x$

$\therefore 35 = 7x$

$\therefore x = \frac{35}{7} = 5$

**Example 4**

Find the number that if we add to the two terms of the ratio 17 : 22 the result will be 6 : 7

**Solution**

Let the required number be  $x$

$\therefore \frac{17+x}{22+x} = \frac{6}{7}$

$\therefore 7(17+x) = 6(22+x)$

$\therefore 119+7x = 132+6x$

$\therefore 7x-6x = 132-119$

$\therefore x$  (The required number)  $= 13$

**TRY 2**

by yourself

Find the real number that if we subtract from both terms of the ratio  $\frac{5}{6}$  it will become  $\frac{3}{2}$

**Property 2**

If  $a \times d = b \times c$ , then  $\frac{a}{b} = \frac{c}{d}$

The reason : If we divide each ratio by  $b d$ , we get :  $\frac{a \times d}{b d} = \frac{b \times c}{b d}$  i.e.  $\frac{a}{b} = \frac{c}{d}$

Also we can deduce that :

• If  $a \times d = b \times c$ , then  $\frac{a}{c} = \frac{b}{d}$

• If  $a \times d = b \times c$ , then  $\frac{b}{a} = \frac{d}{c}$

• If  $a \times d = b \times c$ , then  $\frac{c}{a} = \frac{d}{b}$

**Example 5** In each of the following, find  $\frac{x}{y}$  if :

1  $12 \times x = 3 \times y$

2  $\frac{1}{2} x = \frac{3}{4} y$

**Solution**

1  $\therefore 12 \times x = 3 \times y$

$\therefore \frac{x}{y} = \frac{3}{12} = \frac{1}{4}$

2  $\therefore \frac{1}{2} x = \frac{3}{4} y$

$\therefore \frac{x}{y} = \frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{3}{2}$

**Example 6** If  $4x - 3y : 2x + y = \frac{4}{7}$ , find in the simplest form the ratio  $x : y$

**Solution**

$\therefore \frac{4x - 3y}{2x + y} = \frac{4}{7}$

$\therefore 7(4x - 3y) = 4(2x + y)$

$\therefore 28x - 21y = 8x + 4y$

$\therefore 28x - 8x = 21y + 4y$

$\therefore 20x = 25y$

$\therefore \frac{x}{y} = \frac{25}{20}$

$\therefore \frac{x}{y} = \frac{5}{4}$

**Example 7** If  $2x^2 - 6y^2 = xy$ , find :  $x : y$

**Solution**

$$\therefore 2x^2 - 6y^2 = xy$$

$$\therefore (2x + 3y)(x - 2y) = 0$$

$$\text{, then } 2x = -3y$$

$$\text{or } x - 2y = 0 \text{ , then } x = 2y$$

$$\text{i.e. } \frac{x}{y} = -\frac{3}{2} \text{ or } \frac{x}{y} = \frac{2}{1}$$

$$\therefore 2x^2 - xy - 6y^2 = 0$$

$$\therefore 2x + 3y = 0$$

$$\therefore \frac{x}{y} = \frac{-3}{2}$$

$$\therefore \frac{x}{y} = \frac{2}{1}$$

**TRY 3**

*by yourself*

1 If  $2a - 5b = 0$ , find :  $\frac{a}{b}$

2 If  $\frac{x+2y}{4x-3y} = \frac{7}{6}$ , then prove that :  $\frac{x}{y} = \frac{3}{2}$

3 If  $4a^2 - 9b^2 = 0$ , find  $a : b$

**Property 3**

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } \frac{a}{c} = \frac{b}{d}$$

$$\text{i.e. } \frac{\text{The antecedent of the first ratio}}{\text{The antecedent of the second ratio}} = \frac{\text{The consequent of the first ratio}}{\text{The consequent of the second ratio}}$$

**The reason :** If we multiply each ratio by  $\frac{b}{c}$ , we get :  $\frac{a}{b} \times \frac{b}{c} = \frac{c}{d} \times \frac{b}{c}$

$$\text{i.e. } \frac{a}{c} = \frac{b}{d}$$

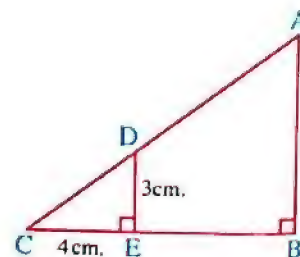
**For example:** If  $\frac{a}{4} = \frac{b}{3}$ , then  $\frac{a}{b} = \frac{4}{3}$  and  $\frac{b}{a} = \frac{3}{4}$

**Example 8** In the opposite figure :

ABC is a right-angled triangle at B in which :

$D \in \overline{AC}$ ,  $E \in \overline{BC}$  where  $\overline{DE} \perp \overline{BC}$

,  $DE = 3$  cm. and  $EC = 4$  cm. Find  $AB : BC$



**Solution**

In  $\triangle ABC$ ,  $\triangle DEC$  :  $m(\angle B) = m(\angle DEC) = 90^\circ$ ,  $\angle C$  is a common angle  
 $\therefore m(\angle A) = m(\angle EDC)$

$$\therefore \triangle ABC \sim \triangle DEC, \text{ then we deduce that : } \frac{AB}{DE} = \frac{BC}{EC}$$

$$\therefore \frac{AB}{3} = \frac{BC}{4}$$

$$\therefore \frac{AB}{BC} = \frac{3}{4} \quad (\text{Q.E.D.})$$

**Property 4**

If  $\frac{a}{b} = \frac{c}{d}$ , then  $a = cm$  and  $b = dm$  (where  $m$  is a constant  $\neq 0$ )

For example: If  $\frac{a}{b} = \frac{3}{4}$ , then  $a = 3m$ ,  $b = 4m$  (where  $m$  is a constant  $\neq 0$ )

**Example 9** If  $a : b = 3 : 5$ , find the ratio  $20a - 7b : 15a + b$

**Solution**

$$\therefore \frac{a}{b} = \frac{3}{5} \quad \therefore a = 3m, \quad b = 5m \quad (\text{where } m \neq 0)$$

Substituting by  $a$  and  $b$  in terms of  $m$ :

$$\therefore \frac{20a - 7b}{15a + b} = \frac{60m - 35m}{45m + 5m} = \frac{25m}{50m} = \frac{1}{2}$$

**Another solution :**

By dividing the terms of the ratio  $\frac{20a - 7b}{15a + b}$  by  $b$

, then substituting by the value  $\frac{a}{b} = \frac{3}{5}$

$$\therefore \frac{20a - 7b}{15a + b} = \frac{20\left(\frac{a}{b}\right) - 7}{15\left(\frac{a}{b}\right) + 1} = \frac{20 \times \frac{3}{5} - 7}{15 \times \frac{3}{5} + 1} = \frac{12 - 7}{9 + 1} = \frac{5}{10} = \frac{1}{2}$$

**Example 10** If  $\frac{a}{b} = \frac{2}{3}$  and  $\frac{x}{y} = \frac{3}{5}$ , prove that :

$(7a x + 4b y), (11a y + b x), 12, 14$  are proportional quantities.

**Solution**

$$\therefore \frac{a}{b} = \frac{2}{3} \quad \therefore a = 2m, \quad b = 3m \quad (\text{where } m \neq 0)$$

$$\therefore \frac{x}{y} = \frac{3}{5} \quad \therefore x = 3k, \quad y = 5k \quad (\text{where } k \neq 0)$$

[Notice that : We used two different constants  $m$  and  $k$ ]

Substituting by  $a, b, x$  and  $y$

$$\begin{aligned} \therefore \frac{7ax + 4by}{11ay + bx} &= \frac{7 \times 2m \times 3k + 4 \times 3m \times 5k}{11 \times 2m \times 5k + 3m \times 3k} \\ &= \frac{42mk + 60mk}{110mk + 9mk} = \frac{102mk}{119mk} = \frac{6}{7} \end{aligned}$$

$$\therefore \frac{12}{14} = \frac{6}{7}$$

$\therefore (7ax + 4by), (11ay + bx), 12$  and  $14$  are proportional quantities.

**TRY 4**

by yourself

If  $\frac{x}{y} = \frac{2}{5}$ , **prove that**  $(2x + y)$ ,  $(x + 2y)$ , 12 and 16 are proportional quantities.

**Example 11**

The ratio between two real numbers is 4 : 7

If we subtract 16 from each of them, then the ratio between the two obtained numbers is 2 : 5 Find the two numbers.

**Solution**

Let the two numbers be  $a$  and  $b$

$$\therefore \frac{a}{b} = \frac{4}{7}$$

$$\therefore a = 4m, b = 7m \text{ (where } m \neq 0)$$

$$\therefore \frac{4m - 16}{7m - 16} = \frac{2}{5}$$

$$\therefore 14m - 32 = 20m - 80$$

$$\therefore 80 - 32 = 20m - 14m$$

$$\therefore 48 = 6m$$

$$\therefore m = \frac{48}{6} = 8$$

$$\therefore a = 4 \times 8 = 32, b = 7 \times 8 = 56 \quad \text{i.e. The two numbers are 32 and 56}$$

**TRY 5**

by yourself

The ratio between two integers is 2 : 5 If 2 is subtracted from the first number and 1 is added to the second, then the ratio becomes 1 : 4 Find the two numbers.

**3** 1  $\frac{2}{5}$ **2** Prove by yourself.**3**  $\frac{2}{3} \neq \frac{2}{3}$ **5** 6, 15**4** Prove by yourself.**2** 8**1** 5**Answers** of try by yourself

## LESSON

# 2

## Follow properties of proportion

In this lesson , we will study the property (5) from properties of proportion , before studying this property , we will study an important remark in proportion to help us solving problems.

### Important remark

If  $a, b, c$  and  $d$  are proportional quantities and we assume that :  $\frac{a}{b} = \frac{c}{d} = m$  , then

$$\textcircled{a} = bm \quad , \quad \textcircled{c} = dm$$

For example:

$$\text{If } \frac{a}{b} = \frac{c}{d} = \frac{3}{4} \text{ , then } a = \frac{3}{4} b \quad , \quad c = \frac{3}{4} d$$

### Generally

If  $a, b, c, d, e, f, \dots$  are proportional quantities and we assume that :

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = m \text{ , then } \textcircled{a} = bm \quad , \quad \textcircled{c} = dm \quad , \quad \textcircled{e} = fm \text{ , } \dots$$

**Example 1** If  $a, b, c$  and  $d$  are proportional quantities , prove that :

$$1 \quad \frac{2a + 3c}{7a - 5c} = \frac{2b + 3d}{7b - 5d}$$

$$2 \quad \frac{a + c}{b + d} = \frac{a^2 + c^2}{ab + cd}$$

### Solution

$$1 \quad \text{Let } \frac{a}{b} = \frac{c}{d} = m \quad \therefore \textcircled{a} = bm \quad , \quad \textcircled{c} = dm$$

$$\text{L.H.S.} = \frac{2bm + 3dm}{7bm - 5dm} = \frac{m(2b + 3d)}{m(7b - 5d)} = \frac{2b + 3d}{7b - 5d} = \text{R.H.S.}$$

2 Let  $\frac{a}{b} = \frac{c}{d} = m \quad \therefore \textcircled{a} = bm, \textcircled{c} = dm$

$$\therefore \frac{a+c}{b+d} = \frac{bm+dm}{b+d} = \frac{m(b+d)}{(b+d)} = m \quad (1)$$

$$\therefore \frac{a^2+c^2}{ab+cd} = \frac{(bm)^2+(dm)^2}{bm \times b + dm \times d} = \frac{b^2 m^2 + d^2 m^2}{b^2 m + d^2 m} = \frac{m^2(b^2+d^2)}{m(b^2+d^2)} = m \quad (2)$$

From (1) and (2) we deduce that :  $\frac{a+c}{b+d} = \frac{a^2+c^2}{ab+cd}$

**Example 2** If  $a, b, c, d, e$  and  $f$  are positive proportional quantities,

prove that :  $\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \frac{a}{b}$

**Solution**

Let  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = m \quad \therefore \textcircled{a} = bm, \textcircled{c} = dm, \textcircled{e} = fm$

$$\begin{aligned} \therefore \sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} &= \sqrt{\frac{(bm)^2+(dm)^2+(fm)^2}{b^2+d^2+f^2}} = \sqrt{\frac{b^2 m^2 + d^2 m^2 + f^2 m^2}{b^2+d^2+f^2}} \\ &= \sqrt{\frac{m^2(b^2+d^2+f^2)}{(b^2+d^2+f^2)}} = \sqrt{m^2} = m \end{aligned} \quad (1)$$

$$\therefore \frac{a}{b} = m \quad (2)$$

From (1) and (2) we deduce that :  $\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}} = \frac{a}{b}$

**TRY 1**  
by yourself

If  $\frac{a}{b} = \frac{c}{d}$ , prove that :  $\frac{5a-2c}{5b-2d} = \frac{4a+3c}{4b+3d}$

**Property 5**

If we consider the proportion :  $\frac{9}{15} = \frac{6}{10} = \frac{3}{5}$

- If we add the antecedents and consequents of the 1<sup>st</sup> and the 2<sup>nd</sup> ratios, we get the ratio

$$\frac{9+6}{15+10} = \frac{15}{25} = \frac{3}{5} \text{ which is one of the given ratios.}$$

- Also if we add the antecedents and consequents of the 2<sup>nd</sup> and the 3<sup>rd</sup> ratios, we get the ratio  $\frac{6+3}{10+5} = \frac{9}{15} = \frac{3}{5}$  = one of the given ratios.

- If we add the antecedents and consequents of the three given ratios, we get the ratio

$$\frac{9+6+3}{15+10+5} = \frac{18}{30} = \frac{3}{5} = \text{one of the given ratios.}$$

- Since the ratio does not change if we multiply its two terms by a non-zero real number , then if we multiply the two terms of the first ratio by any number as 2 and multiply the two terms of the second ratio by any other number as  $(-4)$  , then the previous proportion stays true.

i.e.  $\frac{18}{30} = \frac{-24}{-40} = \frac{3}{5}$

- If we add the antecedents and consequents of the first and the second ratios , we get

the ratio  $\frac{18 - 24}{30 - 40} = \frac{-6}{-10} = \frac{3}{5} =$  one of the given ratios.

- If we add the antecedents and consequents of the three ratios , we get the ratio

$\frac{18 - 24 + 3}{30 - 40 + 5} = \frac{-3}{-5} = \frac{3}{5} =$  one of the given ratios.

From the previous points , we can say that :

If we have some equal ratios , then we can obtain many other ratios , each of them equals any of the initial ratios. This will happen by adding the antecedents and consequents of all the ratios or some of them directly or after multiplying the two terms of each ratio by a non-zero real number.

i.e.

If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$  and  $m_1, m_2, m_3, \dots$  are non-zero real numbers

, then  $\frac{m_1 a + m_2 c + m_3 e + \dots}{m_1 b + m_2 d + m_3 f + \dots} =$  one of the given ratios.

**Remark :** The first problem in example (1) can be solved by using the previous property as follows :

$\therefore a, b, c$  and  $d$  are proportional quantities.

$\therefore \frac{a}{b} = \frac{c}{d}$  multiplying the two terms of the 1<sup>st</sup> ratio by 2 and the 2<sup>nd</sup> by 3

Then the sum of antecedents : The sum of consequents = one of the given ratios.

$\therefore \frac{2a + 3c}{2b + 3d} =$  one of the given ratios. (1)

Multiplying the two terms of the 1<sup>st</sup> ratio by 7 and the 2<sup>nd</sup> by  $(-5)$  then

the sum of antecedents : the sum of consequents = one of the given ratio

$\therefore \frac{7a - 5c}{7b - 5d} =$  one of the given ratios. (2)

From (1) and (2) :  $\therefore \frac{2a + 3c}{2b + 3d} = \frac{7a - 5c}{7b - 5d} \quad \therefore \frac{2a + 3c}{7a - 5c} = \frac{2b + 3d}{7b - 5d}$

**Example 3** If  $\frac{a}{4} = \frac{b}{5} = \frac{c}{3}$ ,

prove that :  $\frac{a-b+c}{a+b-c} = \frac{1}{3}$

**Solution**

Multiplying the two terms of the 2<sup>nd</sup> ratio by  $(-1)$ , then add the antecedents and the consequents of the three ratios :

$\therefore \frac{\text{The sum of antecedents}}{\text{The sum of consequents}} = \text{one of the given ratios.}$

$$\therefore \frac{a-b+c}{4-5+3} = \frac{a-b+c}{2} = \text{one of the given ratios.} \quad (1)$$

Multiplying the two terms of the 3<sup>rd</sup> ratio by  $(-1)$ , then add the antecedents and the consequents of the three ratios :

$$\therefore \frac{a+b-c}{4+5-3} = \frac{a+b-c}{6} = \text{one of the given ratios.} \quad (2)$$

From (1) and (2) :  $\therefore \frac{a-b+c}{2} = \frac{a+b-c}{6}$

$$\therefore \frac{a-b+c}{a+b-c} = \frac{2}{6} = \frac{1}{3}$$

**Example 4** If  $\frac{a+b}{11} = \frac{b+c}{9} = \frac{c+a}{4}$ ,

prove that :  $\frac{a+b+c}{5a+4b+3c} = \frac{6}{25}$

**Solution**

Adding the antecedents and consequents of the three ratios.

$\therefore \frac{\text{The sum of antecedents}}{\text{The sum of consequents}} = \text{one of the given ratios.}$

$$\therefore \frac{2a+2b+2c}{24} = \text{one of the given ratios.}$$

$$\therefore \frac{a+b+c}{12} = \text{one of the given ratios.} \quad (1)$$

Multiplying the two terms of 1<sup>st</sup> ratio by 3 and the third by 2 and adding the antecedents and consequents of the three ratios

$\therefore \frac{\text{The sum of antecedents}}{\text{The sum of consequents}} = \text{one of the given ratios.}$

$$\therefore \frac{3a+3b+b+c+2c+2a}{33+9+8} = \text{one of the given ratios.}$$

$$\therefore \frac{5a+4b+3c}{50} = \text{one of the given ratios.} \quad (2)$$

From (1) and (2) :

$$\therefore \frac{a+b+c}{12} = \frac{5a+4b+3c}{50} \quad \therefore \frac{a+b+c}{5a+4b+3c} = \frac{12}{50} = \frac{6}{25}$$

**Example 5** If  $\frac{a+4b}{x+2y} = \frac{4b+7c}{2y+5z} = \frac{7c+a}{5z+x}$ ,

prove that :  $\frac{a}{2b} = \frac{x}{y}$

**Solution**

Multiplying the two terms of the 2<sup>nd</sup> ratio by  $(-1)$ , then add the antecedents and the consequents of the three ratios :

$$\therefore \frac{a+4b-4b-7c+7c+a}{x+2y-2y-5z+5z+x} = \frac{2a}{2x} = \frac{a}{x} = \text{one of the given ratios.} \quad (1)$$

Multiplying the two terms of the 3<sup>rd</sup> ratio by  $(-1)$ , then add the antecedents and the consequents of the three ratios :

$$\therefore \frac{a+4b+4b+7c-7c-a}{x+2y+2y+5z-5z-x} = \frac{8b}{4y} = \frac{2b}{y} = \text{one of the given ratios.} \quad (2)$$

$$\text{From (1) and (2) : } \therefore \frac{a}{x} = \frac{2b}{y} \qquad \therefore \frac{a}{2b} = \frac{x}{y}$$

**TRY 2**  
by yourself

If  $\frac{x}{a-2b} = \frac{y}{b-2c} = \frac{z}{c-2a}$ ,

prove that :  $\frac{x+2y-z}{3a-5c} = \frac{y+2z}{b-4a}$

- 1** Prove by yourself [Hint :  $a = bm, c = dm$ ]  
**2** Prove by yourself [Hint : \* Multiplying the terms of 2<sup>nd</sup> ratio by 2, the 3<sup>rd</sup> ratio by  $(-1)$  and adding the antecedents and consequents of the three ratios  
 \* Hence multiplying the terms of 3<sup>rd</sup> ratio by 2 and adding the antecedents and consequents of 2<sup>nd</sup> and 3<sup>rd</sup> ratios.]

**Answers** / of try by yourself

## LESSON

# 3

## Continued proportion

### Definition



The quantities  $a$ ,  $b$  and  $c$  are said to be in continued proportion if  $\frac{a}{b} = \frac{b}{c}$  or  $b^2 = ac$

In this proportion,  $a$  is called the **first proportion**,  $c$  is called the **third proportion** and  $b$  is called the **middle proportion** (proportional mean).

For example:

The numbers 4, 6 and 9 form a continued proportion because:  $\frac{4}{6} = \frac{6}{9}$  or because:  $(6)^2 = 4 \times 9$  where 6 is the middle proportion, 4 is the first proportion and 9 is the third proportion.

### Notice that :

- 1 If  $a$ ,  $b$  and  $c$  are in continued proportion, then:  $b^2 = ac$  i.e.  $b = \pm\sqrt{ac}$  and the two quantities  $a$  and  $c$  should be either both positive or both negative.
- 2 For any two positive numbers or any two negative numbers  $x$  and  $y$ , there are two middle proportions ( $\sqrt{xy}$  and  $-\sqrt{xy}$ ).

**Example 1** Find the middle proportion (proportional mean) between each two quantities :

1 5, 20

2  $\ell^2$ ,  $m^2$

3  $2\sqrt{3}$ ,  $6\sqrt{3}$

### Solution

1 The middle proportion  $= \pm\sqrt{5 \times 20} = \pm\sqrt{100} = \pm 10$

2 The middle proportion  $= \pm\sqrt{\ell^2 m^2} = \pm \ell m$

3 The middle proportion  $= \pm\sqrt{2\sqrt{3} \times 6\sqrt{3}} = \pm\sqrt{36} = \pm 6$

**Example 2** Find the third proportion of each two quantities :

1 12 , 18

2  $-8x^2y^2, 4x^3y$

**Solution**

1 Let the third proportion be  $c$  :

$$\therefore \frac{12}{18} = \frac{18}{c}$$

$$\therefore c = \frac{18 \times 18}{12} = 27$$

2 Let the third proportion be  $c$  :

$$\therefore \frac{-8x^2y^2}{4x^3y} = \frac{4x^3y}{c}$$

$$\therefore c = \frac{4x^3y \times 4x^3y}{-8x^2y^2} = \frac{16x^6y^2}{-8x^2y^2} = -2x^4$$

**TRY 1**  
by yourself

1 Find the middle proportion between 32 and 18

2 Find the first proportion of 8 and 16

**Remark**

If  $a, b$  and  $c$  are in continued proportion and we assume that :  $\frac{a}{b} = \frac{b}{c} = m$

, then  $\frac{b}{c} = m$

$\therefore \textcircled{b} = cm$

(1)

,  $\therefore \frac{a}{b} = m$

$\therefore a = bm$

Substituting for  $b$  from (1) :  $\therefore a = (cm) m$

$\therefore \textcircled{a} = cm^2$

”

i.e.

If  $\frac{a}{b} = \frac{b}{c} = m$ , then  $\begin{cases} b = cm \\ a = cm^2 \end{cases}$

**Example 3** If  $a, b$  and  $c$  are in continued proportion ,

prove that :  $\frac{4a^2 - 3b^2}{4b^2 - 3c^2} = \frac{a}{c}$

**Solution**

Let  $\frac{a}{b} = \frac{b}{c} = m$

$\therefore \textcircled{b} = cm, \textcircled{a} = cm^2$

$$\begin{aligned} \therefore \frac{4a^2 - 3b^2}{4b^2 - 3c^2} &= \frac{4(cm^2)^2 - 3(cm)^2}{4(cm)^2 - 3c^2} = \frac{4c^2m^4 - 3c^2m^2}{4c^2m^2 - 3c^2} \\ &= \frac{c^2m^2(4m^2 - 3)}{c^2(4m^2 - 3)} = m^2 \end{aligned} \quad (1)$$

,  $\therefore \frac{a}{c} = \frac{cm^2}{c} = m^2 \quad (2)$

From (1) and (2) we deduce that :  $\frac{4a^2 - 3b^2}{4b^2 - 3c^2} = \frac{a}{c}$

**Another solution :**

$$\therefore \frac{a}{b} = \frac{b}{c}$$

$$\therefore b^2 = ac$$

$$\therefore \text{L.H.S.} = \frac{4a^2 - 3ac}{4ac - 3c^2} = \frac{a(4a - 3c)}{c(4a - 3c)} = \frac{a}{c} = \text{R.H.S.}$$

**Example 4** If  $b$  is a middle proportion between  $a$  and  $c$ , prove that :

$$1 \quad \frac{a-b}{a} = \frac{a-c}{a+b}$$

$$2 \quad ab - c^2 = (b-c)(a+b+c)$$

**Solution**

$\therefore b$  is the middle proportion between  $a$  and  $c$

$\therefore a, b$  and  $c$  are in continued proportion

$$\text{Let } \frac{a}{b} = \frac{b}{c} = m$$

$$\therefore b = cm, \quad a = cm^2$$

$$1 \quad \therefore \frac{a-b}{a} = \frac{cm^2 - cm}{cm^2} = \frac{cm(m-1)}{cm^2} = \frac{m-1}{m} \quad (1)$$

$$, \therefore \frac{a-c}{a+b} = \frac{cm^2 - c}{cm^2 + cm} = \frac{c(m^2 - 1)}{cm(m+1)} = \frac{c(m-1)(m+1)}{cm(m+1)} = \frac{m-1}{m} \quad (2)$$

From (1) and (2) we deduce that :

$$\frac{a-b}{a} = \frac{a-c}{a+b}$$

$$2 \quad \therefore ab - c^2 = cm^2 \times cm - c^2 = c^2 m^3 - c^2 = c^2 (m^3 - 1) \quad (1)$$

$$\begin{aligned} , \therefore (b-c)(a+b+c) &= (cm - c)(cm^2 + cm + c) \\ &= c(m-1) \times c(m^2 + m + 1) \\ &= c^2(m-1)(m^2 + m + 1) = c^2(m^3 - 1) \quad (2) \end{aligned}$$

From (1) and (2) we deduce that :

$$ab - c^2 = (b-c)(a+b+c)$$

**TRY 2**  
 by yourself

If  $a, b$  and  $c$  are in continued proportion, prove that :  $\frac{3c^2 - 4b^2}{3b^2 - 4a^2} = \frac{c^2}{b^2}$

### Generalizing the definition of the continued proportion

The quantities  $a, b, c, d, \dots$  are in continued proportion if:  $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \dots$

For example:

The numbers 16, 24, 36 and 54 are in continued proportion

because:  $\frac{16}{24} = \frac{24}{36} = \frac{36}{54}$  (each ratio =  $\frac{2}{3}$ )

#### Remark

If  $a, b, c$  and  $d$  are in continued proportion and we assume that:  $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$ , then:

$$\frac{c}{d} = m \quad \therefore \textcircled{c} = dm \quad (1)$$

$$\frac{b}{c} = m \quad \therefore b = cm$$

$$\text{Substituting for } c \text{ from (1): } \therefore b = (dm) m \quad \therefore \textcircled{b} = dm^2 \quad (2)$$

$$\frac{a}{b} = m \quad \therefore a = bm$$

$$\text{Substituting for } b \text{ from (2): } \therefore a = (dm^2) m \quad \therefore \textcircled{a} = dm^3$$

i.e.

If  $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$ , then  $\boxed{c = dm}$ ,  $\boxed{b = dm^2}$  and  $\boxed{a = dm^3}$

**Example 5** If  $a, b, c$  and  $d$  are in continued proportion

, prove that:  $\frac{a+d}{b-c+d} = \frac{a-c}{b-c}$

**Solution**

$$\text{Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m \quad \therefore \textcircled{c} = dm, \textcircled{b} = dm^2, \textcircled{a} = dm^3$$

$$\begin{aligned} \therefore \frac{a+d}{b-c+d} &= \frac{dm^3 + d}{dm^2 - dm + d} = \frac{d(m^3 + 1)}{d(m^2 - m + 1)} \\ &= \frac{(m+1)(m^2 - m + 1)}{m^2 - m + 1} = m + 1 \end{aligned} \quad (1)$$

$$\frac{a-c}{b-c} = \frac{dm^3 - dm}{dm^2 - dm} = \frac{dm(m^2 - 1)}{dm(m - 1)} = \frac{(m-1)(m+1)}{(m-1)} = m + 1 \quad (2)$$

From (1) and (2) we deduce that:  $\frac{a+d}{b-c+d} = \frac{a-c}{b-c}$

**TRY 3**  
by yourself

If  $a, b, c$  and  $d$  are in continued proportion, prove that:  $\frac{a+2b}{b+2c} = \frac{c+a}{d+b}$

**Example 6** If the quantities  $a$ ,  $2b$ ,  $3c$  and  $4d$  are in continued proportion, prove that :  $(2b - 3c)$  is a middle proportion between  $(a - 2b)$  and  $(3c - 4d)$

**Solution**

$$\text{Let } \frac{a}{2b} = \frac{2b}{3c} = \frac{3c}{4d} = m \quad \therefore (3c) = 4dm, (2b) = 4dm^2, (a) = 4dm^3$$

proving that :  $(2b - 3c)$  is the middle proportion between  $(a - 2b)$  and  $(3c - 4d)$

means proving that :  $(2b - 3c)^2 = (a - 2b)(3c - 4d)$

$$\begin{aligned} \therefore (2b - 3c)^2 &= (4dm^2 - 4dm)^2 \\ &= (4dm(m - 1))^2 = 16d^2m^2(m - 1)^2 \end{aligned} \quad (1)$$

$$\begin{aligned} (a - 2b)(3c - 4d) &= (4dm^3 - 4dm^2)(4dm - 4d) \\ &= 4dm^2(m - 1) \times 4d(m - 1) = 16d^2m^2(m - 1)^2 \end{aligned} \quad (2)$$

From (1) and (2) we deduce that :  $(2b - 3c)^2 = (a - 2b)(3c - 4d)$

$\therefore (2b - 3c)$  is a middle proportion between  $(a - 2b)$  and  $(3c - 4d)$

**Another solution :**

$\therefore a, 2b, 3c$  and  $4d$  are in continued proportion.

$$\therefore \frac{a}{2b} = \frac{2b}{3c} = \frac{3c}{4d}$$

Subtracting the terms of the 2<sup>nd</sup> ratio from the terms of the 1<sup>st</sup> ratio

$$\therefore \frac{a - 2b}{2b - 3c} = \text{one of the given ratios.} \quad (1)$$

Subtracting the terms of the 3<sup>rd</sup> ratio from the terms of the 2<sup>nd</sup> ratio

$$\therefore \frac{2b - 3c}{3c - 4d} = \text{one of the given ratios.} \quad (2)$$

$$\text{From (1) and (2) we deduce that : } \frac{a - 2b}{2b - 3c} = \frac{2b - 3c}{3c - 4d}$$

$\therefore (2b - 3c)$  is a middle proportion between  $(a - 2b)$  and  $(3c - 4d)$

**3** Prove by yourself | Hint :  $a = dm^3, b = dm^2, c = dm$  |

**2** Prove by yourself | Hint :  $a = cm^2, b = cm$  |

**2** 4

**1** 1 ± 24

**Answers of try yourself**



## LESSON

# 4

## Direct variation and inverse variation

### First

### The direct variation



#### Definition

It is said that  $y$  varies directly as  $X$  and it is written  $y \propto X$  if  $y = mX$

i.e.  $\frac{y}{X} = m$ , where  $m$  is a constant  $\neq 0$

, the relation :  $y = mX$  is represented graphically by a straight line passing through the origin point  $(0, 0)$

#### For example:

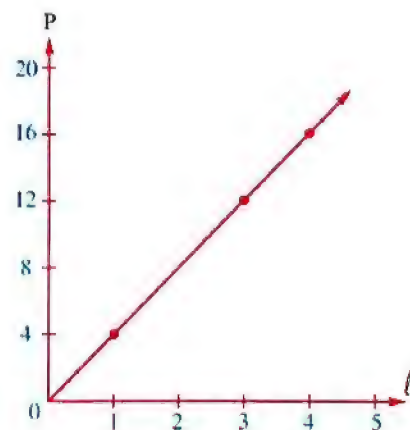
The perimeter of the square ( $P$ ) is varying directly with its side length ( $\ell$ )

Because :  $P = 4\ell$  or  $\frac{P}{\ell} = 4$

and the following table expresses some values of  $\ell$  and values of  $P$  corresponding to them.

Side length ( $\ell$ )	1	3	4
The perimeter ( $P$ )	4	12	16

and the opposite figure represents graphically the relation between  $P$  and  $\ell$



**Example 1** Show which of the following graphs represents a direct variation between  $x$  and  $y$  :

a



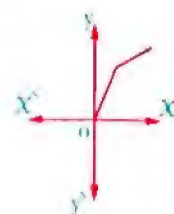
b



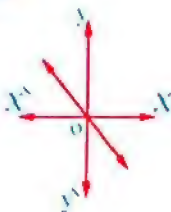
c



d



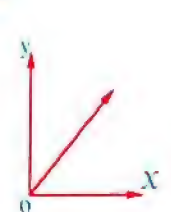
e



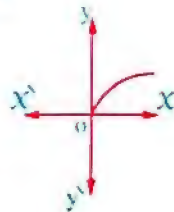
f



g



h



**Solution**

The graphs which represent a direct variation between  $x$  and  $y$  are :

**c** , **e** and **g** because in each of them , the straight line passes through the origin point.

**Example 2** If  $a^2 + 4b^2 = 4ab$  , prove that :  $a \propto b$

**Solution**

To prove that  $a \propto b$  we prove that  $a = mb$  where  $m$  is a constant  $\neq 0$

$$\therefore a^2 + 4b^2 = 4ab$$

$$\therefore a^2 - 4ab + 4b^2 = 0$$

$$\therefore (a - 2b)^2 = 0$$

$$\therefore a - 2b = 0$$

$$\therefore a = 2b$$

$$\therefore a \propto b$$

**TRY**

*by yourself*

If  $\frac{3x-5y}{3x-9y} = \frac{1}{2}$  for every values of  $x \in \mathbb{R}_+$  ,  $y \in \mathbb{R}_+$  , **prove that** :  $x \propto y$

**Property**

If  $y \propto x$  , the variable  $x$  took the two values  $x_1$  and  $x_2$  and  $y$  took the two values  $y_1$  and  $y_2$

respectively , then :  $\frac{y_1}{y_2} = \frac{x_1}{x_2}$

Because :  $\because y \propto X$  then  $y = mX$  where  $m \neq 0$

at  $X = X_1$  ,  $y = y_1$  then  $y_1 = mX_1$  (1)

, at  $X = X_2$  ,  $y = y_2$  then  $y_2 = mX_2$  (2)

Dividing (1) by (2) :  $\therefore \frac{y_1}{y_2} = \frac{mX_1}{mX_2} \quad \therefore \frac{y_1}{y_2} = \frac{X_1}{X_2}$



**Example 3** If  $y \propto X$  and  $y = 20$  when  $X = 7$

, then find the value of  $y$  when  $X = 14$

**Solution**

$\because y \propto X \quad \therefore \frac{y_1}{y_2} = \frac{X_1}{X_2}$

when  $y_1 = 20$  ,  $X_1 = 7$  ,  $y_2 = ?$  ,  $X_2 = 14$

$\therefore \frac{20}{y_2} = \frac{7}{14} \quad \therefore y_2 = \frac{20 \times 14}{7} = 40$

**Another solution :**

$\because y \propto X \quad \therefore y = mX$  ( $m$  is a constant  $\neq 0$ )

$\because y = 20$  as  $X = 7 \quad \therefore 20 = m \times 7$

$\therefore m = \frac{20}{7} \quad \therefore y = \frac{20}{7} X$

, when  $X = 14 \quad \therefore y = \frac{20}{7} \times 14 \quad \therefore y = 40$

**Example 4** If  $X$  and  $y$  are two variables where  $y$  varies directly as the multiplicative inverse of  $\left(\frac{1}{X^3}\right)$  If  $y = 18$  when  $X = 2$

Find the relation between  $X$  and  $y$  , then find the values of  $y$  when

$X \in \{0, 1, 4\}$

**Solution**

$\because y \propto$  the multiplicative inverse of  $\frac{1}{X^3}$

$\therefore y \propto X^3 \quad \therefore y = mX^3$  where  $m$  is a constant  $\neq 0$

$\because y = 18$  as  $X = 2 \quad \therefore 18 = m \times (2)^3 \quad \therefore m = \frac{18}{8} = \frac{9}{4}$

$\therefore y = \frac{9}{4} X^3$  This is the relation between  $X$  and  $y$

as  $X = 0 \quad \therefore y = \frac{9}{4} \times 0 = 0$

as  $X = 1 \quad \therefore y = \frac{9}{4} \times 1 = \frac{9}{4} = 2\frac{1}{4}$

as  $X = 4 \quad \therefore y = \frac{9}{4} \times 64 = 144$

**Example 5** If ( $V$ ) denotes the volume of a right circular cone, its height is constant and if ( $V$ ) varies directly as the square of radius length of the base of the cone ( $r$ ) and the volume of the cone was  $477 \text{ cm}^3$ , when the radius length of its base = 15 cm.

Find the volume of the cone when the base radius length = 10 cm.

**Solution**

$$\therefore V \propto r^2 \qquad \therefore \frac{V_1}{V_2} = \frac{r_1^2}{r_2^2} \qquad \therefore \frac{V_1}{V_2} = \left( \frac{r_1}{r_2} \right)^2$$

where  $V_1 = 477 \text{ cm}^3$ ,  $r_1 = 15 \text{ cm}$ ,  $V_2 = ?$ ,  $r_2 = 10 \text{ cm}$ .

$$\therefore \frac{477}{V_2} = \left( \frac{15}{10} \right)^2 = \frac{9}{4} \qquad \therefore V_2 = \frac{477 \times 4}{9} = 212 \text{ cm}^3$$



If  $X \propto y$  and  $y = 2$  when  $X = 40$ , find the value of  $X$  when  $y = 3$



## Second The inverse variation

### Definition

It is said that  $y$  varies inversely as  $X$  and it is written  $y \propto \frac{1}{X}$  if  $y = \frac{m}{X}$

i.e.  $Xy = m$ , where ( $m$  is a constant  $\neq 0$ )

**For example:**

The uniform velocity ( $v$ ) changes inversely as time ( $t$ ) when the covered distance is constant

Because :  $v = \frac{d}{t}$  or  $vt = d$

, in this case we say that the velocity changes directly as the multiplicative inverse of time and it is written as :  $v \propto \frac{1}{t}$

**Example 6** If  $a^2 b^4 - 10 ab^2 = -25$ , prove that :  $a$  varies inversely as  $b^2$

**Solution**

To prove that  $a$  varies inversely as  $b^2$  we prove that :  $ab^2 = m$  where  $m \neq 0$

$$\therefore a^2 b^4 - 10 ab^2 = -25$$

$$\therefore a^2 b^4 - 10 ab^2 + 25 = 0$$

$$\therefore (ab^2 - 5)^2 = 0$$

$$\therefore ab^2 - 5 = 0$$

$$\therefore ab^2 = 5$$

$$\therefore a \text{ varies inversely as } b^2$$

### TRY 3

by yourself

If  $a^2 b^2 + 49 = 14 ab$  **prove that** :  $a \propto \frac{1}{b}$

### Property

If  $y \propto \frac{1}{x}$ , the variable  $x$  took the two values  $x_1, x_2$  and as a result for that  $y$  took the two values  $y_1$  and  $y_2$  respectively, then :

$$\frac{y_1}{y_2} = \frac{x_2}{x_1}$$

Because :  $\because y \propto \frac{1}{x}$ , then  $y = \frac{m}{x}$  where  $m \neq 0$

at  $x = x_1$ ,  $y = y_1$ , then  $y_1 = \frac{m}{x_1}$  (1)

, at  $x = x_2$ ,  $y = y_2$ , then  $y_2 = \frac{m}{x_2}$  (2)

Dividing (1) by (2) :

$$\therefore \frac{y_1}{y_2} = \frac{m}{x_1} \div \frac{m}{x_2} = \frac{m}{x_1} \times \frac{x_2}{m} = \frac{x_2}{x_1}$$

**Example 7** If the length of a rectangle ( $l$ ) varies inversely as its width ( $w$ ), when the area is constant and  $l = 12$  cm. as  $w = 8$  cm., **find** :  $l$  when  $w = 3$  cm.

### Solution

$$\therefore l \propto \frac{1}{w}$$

$$\therefore \frac{l_1}{l_2} = \frac{w_2}{w_1}, \text{ where } l_1 = 12 \text{ cm.}, w_1 = 8 \text{ cm.}, l_2 = ?, w_2 = 3 \text{ cm.}$$

$$\therefore \frac{12}{l_2} = \frac{3}{8} \qquad \therefore l_2 = \frac{8 \times 12}{3} = 32 \text{ cm.}$$

### Another solution :

$$\therefore l \propto \frac{1}{w}$$

$$\therefore lw = m, \text{ where } m \text{ is a constant } \neq 0$$

$$\therefore l = 12 \text{ cm. as } w = 8 \text{ cm.}$$

$$\text{When } w = 3 \text{ cm.}$$

$$\therefore m = 12 \times 8 = 96$$

$$\therefore 3l = 96$$

$$\therefore lw = 96$$

$$\therefore l = \frac{96}{3} = 32 \text{ cm.}$$

**Example 8** If  $y$  varies inversely as  $X$  and  $y = 6$  as  $X = 2.5$ , find the relation between  $X$  and  $y$ , then find the values of  $y$  if  $X = 5$

**Solution**

$$\therefore y \propto \frac{1}{X}$$

$$\therefore Xy = m, \text{ where } m \text{ is a constant } \neq 0$$

$$\therefore y = 6 \text{ as } X = 2.5$$

$$\therefore m = 6 \times 2.5 = 15$$

$$\therefore \text{The relation between } X \text{ and } y \text{ is } \boxed{Xy = 15}$$

$$\text{, at } X = 5$$

$$\therefore 5y = 15$$

$$\therefore \boxed{y = \frac{15}{5} = 3}$$

**Example 9** If  $y = 1 + b$  where  $b$  varies inversely as  $X^2$  and  $y = 17$  as  $X = \frac{1}{2}$ . Find the relation between  $X$  and  $y$ , then find the value of  $y$  when  $X = 2$

**Solution**

$$\therefore b \propto \frac{1}{X^2}$$

$$\therefore b = \frac{m}{X^2}, \text{ where } m \text{ is a constant } \neq 0$$

$$\therefore y = 1 + \frac{m}{X^2}$$

$$\therefore y = 17 \text{ as } X = \frac{1}{2}$$

$$\therefore 17 = 1 + \frac{m}{\left(\frac{1}{2}\right)^2}$$

$$\therefore 17 = 1 + \frac{m}{\frac{1}{4}}$$

$$\text{Subtracting 1 from both sides : } \therefore 16 = \frac{m}{\frac{1}{4}}$$

$$\therefore m = 16 \times \frac{1}{4} = 4$$

$$\therefore \boxed{y = 1 + \frac{4}{X^2}}$$

$$\text{at } X = 2 : \therefore y = 1 + \frac{4}{2^2} = 1 + \frac{4}{4} = 2$$

**TRY 4**  
by yourself

If  $y$  varies inversely as  $X$  and if  $y = 2$  as  $X = 6$ , calculate the value of  $y$  as  $X = 1$

**Answers**  
1 Prove by yourself that :  $X = \frac{3}{y}$   
3 Prove by yourself that :  $a \cdot b = 7$

4 12

2 60

of try by yourself

# UNIT 3

## Statistics



### ▶ Lessons of the unit :

1. Collecting data.
2. Dispersion.

### ► Unit Objectives :

**By the end of this unit, student should be able to :**

- recognize the different resources of collecting data.
- recognize the methods of collecting data , and the advantages and the disadvantages of each method.
- recognize the concept of the sample.
- recognize the methods of selection of samples.
- recognize the types of the samples.
- choose the best method to select a sample for studying a certain phenomenon.
- use the calculator and the computer for generating random numbers used in the samples.
- recognize the dispersion measurements.
- recognize the advantages and the disadvantages of the range as one of the dispersion measurements.
- calculate the range of a set of individuals.
- calculate the standard deviation of a set of individuals.
- calculate the standard deviation of a simple frequency distribution.
- calculate the standard deviation of a frequency distribution of sets.
- use the calculator to calculate the standard deviation.

## LESSON

# 1

## Collecting data

- The statistical investigator collects , classifies , represents and analyses data in purpose of deducing some results on which he depends in making the suitable decisions.
- The more data is accurate , the more the decisions will be true and reliable.
- Collecting data in such scientific methods will lead to get accurate outcomes when doing operations of statistical inference and proper decision making.
- Collecting statistical data demands knowing the resources of collecting it and determining the methods of collecting it.

### Resources of collecting data is classified into

#### 1 Primary resources (field resources) :

These are the resources from which we get data directly.

#### 2 Secondary resources (historical resources) :

These are the resources from which we get data that previously collected and registered by some authorities , formal organisations or persons.

There are some examples for each resource with representing the advantages and the disadvantages of each one :

	1 Primary resources	2 Secondary resources
Examples :	<ul style="list-style-type: none"> <li>• The personal interview.</li> <li>• Questionnaires (survey).</li> <li>• Observing and measuring.</li> </ul>	<ul style="list-style-type: none"> <li>• Central agency for public mobilization and statistics.</li> <li>• Mass-media and internet.</li> <li>• Documents of data of employees in a company.</li> </ul>
Advantages :	Accuracy.	Saves time , effort and money.
Disadvantages :	It needs more time , effort and money besides it requires more investigators in large societies.	It is less accurate.

### Methods of collecting data

- The method of collecting data depends on the aim of collecting these data and it also depends on the size of the statistical society under study.
- The statistical society is defined as all values which have general common characters.

For example:

- The workers in a factory represent a statistical society , whose value is the worker.
- The pupils of a school represent a statistical society , whose value is the pupil.



We will show two methods of collecting data :

#### 1 Method of mass population :

It is based on collecting the data related to the phenomenon under study from all individuals of the statistical society.

#### 2 Method of samples :

It is based on collecting data related to the phenomenon under study from a representative sample of the society , and applying the research on it , then generalizing the results on the whole society.

There are some examples for each method with representing the advantages and the disadvantages of each one :

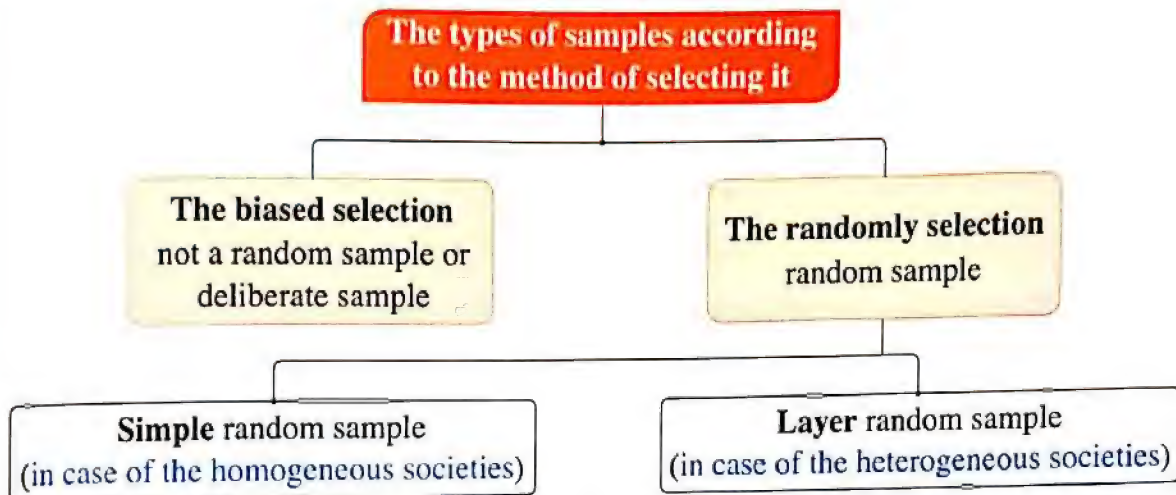
	1 Method of mass population	2 Method of samples
<b>Examples :</b>	<ul style="list-style-type: none"> <li>• Elections.</li> <li>• Census.</li> <li>• Setting up a data base of all employees in an organization.</li> </ul>	<ul style="list-style-type: none"> <li>• A sample of a patient's blood to make some clinical check up.</li> <li>• A sample of some products of a factory to find out if it matches the standard specifications.</li> </ul>
<b>Advantages :</b>	<ul style="list-style-type: none"> <li>• Accuracy.</li> <li>• Inclusiveness.</li> <li>• Neutrality.</li> <li>• Representing all the society individuals.</li> </ul>	<ul style="list-style-type: none"> <li>• Saving time , effort and money.</li> <li>• It is the only method for collecting data about large unlimited societies such as the search on contents of the desert sand.</li> <li>• It is the only method for collecting data about some limited societies in which mass population method leads to a great loss in it such as checking a sample of a patient's blood because of checking the whole blood of the patient leads to death.</li> </ul>
<b>Disadvantages :</b>	<ul style="list-style-type: none"> <li>• Sometimes it needs long time , great effort and a great cost.</li> </ul>	<ul style="list-style-type: none"> <li>• The results sometimes are not accurate specially if the sample doesn't represent the statistical society authentically , in this case the sample is called a biased sample.</li> </ul>

In the following , we will explain the concept of the sample and its types and how we select it :

### The concept of the sample

It is a small part from a large society that looks like the society and represents it well.

### How can we select the sample ?



At the following we explain each type in detail :

### First The biased selection (samples are not randomly selected)

- It means that we select the sample in a way to satisfy the objectives of the research.  
This is called **the deliberate sample**.

For example:

If we want to know how the students understood a lesson in algebra , we must analyze the outcomes of the test by considering the outcomes of a group of students studying the same topic without the other students , this is not a random selection.



- The biased selection is not representing the statistical society.

## Second Random selection (random samples)

It means to select a sample such that every member of the population has an equal chance of having selected.

The following are the most important types of the random samples which are :

- 1 Simple random sample.
- 2 Layer random sample.

### 1 Simple random sample

- It is used for the homogeneous societies which are not naturally divided into groups or classes.
- It is selected by two ways according to the number of individuals of statistical society as the following.

#### A The first method : If the size of the society is small :

- This method will be carried out as follows :

- 1 Each individual of the society takes a number , this number is written on a card such that all cards are identical.  
*i.e.* there is no difference in colour or size.
- 2 Each card is folded well such that the number does not appear , then they are put in a box and mixed well.
- 3 We select the sample by drawing one card from the box blindly , then we turned well the cards and select the next card , and so on till we reach the required number of the sample.



This method is suitable if , for example , we select a sample of 10 workers from a factory that has 50 workers.

#### B The second method : If the size of the society is large :

In this method , every individual of the society has a number , then we select the sample using the property of the random number in the scientific calculator as in the opposite picture.

- We press the following keys respectively from the left :



then a decimal will appear on the display in the field from 0.000 to 0.999


- If we get a 1-decimal digit , add two zeroes to make it a part of 1000

For example:  $(0.2 \rightarrow 0.200)$

- If we get a 2-decimal digit , add one zero to make it a part of 1000



For example:  $(0.64 \rightarrow 0.640)$  and so on.

- Take the number neglecting the decimal point , then the individual who has this number is selected as a member of the sample , then repeat pressing on  to get more numbers.
- We will ignore the numbers which are greater than the number of society under study.
- And we ignore the repeated numbers which we selected before.
- The percentage 10% of the number of the society is suitable for holding the survey.

This method is more suitable for selecting a sample of 25 students from a school that has 900 students.

## 2 Layer random sample

- It is used in the statistical societies which are heterogeneous or made up of qualitative sets that are different in characteristics.
- In this case , we cannot select the sample by the simple random sample method because the sample will not represent the society well because it will not represent all the classes of the society.

Therefore we have to follow the following steps :

- 1 We divide the society into homogeneous sets according to the characteristics forming it , each set is called a **layer**.
- 2 We find the number of individuals of each layer , then we find its ratio referring to the total number of the society.
- 3 To form a sample , we select from each layer a certain number of individuals such that the ratio that represents each layer in the sample is the same ratio of the layer in the whole society , and this by using the following law :

The number of individuals of the layer in the sample

$$= \frac{\text{the total number of individuals in the layer}}{\text{the total number of individuals in the society}} \times \text{the number of individuals of the sample}$$

«approximated the result to the nearest unit»

For example:

When we want to study the educational level of the students of a school of 500 students (boys and girls) and if the ratio between the number of boys to the number of girls is 1 : 4 and we want to select a sample formed from 50 students , we should select 10 students from boys and 40 students from girls , for the sample representing all the society well.

**Example 1** A factory has 300 workers. The people in charge of the monthly magazine of this factory want to develop this magazine by doing a survey of a sample representing 10% of the total number of the workers in this factory. Show how the selection of this sample can be carried out using the calculator.

**Solution**

The number of workers in the factory = 300 workers.





$\therefore$  The number of the random sample =  $\frac{10}{100} \times 300 = 30$  workers.

Then we want to select 30 workers to hold this survey.

**The selection operation can be carried out as follows :**

- 1 Each worker in the factory is given a number from 1 to 300
- 2 Use the calculator to select 30 numbers randomly , such that these numbers are included between 0 and 301 and the number that is above 300 should be ignored.

For example:

By pressing the keys     successively from left to right.

- If we get the decimal 0.049 , then the number of the selected person is 49
- If we get the decimal 0.132 , then the number of the selected person is 132
- If we get the decimal 0.12 , then the number of the selected person is 120
- If we get the decimal 0.453 , it must be ignored because 453 is above 300 and so on till we get 30 numbers.
- Assuming that the calculator gave us the shown numbers in the opposite table , then the workers who carry these numbers are the selected sample to carry out this survey.

49	132	120	141	249	272
254	256	4	213	74	198
131	2	156	47	172	13
8	3	85	82	9	38
41	14	34	279	118	103

**Example 2** A factory produced 200 TV sets from the type A , 300 TV sets from the type B and 500 TV sets from the type C , if we want to select a layer sample formed from 50 TV sets such that it represents all the types to examine them.



**Calculate the number of TV sets which should be selected from each kind.**

**Solution**

- The total number of TV sets =  $200 + 300 + 500 = 1000$  TV sets.
- The number of TV sets of the type A in the sample =  $\frac{200}{1000} \times 50 = 10$  TV sets.
- The number of TV sets of the type B in the sample =  $\frac{300}{1000} \times 50 = 15$  TV sets.
- The number of TV sets of the type C in the sample =  $\frac{500}{1000} \times 50 = 25$  TV sets.

**TRY**


*by yourself*

A school has 300 male students and 500 female students wanted to do a survey on a sample of 24 male and female students representing each layer according to its size. Calculate the number of students of each layer in the sample.

The number of female students in the sample is 15

The number of male students in the sample is 9

**Answers** *of try by yourself*



## LESSON

# 2

## Dispersion

### Prelude

- You studied before some of statistical measures which were known as **“measures of central tendency”** as the mean , the median and the mode.
- And we know that each of them describe the frequency distributions and the statistical data by identifying one numerical value , where the left values centralize about it.
- But in some cases the measures of central tendency are not enough to describe clearly the data.

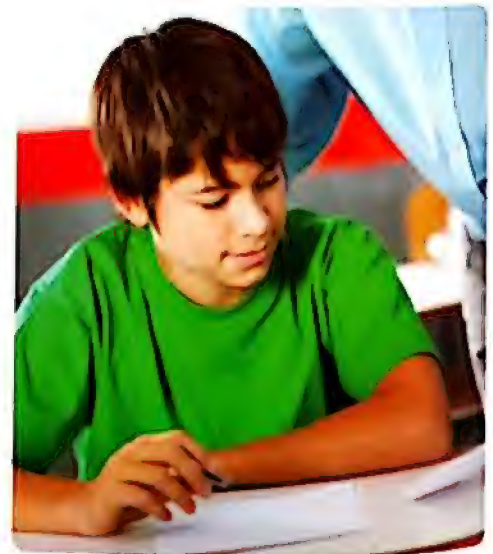
**To explain that , let's study the following case :**

Two sets of 5 students each , an exam of maximum mark 50 marks is given for each sets , the marks of the students were as follows :

**The set A :** 29 , 26 , 35 , 35 , 35

**The set B :** 8 , 35 , 49 , 35 , 33

At calculating the mean ,  
the median and the mode of the marks of the  
students in each set alone , we find the shown  
results in the following table :





### Remember that

	mean	median	mode
Set A	32	35	35
Set B	32	35	35

- The mean =  $\frac{\text{the sum of values}}{\text{the number of this values}}$
- The median of a set of values is the value which lies at the middle of the set of values after ordering them.
- The mode of a set of values is the most common value in the set.

• In the previous case, the two sets are different, and in spite of that, we found that they have the same mean, median and mode, which don't mean that these sets are necessarily homogeneous.

• Therefore, the measures of central tendency only are unable to describe all the characteristics a set of frequency distributions and statistical data.

So we need besides the measures of central tendency that depends on determining one value that the other data centralize around it, another kind of measures which depends on determining a degree of convergence or divergence of data.

For example:

In the previous example, the marks of the set A are convergent because their values are included between 26 and 35 marks while the marks of the set B are divergent because their values are included between 8 and 49 marks.

i.e. The marks of the set B are more divergent than the marks of the set A

• These new measures are called the measures of dispersion. We will study each of the range and the standard deviation.

### Dispersion of a set of values

It means the divergence or the differences among its values.

• The dispersion is small if the difference among the values is little while the dispersion is great if the difference among the values is great, the dispersion is zero if all the values are equal.

i.e. The dispersion of a set of values is a measure of the degree to which these values spread out and that expresses how much the sets are homogeneous.

**Dispersion measurements****1 The range (the simplest measure of dispersion) :**

It is the difference between the greatest value and the smallest value in the set.

The range = the greatest value – the smallest value

For example:

- If the values of set A are 60 , 58 , 62 , 61 and 59

∴ The range =  $62 - 58 = 4$

- If the values of set B are 72 , 78 , 46 , 65 and 39

∴ The range =  $78 - 39 = 39$

So the set B is more divergent than the set A

**The advantages of range :**

- It is an easy and simple method that gives a quick idea about the divergence or convergence of the values.
- It is considered as the simplest and the easiest method to measure dispersion.

**The disadvantages of range :**

- It does not reflect the influence of all values because its measure depends on the greatest and smallest values only , therefore it does not give a full idea of the dispersion of the set of values.
- It is influenced greatly by the outlier.

For example:

- The range of the set of values : 21 , 22 , 61 , 24 and 26 is ( $61 - 21 = 40$ )

- While if we ignore the value 61 from the set , then the range becomes ( $26 - 21 = 5$ )

i.e. The range equals  $\frac{1}{8}$  the previous range , therefore the range is an approximated measure and we cannot depend on it.

## 2 Standard deviation :

It is the most important , common and accurate measure of dispersion. We can calculate it by calculating the positive square root of the average of squares of deviations of the values from their mean. It is denoted by  $\sigma$  and it is read as (sigma).

### First

### Calculating the standard deviation of a set of values :

$$\text{The standard deviation } \sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

Where :

$x$  denotes a value of the values ,

$\bar{x}$  denotes the mean of the values and it is read as  $x$  bar ,

$n$  denotes the number of values ,

$\sum$  denotes the summation operation.

**Example 1** Calculate the standard deviation of the values : 8 , 9 , 7 , 6 and 5

### Solution

1 We find the mean of the values  $(\bar{x}) = \frac{\sum x}{n} = \frac{8+9+7+6+5}{5} = 7$

2 We form the opposite table :

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
8	$8 - 7 = 1$	1
9	$9 - 7 = 2$	4
7	$7 - 7 = 0$	0
6	$6 - 7 = -1$	1
5	$5 - 7 = -2$	4
Total		10

3 We calculate the standard deviation as follows :

$$\text{The standard deviation } (\sigma) = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\frac{10}{5}} = \sqrt{2} \approx 1.41$$

### TRY 1

by yourself

If 25 , 24 , 25 , 30 , 28 and 30 represent the marks of one of the pupils in examination of algebra in different months , find :

1 The mean.

2 The standard deviation.

## Second Calculating the standard deviation of a frequency distribution :

For any frequency distribution : The standard deviation  $\sigma = \sqrt{\frac{\sum (X - \bar{X})^2 k}{\sum k}}$

Where :

$X$  represents the value or the centre of the set ,

$k$  represents the frequency of the value or the set ,

$\sum k$  is the sum of frequencies and  $\bar{X}$  (the mean) =  $\frac{\sum (X \times k)}{\sum k}$

### A Calculating the standard deviation of a simple frequency distribution :

**Example 2** The following table shows the distribution of ages of 20 persons in years :

The age	15	20	22	23	25	30	Total
Number of persons	2	3	5	5	1	4	20

Find the standard deviation of the ages.

#### Solution

**1** We find the mean of the ages ( $\bar{X}$ ) by using the following table :

The age ( $X$ )	Number of persons ( $k$ )	$X \times k$
15	2	30
20	3	60
22	5	110
23	5	115
25	1	25
30	4	120
<b>Total</b>	<b>20</b>	<b>460</b>

The mean ( $\bar{X}$ ) =  $\frac{\sum (X \times k)}{\sum k} = \frac{460}{20} = 23$  years.

- 2 We form the following table :

$x$	$k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
15	2	$15 - 23 = -8$	64	128
20	3	$20 - 23 = -3$	9	27
22	5	$22 - 23 = -1$	1	5
23	5	$23 - 23 = 0$	0	0
25	1	$25 - 23 = 2$	4	4
30	4	$30 - 23 = 7$	49	196
<b>Total</b>	<b>20</b>			<b>360</b>

- 3 We calculate the standard deviation as follows :

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{\sum (x - \bar{x})^2 \times k}{\sum k}} = \sqrt{\frac{360}{20}} = \sqrt{18} = 4.24 \text{ years.}$$

### TRY 2

by yourself

The following frequency distribution shows the number of days of absentees in a class :

Number of absence days	0	1	2	3	4	Total
Number of pupils	5	7	7	5	6	30

Calculate the mean and the standard deviation for the number of days of absence.

## B Calculating the standard deviation of a frequency distribution of sets :

**Example 3** The following is the frequency distribution of weekly incentives of 100 workers in a factory :

Incentives in pounds	35 –	45 –	55 –	65 –	75 –	85 –
Number of workers	10	14	20	28	20	8

Find the standard deviation of this distribution.

**Solution**

1 We find the mean ( $\bar{X}$ )  **Remember that**

by using the following table :

$$\text{The centre of the set} = \frac{\text{lower limit} + \text{upper limit}}{2}$$

Sets	Centres of sets ( $X$ )	Frequency ( $k$ )	$X \times k$
35 –	40	10	400
45 –	50	14	700
55 –	60	20	1200
65 –	70	28	1960
75 –	80	20	1600
85 –	90	8	720
<b>Total</b>		<b>100</b>	<b>6580</b>

$$\therefore \text{The mean } (\bar{X}) = \frac{\sum (X \times k)}{\sum k} = \frac{6580}{100} = 65.8 \text{ pounds.}$$

2 We form the following table :

$X$	$k$	$X - \bar{X}$	$(X - \bar{X})^2$	$(X - \bar{X})^2 \times k$
40	10	$40 - 65.8 = -25.8$	665.64	6656.4
50	14	$50 - 65.8 = -15.8$	249.64	3494.96
60	20	$60 - 65.8 = -5.8$	33.64	672.8
70	28	$70 - 65.8 = 4.2$	17.64	493.92
80	20	$80 - 65.8 = 14.2$	201.64	4032.8
90	8	$90 - 65.8 = 24.2$	585.64	4685.12
<b>Total</b>	<b>100</b>			<b>20036</b>

3 We calculate the standard deviation as follows :

$$\text{Standard deviation } (\sigma) = \sqrt{\frac{\sum (X - \bar{X})^2 \times k}{\sum k}} = \sqrt{\frac{20036}{100}} = 14.15 \text{ pounds.}$$

### Remarks

- The standard deviation is influenced by all values not by the two terminal values only (the smallest and the greatest value) as the range, therefore it represents the dispersion better than the range.
- The standard deviation has the same measuring units of the original data.
- The values which are more homogeneous have less dispersion and their standard deviation is small.
- If the standard deviation equals zero that means the all values are equal, it is the perfect homogeneous case (the vanished dispersion)

### TRY 3

For the following frequency distribution, calculate :

1 The mean.

2 The standard deviation.

Sets	1 -	3 -	5 -	7 -	9 - 11
Frequency	7	3	5	3	2

3 1 5 2 2.72 (approximately)

The standard deviation ( $\sigma$ )  $\approx$  1.37 days

2 The mean ( $\bar{X}$ ) = 2 days

2 2.45 marks (approximately)

1 1 27 marks

Answers of try by yourself

### Using the calculator to calculate the standard deviation :

- We can use the calculator CASIO ( $fX-82$  ES ,  $fX-85$  ES ,  $fX-500$  ES ,  $fX-95$  ES Plus ,  $fX-991$  ES Plus) to calculate the standard deviation.
- The following steps show how to solve the previous example (example 3) using the calculator :
- We will use the calculator (  $fX-95$  ES Plus)

#### Step (1)

before inserting the data of the previous example, we should set the calculator system by pressing the following keys from left :



Then the screen will appear as in the opposite figure.

#### Step (2)

- we insert the values ( $X$ ) in the case of simple frequency distribution or the centres of sets ( $X$ ) in the case of frequency distribution of sets in the first column ( $X$ )
- With respect to the previous example :


We insert the centres of sets :

40 , 50 , 60 , 70 , 80 and 90 by pressing the following keys from left as follows :



Then the screen will appear as in the opposite figure.

**Step (3)**

Use the key  to move to the second column (FREQ), then insert frequencies






10 , 14 , 20 , 28 , 20 and 8 by pressing the following keys from left as follows :

Thus we insert the data of the previous example on the calculator.

**Step (4)**

For finding the value of the standard deviation , we press the following keys from left :

   (VAR)  ( $\sigma x$ ) 

Then the screen will appear as in the opposite figure.

$\therefore$  Standard deviation  $\sigma \approx 14.15$



**Second**

# Trigonometry and Geometry



**UNIT 4** Trigonometry..... 93

**UNIT 5** Analytical geometry..... 107

# UNIT 4

## Trigonometry



### ▶ Lessons of the unit :

1. The main trigonometrical ratios of the acute angle.
2. The main trigonometrical ratios of some angles.

▶ Use your smart phone or tablet to scan the QR Code and enjoy watching videos.



## ► Unit Objectives :

By the end of this unit, student should be able to :

- recognize the main trigonometrical ratios of the acute angle.
  - recognize the main trigonometrical ratios of the angles of measures  $30^\circ$  ,  $60^\circ$  and  $45^\circ$
  - find the main trigonometrical ratios of a given angle.
  - find the measure of the angle if one of its trigonometrical ratios is given.
  - use the calculator to find the main trigonometrical ratios.
- 

## Enriching information

Trigonometry is one of mathematics branches and it is one of the general geometry branches , it concerns studying the relations between the sides and angles of the triangle and the trigonometric ratios as the sine and cosine of the angle.

Ancient Egyptians were the first to use the trigonometric theorems and rules in building pyramids and temples.

Trigonometry has many applications in surveying roads and manufacturing motors , TV sets , football playgrounds , calculating geographic distances and astronomy discovering.

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## LESSON

# 1

## The main trigonometrical ratios of the acute angle

### Prelude

- You studied before the units of measure of the angle and they are :

The degree which is denoted by  $1^\circ$  , the minute which is denoted by  $1'$  and the second which is denoted by  $1''$

For example:

The angle whose measure is 22 degrees , 36 minutes and 48 seconds is written as  $22^\circ 36' 48''$

### The relation between each of the degrees, the minutes and the seconds

$$1^\circ = 60'$$

$$1' = 60''$$

$$\text{i.e. } 1^\circ = 60 \times 60 = 3600''$$

#### Example 1

1 Write in degrees :  $22^\circ 36' 48''$

2 Write in degrees , minutes and seconds :  $45.18^\circ$

#### Solution

1 We convert the minutes into degrees , as the following :

$$36' = \frac{36}{60} = 0.6^\circ$$

We convert the seconds into degrees , as the following :

$$48'' = \frac{48}{3600} = 0.013^\circ$$

$$\text{i.e. } 22^\circ 36' 48'' = 22^\circ + 0.6^\circ + 0.013^\circ = 22.613^\circ$$



#### Remember that

$0.00\dot{3}$  is read as the recurring decimal  $0.003$

### Another solution by using the scientific calculator :

Press the keys in sequence from left as follows :



Then the result will be  $22.61333333$

2 We convert  $0.18^\circ$  into minutes as the following :  $0.18 \times 60 = 10.8$

We convert  $0.8$  into seconds as the following :  $0.8 \times 60 = 48$

i.e.  $45.18^\circ = 45^\circ 10' 48''$

### Another solution by using the scientific calculator :

Press the keys in sequence from left as follows :



Then the result will be  $45^\circ 10' 48''$

**Example 2** If the ratio between the measures of two complementary angles is  $7 : 9$ , find the degree measure of each of them.

#### Solution

Let the measures of the two angles be :

$7x$  and  $9x$

$$\therefore 7x + 9x = 90^\circ$$

$$\therefore 16x = 90^\circ$$

$$\therefore x = \frac{90^\circ}{16} = 5.625^\circ$$

$$\begin{aligned} \therefore \text{The measure of the first angle} \\ &= 5.625^\circ \times 7 = 39.375^\circ \\ &= 39^\circ 22' 30'' \end{aligned}$$

$$\therefore \text{the measure of the second angle} = 5.625^\circ \times 9 = 50.625^\circ = 50^\circ 37' 30''$$



#### Remember that

- The sum of measures of two complementary angles  $= 90^\circ$
- The sum of measures of two supplementary angles  $= 180^\circ$
- The sum of measures of the interior angles of any triangle  $= 180^\circ$

#### TRY 1

by yourself

If the ratio between the measures of two supplementary angles is  $5 : 11$ , find the degree measure of each of them.

## The main trigonometrical ratios of the acute angle



### The trigonometrical ratio of the acute angle

It is the ratio between two side lengths of the right-angled triangle that contains this angle.

There are three main trigonometrical ratios of the acute angle and they are :

**1 The sine of the angle :**

abbreviated (**sin**) and equals

$\frac{\text{the length of the opposite side to the angle}}{\text{the length of the hypotenuse}}$

**2 The cosine of the angle :**

abbreviated (**cos**) and equals

$\frac{\text{the length of the adjacent side to the angle}}{\text{the length of the hypotenuse}}$

**3 The tangent of the angle :**

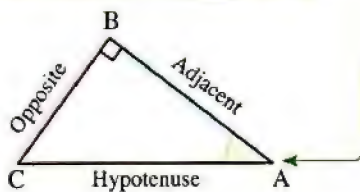
abbreviated (**tan**) and equals

$\frac{\text{the length of the opposite side to the angle}}{\text{the length of the adjacent side to the angle}}$

i.e.

If  $\Delta ABC$  is a right-angled triangle at B , then :

According to angle A

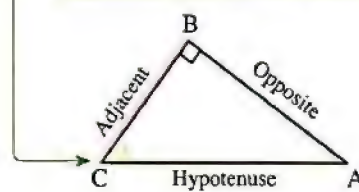


$$1 \quad \sin A = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$2 \quad \cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$3 \quad \tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{BC}{AB}$$

According to angle C



$$1 \quad \sin C = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$2 \quad \cos C = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$3 \quad \tan C = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{AB}{BC}$$

For example:

**In the opposite figure :**

If  $\Delta ABC$  is a right-angled triangle at B ,

AB = 3 cm. , BC = 4 cm. and AC = 5 cm. , then :

$$1 \quad \sin A = \frac{4}{5}$$

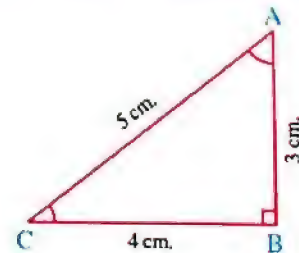
$$2 \quad \cos A = \frac{3}{5}$$

$$3 \quad \tan A = \frac{4}{3}$$

$$1 \quad \sin C = \frac{3}{5}$$

$$2 \quad \cos C = \frac{4}{5}$$

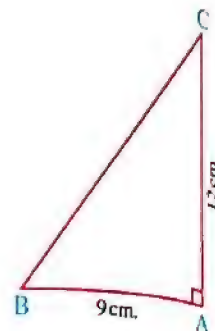
$$3 \quad \tan C = \frac{3}{4}$$



**Example 3** In the opposite figure :

$\Delta ABC$  is right-angled at A where  
 $AB = 9$  cm. and  $AC = 12$  cm.

- Find each of :  $\sin B$ ,  $\cos B$ ,  $\tan B$ ,  $\sin C$ ,  $\cos C$  and  $\tan C$
- Prove that :  $\sin B \cos C + \cos B \sin C = 1$

**Solution**

$\therefore$  In  $\Delta ABC$  :  $m(\angle A) = 90^\circ$

$\therefore (BC)^2 = (AB)^2 + (AC)^2$  (Pythagoras' theorem)

$\therefore (BC)^2 = 81 + 144 = 225 \quad \therefore BC = 15$  cm.

$$1 \quad \sin B = \frac{AC}{BC} = \frac{12}{15} = \frac{4}{5},$$

$$\cos B = \frac{AB}{BC} = \frac{9}{15} = \frac{3}{5},$$

$$\tan B = \frac{AC}{AB} = \frac{12}{9} = \frac{4}{3},$$

$$\sin C = \frac{AB}{BC} = \frac{9}{15} = \frac{3}{5},$$

$$\cos C = \frac{AC}{BC} = \frac{12}{15} = \frac{4}{5},$$

$$\tan C = \frac{AB}{AC} = \frac{9}{12} = \frac{3}{4}$$

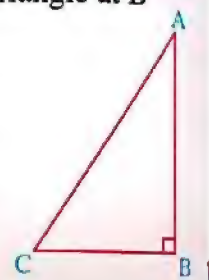
**Remember Pythagoras' theorem :**

If ABC is a right-angled triangle at B  
, then :

$$\bullet (AC)^2 = (AB)^2 + (BC)^2$$

$$\bullet (AB)^2 = (AC)^2 - (BC)^2$$

$$\bullet (BC)^2 = (AC)^2 - (AB)^2$$



$$2 \quad \sin B \cos C + \cos B \sin C = \frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5} = \frac{16}{25} + \frac{9}{25} = \frac{25}{25} = 1$$

**Remarks**

In the previous example, note that :

$$1 \quad \sin B = \cos C = \frac{4}{5}, \quad \sin C = \cos B = \frac{3}{5}$$

and by noticing :  $m(\angle B) + m(\angle C) = 90^\circ$  "Complementary angles"

We can deduce that :

The **sine** of any acute angle equals the **cosine** of its complementary

i.e. If  $m(\angle A) + m(\angle B) = 90^\circ$

$$\text{, then } \sin A = \cos B$$

$$\sin B = \cos A$$

and vice versa i.e. If  $\angle A$  and  $\angle B$  are acute angles and  $\sin A = \cos B$   
then  $m(\angle A) + m(\angle B) = 90^\circ$

$$\textcircled{2} \frac{\sin B}{\cos B} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}, \tan B = \frac{4}{3} \quad \therefore \tan B = \frac{\sin B}{\cos B}$$

$$, \frac{\sin C}{\cos C} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}, \tan C = \frac{3}{4} \quad \therefore \tan C = \frac{\sin C}{\cos C}$$

**Generally**

The tangent of the angle =  $\frac{\text{The sine of the angle}}{\text{The cosine of the angle}}$

”

**TRY 2**  
by yourself

XYZ is a right-angled triangle at Y, XY = 4 cm. and XZ = 5 cm.

- ① Find the value of :  $2 \sin X \cos X$
- ② Prove that :  $\sin X \cos Z + \cos X \sin Z = 1$

**Example 4** ABC is a triangle in which : AB = AC = 10 cm. , BC = 12 cm. ,  
 $\overline{AD}$  is drawn to be perpendicular to  $\overline{BC}$  to cut it at D

- 1 Find the value of :  $\sin B + \cos C$
- 2 Find the value of :  $\tan (\angle CAD)$
- 3 Show that :  $\sin C + \cos C > 1$  and find the value of :  $\sin^2 C + \cos^2 C$   
 and deduce that :  $\sin^2 C + \cos^2 C < \sin C + \cos C$

**Solution**

$$\therefore \overline{AD} \perp \overline{BC} \text{ and } AB = AC$$

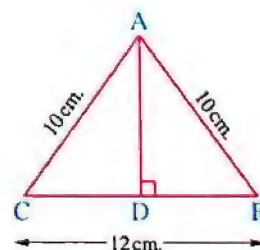
$$\therefore D \text{ is the midpoint of } \overline{BC}$$

$$\therefore BD = DC = 6 \text{ cm.}$$

$$\text{In } \triangle ADB : \therefore m(\angle ADB) = 90^\circ$$

$$\therefore (AD)^2 = (AB)^2 - (BD)^2 \text{ (Pythagoras' theorem)}$$

$$\therefore (AD)^2 = 100 - 36 = 64 \quad \therefore AD = 8 \text{ cm.}$$



$$\textcircled{1} \therefore \sin B = \frac{AD}{AB} = \frac{8}{10} = \frac{4}{5}, \cos C = \frac{CD}{AC} = \frac{6}{10} = \frac{3}{5}$$

$$\therefore \sin B + \cos C = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}$$

$$\textcircled{2} \tan (\angle CAD) = \frac{CD}{AD} = \frac{6}{8} = \frac{3}{4}$$

$$3 \quad \therefore \sin C = \frac{AD}{AC} = \frac{8}{10} = \frac{4}{5}, \cos C = \frac{3}{5}$$

$$\therefore \sin C + \cos C = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}$$

$$\therefore \sin C + \cos C > 1$$

$$\sin^2 C + \cos^2 C = \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = \frac{16}{25} + \frac{9}{25} = 1$$

$$\therefore \sin^2 C + \cos^2 C < \sin C + \cos C$$

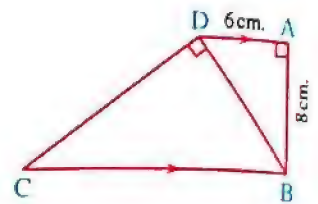
### Example 5 In the opposite figure :

ABCD is a quadrilateral in which :

$$m(\angle A) \cong m(\angle BDC) = 90^\circ$$

,  $\overline{AD} \parallel \overline{BC}$ ,  $AD = 6$  cm. and  $AB = 8$  cm.

Find the length of  $\overline{DC}$



#### Solution

In  $\triangle ABD$  :  $\therefore m(\angle A) = 90^\circ$

$$\therefore (DB)^2 = (AB)^2 + (AD)^2 = 64 + 36 = 100$$

$$\therefore DB = 10 \text{ cm.}$$

,  $\therefore \overline{AD} \parallel \overline{BC}$  and  $\overline{BD}$  is a transversal

$$\therefore m(\angle ADB) = m(\angle DBC) \text{ "Alternate angles"}$$

$$\therefore \tan(\angle ADB) = \tan(\angle DBC)$$

$$\therefore \frac{AB}{AD} = \frac{DC}{BD}$$

$$\therefore \frac{8}{6} = \frac{DC}{10}$$

$$\therefore DC = \frac{10 \times 8}{6} = 13\frac{1}{3} \text{ cm.}$$

#### Notice that :

Also , we can solve this example by using the similarity

### TRY 3 by yourself

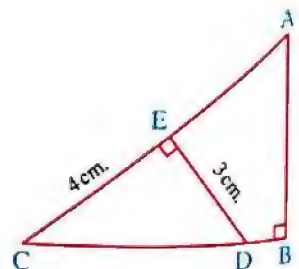
In the opposite figure :

ABC is a triangle in which :

$$m(\angle B) = 90^\circ, D \in \overline{BC}, E \in \overline{AC}$$

where  $\overline{DE} \perp \overline{AC}$ ,  $DE = 3$  cm. and  $EC = 4$  cm.

**Prove that :**  $\sin A \cos C + \sin C \cos(\angle EDC) = 1$

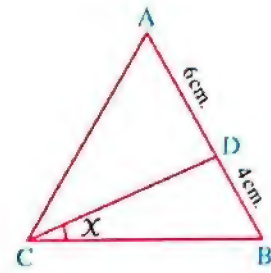


**Example 6** In the opposite figure :

ABC is an equilateral triangle ,  $D \in \overline{AB}$

where :  $AD = 6 \text{ cm.}$  ,  $DB = 4 \text{ cm.}$  ,

if  $k \tan X = \sqrt{3}$  , find the value of : k

**Solution**

**Construction :** Draw  $\overline{DE} \perp \overline{BC}$  and intersects it at E

**Proof :**  $\because$  ABC is an equilateral triangle

$$\therefore m(\angle B) = 60^\circ$$

$$\text{In } \triangle BDE : \because \overline{DE} \perp \overline{BC} \therefore m(\angle DEB) = 90^\circ$$

$$\therefore m(\angle BDE) = 180^\circ - (60^\circ + 90^\circ) = 30^\circ$$

$$\therefore BE = \frac{1}{2} BD = 2 \text{ cm.}$$

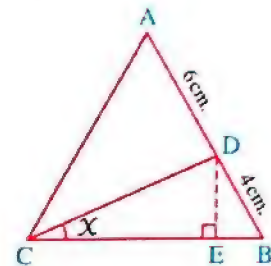
$$\therefore (DE)^2 = (DB)^2 - (BE)^2 \text{ (Pythagoras' theorem)}$$

$$\therefore (DE)^2 = 16 - 4 = 12 \quad \therefore DE = \sqrt{12} = 2\sqrt{3} \text{ cm.}$$

$$\therefore BC = BA = 10 \text{ cm.} \quad \therefore EC = BC - BE = 10 - 2 = 8 \text{ cm.}$$

$$\therefore \tan X = \frac{DE}{EC} = \frac{2\sqrt{3}}{8} = \frac{\sqrt{3}}{4} \quad , \therefore k \tan X = \sqrt{3}$$

$$\therefore k \times \frac{\sqrt{3}}{4} = \sqrt{3} \quad \therefore k = \sqrt{3} \times \frac{4}{\sqrt{3}} = 4 \quad \text{(The req.)}$$

**TRY 4**  
by yourself

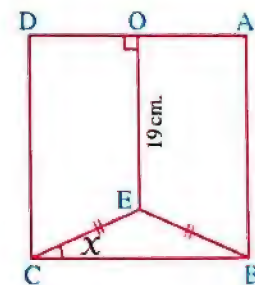
In the opposite figure :

ABCD is a square of side length is 24 cm. ,

E is a point inside it where :  $BE = CE$  ,  $OE = 19 \text{ cm.}$

,  $\overline{OE} \perp \overline{AD}$

If  $k(\cos X - \sin X) = \frac{1}{13}$  , find the value of : k



4  $\frac{7}{1}$

3 Prove by yourself [Hint :  $DC = 5 \text{ cm.}$  ,  $m(\angle A) = m(\angle EDC)$ ]

2  $\frac{25}{24}$  Prove by yourself

1  $56^\circ 15'$  ,  $123^\circ 45'$

Answers of try by yourself

## LESSON

# 2

## The main trigonometrical ratios of some angles

### The main trigonometrical ratios of the angles measuring $30^\circ$ and $60^\circ$

In the opposite figure :

ABC is a right-angled triangle at B in which :  $m(\angle A) = 60^\circ$  and  $m(\angle C) = 30^\circ$  and it is called "thirty and sixty triangle".

And in it , the length of the side opposite to the angle of measure  $30^\circ$  equals half the length of the hypotenuse.

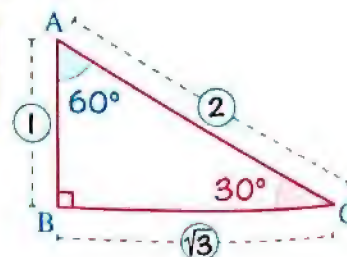
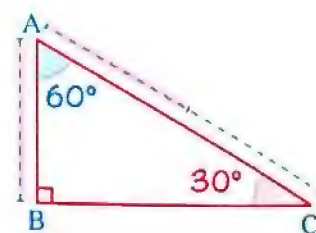
$$\text{i.e. } AB = \frac{1}{2} AC$$

Assume that : The length of  $\overline{AB} = l$  unit length , then the length of  $\overline{AC} = 2l$  unit length. By applying Pythagoras' theorem to find the length of  $\overline{BC}$  , we find that :

$$BC = \sqrt{(AC)^2 - (AB)^2} = \sqrt{4l^2 - l^2} = \sqrt{3l^2} = \sqrt{3}l \text{ unit length.}$$

$$\text{i.e. } AB : AC : BC = l : 2l : \sqrt{3}l = 1 : 2 : \sqrt{3}$$

And from  $\Delta ABC$  , we can find the main trigonometrical ratios of the angles measuring  $30^\circ$  and  $60^\circ$  as follows :



$30^\circ$ $\sin 30^\circ = \frac{AB}{AC} = \frac{1}{2}$	$\cos 30^\circ = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$	$\tan 30^\circ = \frac{AB}{BC} = \frac{1}{\sqrt{3}}$
$60^\circ$ $\sin 60^\circ = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$	$\cos 60^\circ = \frac{AB}{AC} = \frac{1}{2}$	$\tan 60^\circ = \frac{BC}{AB} = \sqrt{3}$

### The main trigonometrical ratios of the angle measuring $45^\circ$

In the opposite figure :

ABC is an isosceles triangle where  $AC = BC = l$  unit length

and  $m(\angle C) = 90^\circ$

$$\therefore m(\angle A) = m(\angle B) = 45^\circ$$

By applying Pythagoras' theorem to find the length of  $\overline{AB}$

, we find that :

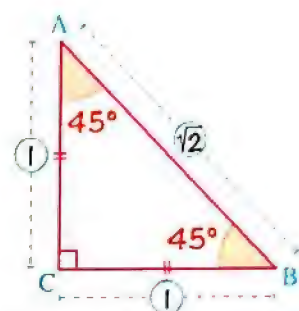
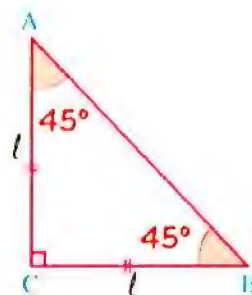
$$AB = \sqrt{(AC)^2 + (BC)^2}$$

$$= \sqrt{l^2 + l^2} = \sqrt{2l^2} = \sqrt{2} l \text{ unit length.}$$

i.e.  $AC : BC : AB = l : l : \sqrt{2} l = 1 : 1 : \sqrt{2}$

From  $\triangle ABC$ , we can find the main trigonometrical ratios of the angle measuring  $45^\circ$  as follows :

$45^\circ$	$\sin 45^\circ = \frac{1}{\sqrt{2}}$	$\cos 45^\circ = \frac{1}{\sqrt{2}}$	$\tan 45^\circ = 1$
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\* And the following table summarizes to us the main trigonometrical ratios of the angles whose measures are  $30^\circ$ ,  $60^\circ$  and  $45^\circ$

The trigonometrical ratio \ The measure of the angle	$30^\circ$	$60^\circ$	$45^\circ$
sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$
tan	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1

**Example 1** Find the value of :  $\sin 30^\circ \cos 60^\circ + \cos^2 30^\circ + 5 \tan 45^\circ - 10 \cos^2 45^\circ$

**Solution**

$$\begin{aligned} \text{The expression} &= \frac{1}{2} \times \frac{1}{2} + \left(\frac{\sqrt{3}}{2}\right)^2 + 5 \times 1 - 10 \times \left(\frac{1}{\sqrt{2}}\right)^2 \\ &= \frac{1}{4} + \frac{3}{4} + 5 - \frac{10}{2} = 1 + 5 - 5 = 1 \end{aligned}$$

**Example 2** Prove that :  $\sin^2 60^\circ + \sin^2 45^\circ + \sin^2 30^\circ = \cos^2 30^\circ + \frac{1}{3} \tan^2 60^\circ - \cos^2 60^\circ$

**Solution**

$$\text{L.H.S.} = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{2} + \frac{1}{4} = \frac{3}{2}$$

$$\text{R.H.S.} = \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{3} (\sqrt{3})^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} + 1 - \frac{1}{4} = \frac{3}{2}$$

$\therefore$  The two sides are equal.

**TRY 1**

by yourself

- 1 Find the value of : (1)  $\cos 60^\circ + \sin 30^\circ - \tan 45^\circ$  (2)  $\sin^2 30^\circ + \sin^2 60^\circ$   
 2 Prove that :  $2 \sin 30^\circ + 4 \cos 60^\circ = \tan^2 60^\circ$

**Example 3** Find the value of  $X$  which satisfies :

- 1  $X \sin 30^\circ \cos^2 45^\circ = \cos^2 30^\circ$   
 2  $2 \sin X = \tan^2 60^\circ - 2 \tan 45^\circ$  where  $X$  is the measure of an acute angle.

**Solution**

- 1  $\therefore X \sin 30^\circ \cos^2 45^\circ = \cos^2 30^\circ \therefore X \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$   
 $\therefore \frac{1}{4} X = \frac{3}{4} \therefore X = 3$   
 2  $\therefore 2 \sin X = \tan^2 60^\circ - 2 \tan 45^\circ \therefore 2 \sin X = (\sqrt{3})^2 - 2 \times 1 = 3 - 2 = 1$   
 $\therefore \sin X = \frac{1}{2} \therefore X = 30^\circ$

**TRY 2**

by yourself

Find the value of  $X$  which satisfies :

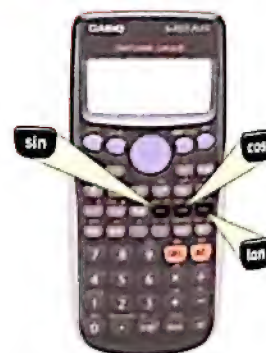
- 1  $X \cos 30^\circ = \tan 60^\circ$  2  $\tan X = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$ , such that  $X$  is an acute angle.

**Using the calculator****First****Finding the main trigonometrical ratios of a given angle :**

In the calculator , there are three keys : **sin** , **cos** , **tan**

- 1 The key **sin** means sine.  
 2 The key **cos** means cosine.  
 3 The key **tan** means tangent.

By using these keys we can find the main trigonometrical ratios of any angle if its measure is known.



**Example 4** By using the calculator , find the value of each of the following approximated to the nearest 4 decimals :

- 1  $\sin 36^\circ$  2  $\cos 72^\circ 35'$  3  $\tan 50^\circ 46' 25''$

**Solution**

Use the keys of the calculator as the following sequence from left :

1    

$$\therefore \sin 36^\circ \approx 0.5878$$

2        

$$\therefore \cos 72^\circ 35' \approx 0.2993$$

3           

$$\therefore \tan 50^\circ 46' 25'' \approx 1.2250$$

**TRY 3**

by yourself

By using the calculator, find the value of each of the following approximated to the nearest 3 decimals :

1  $\sin 35^\circ 12'$

2  $\tan 58^\circ 24'$

**Second**

**Finding the measure of the angle if one of its trigonometrical ratios is given :**

If  $\sin A = 0.6218$  , then A is the measure of the angle whose sine is 0.6218

To find the measure of this angle , we can use the calculator as the following sequence from left :

          Then  $A \approx 38^\circ 26' 52''$

**Example 5** Find A in each of the following , where A is the measure of an acute angle :

1  $\sin A = 0.8$

2  $\cos A = 0.7152$

3  $\tan A = 1.5156$

**Solution**

Use the keys of the calculator as the following sequence from left :

1       

$$\therefore A \approx 53^\circ 7' 48''$$

2          

$$\therefore A \approx 44^\circ 20' 25''$$

3           

$$\therefore A \approx 56^\circ 34' 59''$$

**TRY 4**

by yourself

Using the calculator , find A in each of the following where A is the measure of an acute angle :

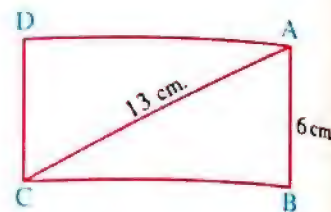
1  $\sin A = 0.3945$

2  $\cos A = 0.3824$

**Example 6** In the opposite figure :

ABCD is a rectangle in which :  
 $AB = 6$  cm. and  $AC = 13$  cm. **Find :**

- 1  $m(\angle ACB)$
- 2 The area of the rectangle ABCD to the nearest one decimal digit.

**Solution**

$\therefore$  ABCD is a rectangle.  $\therefore m(\angle B) = 90^\circ$

$$\text{In } \triangle ABC : \sin(\angle ACB) = \frac{AB}{AC} = \frac{6}{13}$$

And by using the calculator :  $\therefore m(\angle ACB) \approx 27^\circ 29' 11''$  (First req.)

$$\therefore \cos(\angle ACB) = \frac{BC}{AC}$$

$$\therefore \cos 27^\circ 29' 11'' = \frac{BC}{13}$$

$$\therefore BC = 13 \times \cos 27^\circ 29' 11''$$

$\therefore$  The area of the rectangle ABCD  $= AB \times BC$

$$= 6 \times 13 \times \cos 27^\circ 29' 11'' \approx 69.2 \text{ cm}^2$$

(Second req.)

**Notice that :**

Also, we can find the length of  $\overline{BC}$  by using Pythagoras' theorem in  $\triangle ABC$

**TRY 5**

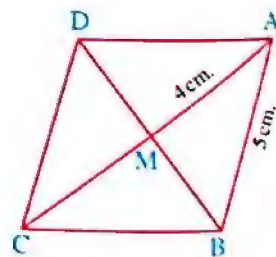
by yourself

In the opposite figure :

ABCD is a rhombus, whose diagonals intersect at M, if  $AB = 5$  cm.,  $AM = 4$  cm.

**Find :**

- 1  $m(\angle BAD)$
- 2 The area of the rhombus ABCD



- Answers**
- 1 (1) zero (2) 1
- 2 1 2
- 3 1 0.576
- 4 1 23° 14' 5" (approximately)
- 5 1 73° 44' 23" (approximately)

- 2 Prove by yourself
- 2 60°
- 2 1.625
- 2 67° 31' 3" (approximately)
- 2 24 cm²

or try by yourself

# UNIT 5

## Analytical geometry



### ▶ Lessons of the unit :

1. Distance between two points.
2. The two coordinates of the midpoint of a line segment.
3. The slope of the straight line.
4. The equation of the straight line given its slope and the intercepted part of y-axis.

▶ Use your smart phone or tablet to scan the QR Code and enjoy watching videos.



## ► Unit Objectives :

**By the end of this unit, student should be able to :**

- find the distance between two points in the coordinates plane.
- find the two coordinates of the midpoint of a line segment.
- recognize the slope of the straight line.
- find the slope of the straight line given the measure of the positive angle which this straight line makes with the positive direction of the  $x$ -axis.
- recognize the relation between the two slopes of the two parallel straight lines.
- recognize the relation between the two slopes of the two perpendicular straight lines.
- find the slope of the straight line and the length of the intercepted part from  $y$ -axis given the equation of the straight line.
- find the equation of the straight line given its slope and the length of the intercepted part from  $y$ -axis.
- use the slope of the straight line for solving some life problems.

## LESSON

# 1

## Distance between two points

Let  $M(x_1, y_1)$  and  $N(x_2, y_2)$  be two points in the same coordinates plane.

From the geometry of the figure we find that :



$$NL = NB - LB = y_2 - y_1$$

$$\text{Generally } NL = |y_2 - y_1|$$

$$\text{Similarly } LM = BO - AO = x_2 - x_1$$

$$\text{Generally } LM = |x_2 - x_1|$$

$\therefore \triangle NLM$  is right-angled at  $L$

$$\therefore (MN)^2 = (LM)^2 + (NL)^2$$

$$\therefore (MN)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

i.e.

The distance between the two points  $M$  and  $N$  equals  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

and we know that :

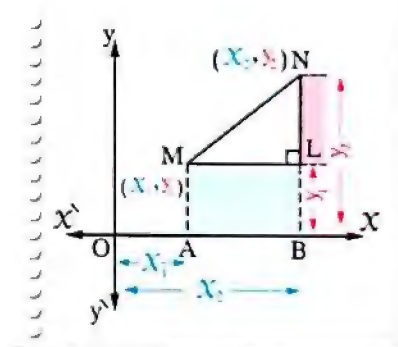
$$(x_2 - x_1)^2 = (x_1 - x_2)^2, \text{ and similarly : } (y_2 - y_1)^2 = (y_1 - y_2)^2, \text{ therefore :}$$

The distance between the two points  $M$  and  $N$  equals also  $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Generally :

The distance between two points =

$\sqrt{\text{square of the difference between } x\text{-coordinates} + \text{square of the difference between } y\text{-coordinates}}$



**Example 1** If A (3, 6), B (-1, 4), find the length of  $\overline{AB}$

**Solution**

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1 - 3)^2 + (4 - 6)^2} = \sqrt{(-4)^2 + (-2)^2} \\ = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5} \text{ length unit.}$$

Another solution :  $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(3 - (-1))^2 + (6 - 4)^2} \\ = \sqrt{4^2 + 2^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5} \text{ length unit.}$

**Example 2** If the distance between the two points (a, 5) and (3a - 1, 1) equals 5 length units. Find the value of : a

**Solution**

$$\therefore \sqrt{(3a - 1 - a)^2 + (1 - 5)^2} = 5$$

$$\therefore \sqrt{(2a - 1)^2 + (-4)^2} = 5 \quad \text{"Squaring the two sides"}$$

$$\therefore (2a - 1)^2 + 16 = 25$$

$$\therefore (2a - 1)^2 = 9 \quad \text{"Taking the square root of the two sides"}$$

$$\therefore 2a - 1 = \pm 3$$

$$\therefore 2a - 1 = 3$$

$$\text{thus, } 2a = 4$$

$$\therefore a = 2$$

$$\text{or } 2a - 1 = -3$$

$$\text{thus, } 2a = -2$$

$$\therefore a = -1$$

**TRY 1**  
by yourself

If A (2, 5) and B (-1, 1) Find the length of :  $\overline{AB}$

**Example 3** If ABC is a triangle where A (0, 0), B (3, 4) and C (-4, 3), find the perimeter of  $\triangle ABC$

**Solution**

$$\therefore \text{The perimeter of } \triangle ABC = AB + BC + CA$$

$$, AB = \sqrt{(3 - 0)^2 + (4 - 0)^2} = \sqrt{3^2 + 4^2} \\ = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ length unit.}$$

$$, BC = \sqrt{(-4 - 3)^2 + (3 - 4)^2} \\ = \sqrt{(-7)^2 + (-1)^2} \\ = \sqrt{49 + 1} = \sqrt{50} = 5\sqrt{2} \text{ length unit.}$$

$$, CA = \sqrt{(-4)^2 + (3)^2} \quad \text{"according to the previous beside noticing"} \\ = \sqrt{16 + 9} = \sqrt{25} = 5 \text{ length unit.}$$

$$\therefore \text{The perimeter of } \triangle ABC = 5 + 5\sqrt{2} + 5 = (10 + 5\sqrt{2}) \text{ length unit.}$$

**Notice that :**

The distance between the point M (x, y) and the origin point O (0, 0) is  $OM = \sqrt{x^2 + y^2}$

**Example 4** Prove that :  $\Delta ABC$  is an equilateral triangle where :  $A(6, 0)$  ,  $B(2, 0)$  and  $C(4, 2\sqrt{3})$  , then find its area.

**Solution**

$$\therefore AB = \sqrt{(6-2)^2 + (0-0)^2} = \sqrt{16+0} = \sqrt{16} = 4 \text{ length unit.}$$

$$, BC = \sqrt{(2-4)^2 + (0-2\sqrt{3})^2} = \sqrt{4+12} = \sqrt{16} = 4 \text{ length unit.}$$

$$\begin{aligned} \text{and } AC &= \sqrt{(6-4)^2 + (0-2\sqrt{3})^2} \\ &= \sqrt{4+12} = \sqrt{16} = 4 \text{ length unit.} \end{aligned}$$

$$\therefore AB = BC = AC \quad \therefore \Delta ABC \text{ is equilateral}$$

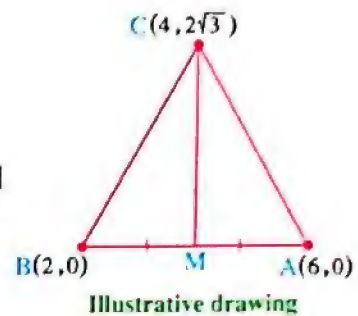
, let M be the midpoint of the base  $\overline{AB}$

$$\therefore \overline{CM} \perp \overline{AB}$$

$\therefore$  By using Pythagoras' theorem , we find that :

$$\therefore \text{The height } MC = \sqrt{(AC)^2 - (AM)^2} = \sqrt{4^2 - 2^2} = \sqrt{12} = 2\sqrt{3} \text{ length unit}$$

$$\therefore \text{The area of } \Delta ABC = \frac{1}{2} \times AB \times MC = \frac{1}{2} \times 4 \times 2\sqrt{3} = 4\sqrt{3} \text{ square unit.}$$



**TRY 2**  
yourself

**Prove that :**  $\Delta ABC$  is an isosceles triangle where :  $A(3, 3)$  ,  $B(5, 9)$  and  $C(-1, 7)$

### Remark 1

To prove that three given points are collinear (i.e. they lie on one straight line) we can find the distance between each two of these points , then prove that the greatest distance equals the sum of the two other distances.

**Example 5** Prove that : The points  $A(-2, 7)$  ,  $B(-3, 4)$  and  $C(1, 16)$  are collinear.

**Solution**

$$\therefore AB = \sqrt{(-2+3)^2 + (7-4)^2} = \sqrt{1+9} = \sqrt{10} \text{ length unit.}$$

$$, BC = \sqrt{(-3-1)^2 + (4-16)^2} = \sqrt{16+144} = \sqrt{160} = 4\sqrt{10} \text{ length unit.}$$

$$\text{and } AC = \sqrt{(-2-1)^2 + (7-16)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10} \text{ length unit.}$$

$$\therefore BC = AB + AC$$

$\therefore A, B$  and  $C$  are collinear.

**Remark 2**

- To prove that the points : A , B and C are vertices of a triangle , we can find AB , BC and AC , then prove that the sum of the smaller two lengths is greater than the third length.
- To determine the type of the triangle ABC according to its angle measures (where  $\overline{AC}$  is the longest side of the triangle ABC )

We compare between  $(AC)^2$  and  $(AB)^2 + (BC)^2$  as the following :

- 1 If  $(AC)^2 > (AB)^2 + (BC)^2$  , then the triangle is obtuse-angled at B
- 2 If  $(AC)^2 = (AB)^2 + (BC)^2$  , then the triangle is right-angled at B
- 3 If  $(AC)^2 < (AB)^2 + (BC)^2$  , then the triangle is acute-angled.

**Example 6** Prove that : The triangle whose vertices are A (3 , 2) , B (-4 , 1) and C (2 , -1) is right-angled , then find its area.

**Solution**

$$\therefore AB = \sqrt{(3+4)^2 + (2-1)^2}$$

$$= \sqrt{49+1} = \sqrt{50} \text{ length unit.}$$

$$\therefore BC = \sqrt{(-4-2)^2 + (1+1)^2}$$

$$= \sqrt{36+4} = \sqrt{40} \text{ length unit.}$$

$$\text{and } AC = \sqrt{(3-2)^2 + (2+1)^2}$$

$$= \sqrt{1+9} = \sqrt{10} \text{ length unit.}$$

$$\therefore (AC)^2 + (BC)^2 = 10 + 40 = 50$$

$$\therefore (AB)^2 = 50$$

$$\therefore (AC)^2 + (BC)^2 = (AB)^2$$

$$\therefore \Delta ABC \text{ is right-angled at C}$$

$$\therefore \text{The area of the triangle } ABC = \frac{1}{2} AC \times BC$$

$$= \frac{1}{2} \times \sqrt{10} \times \sqrt{40}$$

$$= \frac{1}{2} \times \sqrt{10} \times 2\sqrt{10} = 10 \text{ square unit.}$$

**TRY 3**  
by yourself

If A (-1 , -1) , B (2 , 3) and C (6 , 0)

**Prove that :**  $\Delta ABC$  is right-angled triangle , then find its area.

### Remark 3

If ABCD is a quadrilateral :

- 1 To prove that ABCD is a parallelogram , we prove that :  $AB = CD$  ,  $BC = AD$
- 2 To prove that ABCD is a rhombus , we prove that :  $AB = BC = CD = DA$
- 3 To prove that ABCD is a rectangle , we prove that :  $AB = CD$  ,  $BC = AD$  ,  $AC = BD$
- 4 To prove that ABCD is a square , we prove that :  $AB = BC = CD = DA$  ,  $AC = BD$  ”

**Example 7** If A (3 , -2) , B (-5 , 0) , C (0 , -7) and D (8 , -9) ,  
prove that : ABCD is a parallelogram.

**Solution**

$$\begin{aligned}\therefore AB &= \sqrt{(3+5)^2 + (-2-0)^2} = \sqrt{64+4} \\ &= \sqrt{68} \text{ length unit.}\end{aligned}$$

$$\begin{aligned}, BC &= \sqrt{(-5-0)^2 + (0+7)^2} = \sqrt{25+49} \\ &= \sqrt{74} \text{ length unit.}\end{aligned}$$

$$\begin{aligned}, CD &= \sqrt{(0-8)^2 + (-7+9)^2} = \sqrt{64+4} \\ &= \sqrt{68} \text{ length unit.}\end{aligned}$$

$$\text{and } DA = \sqrt{(8-3)^2 + (-9+2)^2} = \sqrt{25+49} = \sqrt{74} \text{ length unit.}$$

$$\therefore AB = CD , BC = DA \quad \therefore ABCD \text{ is a parallelogram.}$$

**Example 8** Prove that : The points A (-1 , 4) , B (1 , 1) , C (-1 , -2) and D (-3 , 1) are vertices of a rhombus and graph it , then find its area.

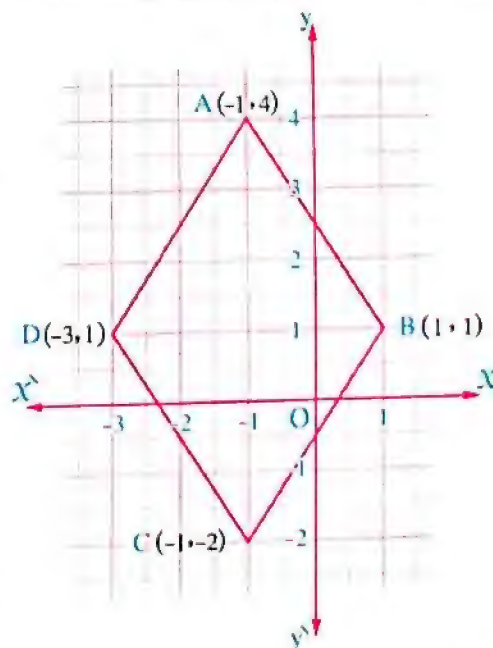
**Solution**

$$\begin{aligned}\therefore AB &= \sqrt{(-1-1)^2 + (4-1)^2} \\ &= \sqrt{4+9} = \sqrt{13} \text{ length unit.}\end{aligned}$$

$$\begin{aligned}, BC &= \sqrt{(1+1)^2 + (1+2)^2} \\ &= \sqrt{4+9} = \sqrt{13} \text{ length unit.}\end{aligned}$$

$$\begin{aligned}, CD &= \sqrt{(-1+3)^2 + (-2-1)^2} \\ &= \sqrt{4+9} = \sqrt{13} \text{ length unit.}\end{aligned}$$

$$\begin{aligned}\text{and } DA &= \sqrt{(-3+1)^2 + (1-4)^2} \\ &= \sqrt{4+9} = \sqrt{13} \text{ length unit.}\end{aligned}$$



$$\therefore AB = BC = CD = DA$$

$\therefore$  The quadrilateral ABCD is a rhombus.

$$\therefore AC = \sqrt{(-1+1)^2 + (4+2)^2} = \sqrt{0+36} = \sqrt{36} = 6 \text{ length unit.}$$

$$, BD = \sqrt{(1+3)^2 + (1-1)^2} = \sqrt{16+0} = \sqrt{16} = 4 \text{ length unit.}$$

$$\therefore \text{The area of the rhombus ABCD} = \frac{1}{2} \times 6 \times 4 = 12 \text{ square unit.}$$

### TRY 4

by yourself

**Prove that :** The points A (− 1 , 3) , B (5 , 1) , C (6 , 4) and D (0 , 6) are vertices of a rectangle , then find its area.

### Remark 4

- The axis of symmetry of a line segment is the straight line that perpendicular to it at its middle.
- Any point on the axis of symmetry of a line segment is at equal distances from its terminals.

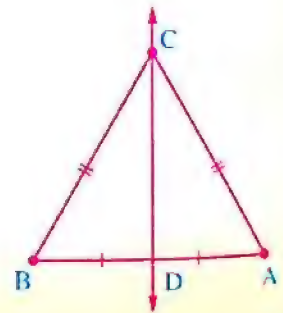
The converse is true , **i.e.** If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.

**For example:**

**In the opposite figure :**

If  $CA = CB$

, then  $C \in$  axis of symmetry of  $\overline{AB}$



**Example 9** If A (1 , − 1) , B (1 , 3)

**Prove that :** The point C (− 1 , 1) lies on the axis of symmetry of  $\overline{AB}$

**Solution**

$$\therefore CA = \sqrt{(-1-1)^2 + (1+1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ length unit.}$$

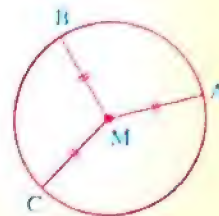
$$, \therefore CB = \sqrt{(-1-1)^2 + (1-3)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ length unit.}$$

$$\therefore CA = CB$$

$\therefore$  The point C lies on the axis of symmetry of  $\overline{AB}$

### Remark 5

- To prove that : Three points as A , B and C lie on a same circle of centre M  
we prove that :  $MA = MB = MC$
- If  $A \in$  the circle M , then the radius length of this circle ( $r$ ) = MA
- Remember that :
  - Circumference of the circle =  $2 \pi r$
  - Area of the circle =  $\pi r^2$



**Example 10** Prove that : The points A (− 6 , 2) , B (0 , 8) and C (− 8 , 4) lie on the circle whose centre is M (− 4 , 6) and find its area where  $\pi \approx 3.14$

#### Solution

$$\therefore MA = \sqrt{(-6 + 4)^2 + (2 - 6)^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5} \text{ length units.}$$

$$, MB = \sqrt{(0 + 4)^2 + (8 - 6)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5} \text{ length units.}$$

$$\text{and } MC = \sqrt{(-8 + 4)^2 + (4 - 6)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5} \text{ length units.}$$

$$\therefore MA = MB = MC$$

$\therefore$  The points A , B and C lie on the circle M whose radius length

$$r = 2\sqrt{5} \text{ length units.}$$

$$\therefore \text{The area of the circle M} = \pi r^2 \approx 3.14 \times (2\sqrt{5})^2 \approx 62.8 \text{ square units.}$$

### TRY 5

**Prove that :** The points A (− 2 , 0) , B (5 , 1) , C (6 , − 6) passing through a circle whose centre is M (2 , − 3) and find the circumference of the circle in terms of  $\pi$

**5** Prove by yourself [Hint : prove that :  $MA = MB = MC$ , the circumference =  $10 \pi$  length units.]

**4** Prove by yourself [Hint : prove that :  $AB = CD$ ,  $BC = AD$ ,  $AC = BD$ , the area = 20 square units.]

**3** Prove by yourself [Hint : prove that  $(AC)^2 = (AB)^2 + (BC)^2$ , the area = 12.5 square units.]

**2** Prove by yourself [Hint : find AB , BC and CA]

**1** 5 length units.

of try by yourself

Answers

## LESSON

# 2

## The two coordinates of the midpoint of a line segment

If A  $(x_1, y_1)$  and B  $(x_2, y_2)$  are two points in a coordinates plane and M  $(x, y)$  is the midpoint of  $\overline{AB}$



From the opposite figure :

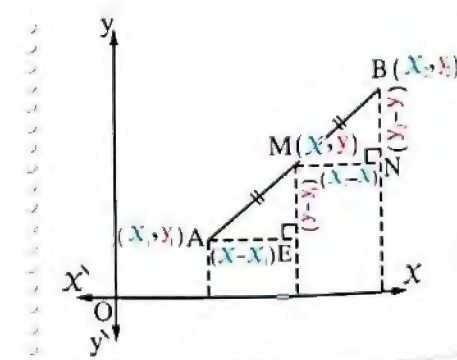
$\triangle AEM, \triangle MNB$  are congruent

$$\therefore AE = MN, \quad EM = NB$$

$$\therefore x - x_1 = x_2 - x, \quad y - y_1 = y_2 - y$$

$$\therefore 2x = x_1 + x_2, \quad 2y = y_1 + y_2$$

$$\therefore x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}$$



$$\therefore M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

For example:

If X  $(3, -2)$  , Y  $(-1, -4)$  and M is the midpoint of  $\overline{XY}$  , then :

$$M = \left( \frac{3 + (-1)}{2}, \frac{-2 + (-4)}{2} \right) = (1, -3)$$

**Example 1** If C (10, -4) is the midpoint of  $\overline{AB}$  where A (4, -2), find the point B

**Solution**

Let B (X, y)

$\therefore$  C is the midpoint of  $\overline{AB}$

$$\therefore (10, -4) = \left( \frac{X+4}{2}, \frac{y+(-2)}{2} \right)$$

$$\therefore \frac{X+4}{2} = 10$$

$$\therefore X+4 = 20$$

$$\therefore X = 16$$

$$\therefore \frac{y-2}{2} = -4$$

$$\therefore y-2 = -8$$

$$\therefore y = -6$$

$$\therefore B = (16, -6)$$

**Notice that :**

If (a, b) = (c, d), then

a = c, b = d

**TRY**

**1**

If C is the midpoint of  $\overline{AB}$ , then find the value of each of X and y in each of the following :

1 A (2, 5), B (-2, -3), C (X, y)

2 A (X, 4), B (-1, -6), C (-2, y)

**Remark**

If  $\overline{AB}$  is a diameter in a circle of centre M, then M is the midpoint of  $\overline{AB}$

**Example 2** If  $\overline{AB}$  is a diameter in the circle M where A (4, -1), B (-2, 7), find the point M, then find the circumference and the area of the circle.

**Solution**

$\therefore \overline{AB}$  is a diameter in the circle M  $\therefore$  M is the midpoint of  $\overline{AB}$

$$\therefore \text{The point M} = \left( \frac{4+(-2)}{2}, \frac{-1+7}{2} \right) = (1, 3)$$

$$\therefore r = AM = \sqrt{(1-4)^2 + (3+1)^2} = \sqrt{9+16} = \sqrt{25} = 5 \text{ length units.}$$

$$\therefore \text{The circumference of the circle} = 2\pi r = 2\pi \times 5 = 10\pi \text{ length units.}$$

$$\therefore \text{the area of the circle} = \pi r^2 = \pi \times 5^2 = 25\pi \text{ square units.}$$

**Another method to calculate the radius length of the circle :**

$$\therefore AB = \sqrt{(-2-4)^2 + (7+1)^2} = \sqrt{36+64} = \sqrt{100} = 10 \text{ length units.}$$

$$\therefore \overline{AB} \text{ is a diameter} \therefore r = \frac{1}{2} AB = 5 \text{ length units.}$$

$\therefore$  then complete the solution to find the circumference and the area of the circle.

**TRY**

**2**

If  $\overline{AB}$  is a diameter in the circle M where A (4, 1) and B (-6, 3), then find the point M

**Example 3** Prove that : The quadrilateral ABCD is a parallelogram where A (4 , 3) , B (0 , 2) , C (- 2 , - 3) and D (2 , - 2)

**Solution**

$\therefore$  The two diagonals of the quadrilateral are  $\overline{AC}$  and  $\overline{BD}$

, the midpoint of  $\overline{AC} = \left( \frac{4 + (-2)}{2}, \frac{3 + (-3)}{2} \right) = (1, 0)$

and the midpoint of  $\overline{BD} = \left( \frac{0 + 2}{2}, \frac{2 + (-2)}{2} \right) = (1, 0)$

$\therefore$  The midpoint of  $\overline{AC}$  is the same midpoint of  $\overline{BD}$

$\therefore$  The two diagonals bisect each other.

$\therefore$  ABCD is a parallelogram.

**Notice that :**

You can solve this example by using the distance between two points as the previous.

**Example 4** Prove that : The points A (5 , 1) , B (1 , - 3) and C (- 5 , 3) are the vertices of a right-angled triangle at B , then find the point D that makes the figure ABCD a rectangle.

**Solution**

$\therefore AB = \sqrt{(1-5)^2 + (-3-1)^2} = \sqrt{16+16} = \sqrt{32}$  length unit.

,  $BC = \sqrt{(-5-1)^2 + (3+3)^2} = \sqrt{36+36} = \sqrt{72}$  length unit.

,  $AC = \sqrt{(-5-5)^2 + (3-1)^2} = \sqrt{100+4} = \sqrt{104}$  length unit.

$\therefore (AB)^2 + (BC)^2 = 32 + 72 = 104 = (AC)^2$

$\therefore \Delta ABC$  is a right-angled triangle at B

Let : D (X , y) in which the figure ABCD is a rectangle.

$\therefore \overline{AC}$  and  $\overline{BD}$  bisect each other.

$\therefore$  The midpoint of  $\overline{AC}$  = the midpoint of  $\overline{BD}$

,  $\therefore$  the midpoint of  $\overline{AC} = \left( \frac{5-5}{2}, \frac{1+3}{2} \right) = (0, 2)$

, the midpoint of  $\overline{BD} = \left( \frac{X+1}{2}, \frac{y-3}{2} \right)$

$\therefore \left( \frac{X+1}{2}, \frac{y-3}{2} \right) = (0, 2)$

$$\therefore \frac{x+1}{2} = 0$$

$$\therefore x+1=0$$

$$\therefore x = -1$$

$$\therefore \frac{y-3}{2} = 2$$

$$\therefore y-3=4$$

$$\therefore y = 7$$

$$\therefore D = (-1, 7)$$

**Example 5** Prove that : The triangle whose vertices are A (-1, 4) , B (3, 1) and C (-5, 1) is an isosceles triangle , then find its area.

**Solution**

$$\therefore AB = \sqrt{(3+1)^2 + (1-4)^2} = \sqrt{16+9}$$

$$= 5 \text{ length unit.}$$

$$\therefore BC = \sqrt{(3+5)^2 + (1-1)^2} = \sqrt{64} = 8 \text{ length unit.}$$

$$\therefore AC = \sqrt{(-5+1)^2 + (1-4)^2} = \sqrt{16+9}$$

$$= 5 \text{ length unit.}$$

$$\therefore AB = AC$$

$\therefore \triangle ABC$  is an isosceles triangle.

Let D (x, y) be the midpoint of  $\overline{BC}$

$$\therefore D = \left( \frac{3-5}{2}, \frac{1+1}{2} \right) = (-1, 1)$$

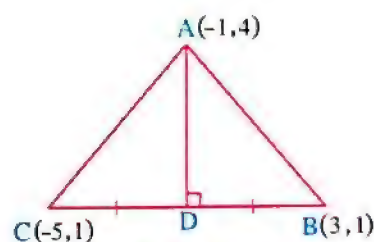
$\therefore D$  is the midpoint of  $\overline{BC}$

$$\therefore \overline{AD} \perp \overline{BC}$$

$$\therefore AD = \sqrt{(-1+1)^2 + (1-4)^2} = \sqrt{9} = 3 \text{ length unit.}$$

$$\therefore BC = 8 \text{ length unit}$$

$$\therefore \text{The area of } \triangle ABC = \frac{1}{2} BC \times AD = \frac{1}{2} \times 8 \times 3 = 12 \text{ square unit.}$$



Illustrative drawing

**TRY 3**

yourself

If C is the midpoint of  $\overline{AB}$  where A (2, 3) , B (4, -7) and C is the midpoint of  $\overline{DE}$  where D (-3, 5) , find the point E

$$(6, -6) \quad \boxed{3}$$

$$(7, 1) \quad \boxed{2}$$

$$1 = x, 0 = y \quad \boxed{1}$$

$$1 = x, -3 = y \quad \boxed{2}$$

of try by yourself

Answers

## LESSON

# 3

## The slope of the straight line

You studied before the slope of the straight line given two points on it.

If A and B are two points in the coordinates plane where A ( $x_1, y_1$ ) and B ( $x_2, y_2$ )

, then : The slope of the straight line  $\overleftrightarrow{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

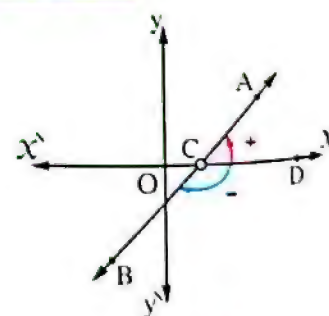
In this lesson , you will learn how to find the slope of the straight line given the measure of the positive angle which this straight line makes with the positive direction of the X-axis.

### The positive measure and the negative measure of an angle

In the opposite figure :

If  $\overleftrightarrow{AB}$  intersects the X-axis at the point C , then  $\overleftrightarrow{AB}$  makes two angles with the positive direction of the X-axis.

- One of them is positive (i.e. it has a positive measure) taken from the positive direction of the X-axis to the straight line in the direction of anticlockwise and it is  $\angle DCA$
- The another one is negative (i.e. it has a negative measure) taken from the positive direction of the X-axis to the straight line in the direction of clockwise and it is  $\angle DCB$



## The slope of the straight line

### Definition

The slope of the straight line is the tangent of the positive angle which this straight line makes with the positive direction of the  $X$ -axis.

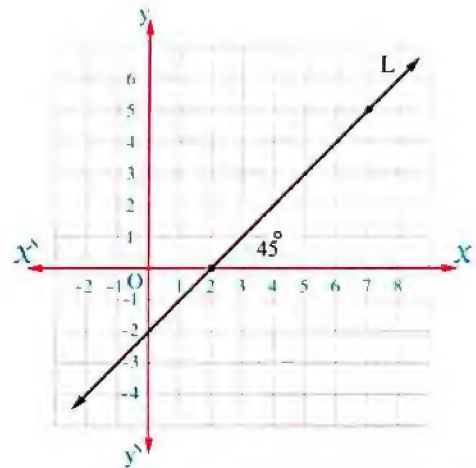
i.e. The slope of the straight line  $= \tan \theta$  where  $\theta$  is the measure of the positive angle which the straight line makes with the positive direction of the  $X$ -axis.

For example:

In the opposite figure :

The straight line  $L$  makes an angle of measure  $45^\circ$  with the positive direction of the  $X$ -axis, then :

the slope of the straight line  $L = \tan 45^\circ = 1$



### Notice that :

The straight line passes through the two points  $(2, 0)$  and  $(7, 5)$ , then :

$$\text{the slope of the straight line } L = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{7 - 2} = \frac{5}{5} = 1$$

### Remark

The angle which the straight line  $L$  makes with the positive direction of the  $X$ -axis takes one of the following cases :

① Acute angle	② Obtuse angle	③ Zero angle	④ Right angle
The slope is positive	The slope is negative	The slope is zero	The slope is undefined

**Example 1** Find the slope of the straight line which makes a positive angle with the positive direction of X-axis where the measure of the angle is :

1  $45^\circ$

2  $124^\circ 15' 12''$

**Solution**

1 The slope of the straight line  $= \tan 45^\circ = 1$

Start  $\rightarrow$  

2 The slope of the straight line  $= \tan 124^\circ 15' 12'' \approx -1.4685$

Start  $\rightarrow$  

**Example 2** Find the measure of the positive angle ( $\theta$ ) which the straight line makes with the positive direction of X-axis if the slope of the straight line is :

1 1.486

2  $-\frac{1}{\sqrt{3}}$

**Solution**

1  $\therefore m = \tan \theta$

$\therefore \tan \theta = 1.486$

$\therefore$  The slope is positive

$\therefore \angle \theta$  is an acute angle.

Start  $\rightarrow$  

$\therefore m(\angle \theta) \approx 56^\circ 3' 41''$

2  $\therefore m = \tan \theta$

$\therefore \tan \theta = -\frac{1}{\sqrt{3}}$

$\therefore$  The slope is negative

$\therefore \angle \theta$  is an obtuse angle.

By using the calculator as follows :

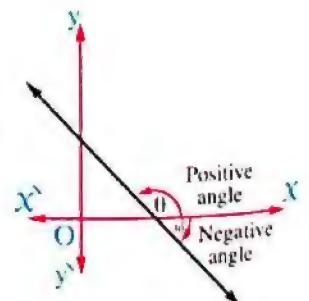
Start  $\rightarrow$  

We will find the calculator gives the result  $-30^\circ$

Where the calculator is programmed to get the acute angle only either negative or positive.

But the required is the positive angle, so we find  $m(\angle \theta)$  by finding the supplementary of the angle of measure  $30^\circ$

Then :  $m(\angle \theta) = 180^\circ - 30^\circ = 150^\circ$



**Example 3** Find the measure of the positive angle ( $\theta$ ) which the straight line L makes with the positive direction of X-axis if the straight line (L) passes through the two points :

1  $(-2, \sqrt{3}), (1, 4\sqrt{3})$

2  $(-2, 3), (-3, 4)$

**Solution**

1  $\therefore$  The straight line L passes through the two points  $(-2, \sqrt{3}), (1, 4\sqrt{3})$

$\therefore$  The slope of the straight line L

$$= \frac{4\sqrt{3} - \sqrt{3}}{1 - (-2)} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

**Notice that :**

The slope is positive, then the angle is acute.

Start  $\rightarrow$  

$\therefore m(\angle \theta) = 60^\circ$

2  $\therefore$  The straight line passes through the two points  $(-2, 3)$  and  $(-3, 4)$

$\therefore$  The slope of the straight line L

$$= \frac{4 - 3}{-3 - (-2)} = -1$$

**Notice that :**

The slope is negative, then the angle is obtuse.

By using the calculator as follows :

Start  $\rightarrow$  

We will find that, the calculator gives the result  $-45^\circ$  (a negative acute angle)

We will find the positive obtuse angle as follows :

$$m(\angle \theta) = 180^\circ - 45^\circ = 135^\circ$$

**TRY 1**  
yourself

1 Find the slope of the straight line which makes a positive angle with the positive direction of X-axis with measure :

(1)  $30^\circ$

(2)  $54^\circ 30' 6''$

(3)  $120^\circ$

2 Find the measure of the positive angle which the straight line makes with the positive direction of X-axis if the slope of the straight line = 6.2

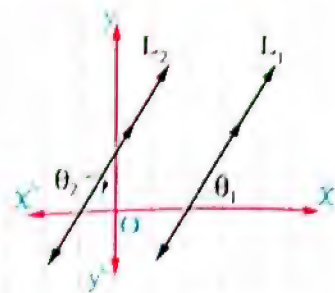
3 Find the measure of the positive angle ( $\theta$ ) which the straight line L makes with the positive direction of X-axis if the straight line L passes through the two points  $(4, -1)$  and  $(5, -3)$

## The relation between the two slopes of the two parallel straight lines

**In the opposite figure :**

If  $L_1$  and  $L_2$  are two parallel straight lines of slopes  $m_1$  and  $m_2$  respectively and make two positive angles with the positive direction of  $X$ -axis of measures  $\theta_1$  and  $\theta_2$  respectively , then

$$\begin{aligned} \therefore L_1 // L_2 & \qquad \qquad \qquad \therefore \theta_1 = \theta_2 \text{ corresponding angles} \\ \therefore \tan \theta_1 = \tan \theta_2 & \qquad \qquad \qquad \therefore m_1 = m_2 \end{aligned}$$



**thus we deduce the following :**

**If**  $L_1 // L_2$  , **then**  $m_1 = m_2$

**i.e.** If two straight lines are parallel , then their slopes are equal.

**Also , we can deduce the opposite :**

**If**  $m_1 = m_2$  , **then**  $L_1 // L_2$

**i.e.** If the two straight lines have equal slopes , then the two straight lines are parallel.

**Example 4** **Prove that :** The straight line which passes through the two points  $(2, 3)$  and  $(-1, 6)$  is parallel to the straight line which makes with the positive direction of  $X$ -axis a positive angle of measure  $135^\circ$

**Solution**

$$\text{The slope of the first straight line } m_1 = \frac{6-3}{-1-2} = \frac{3}{-3} = -1$$

$$\text{, the slope of the second straight line } m_2 = \tan 135^\circ = -1$$

$$\therefore m_1 = m_2$$

$\therefore$  The two straight lines are parallel.

**Example 5** If  $A(-1, 2)$  ,  $B(2, 3)$  ,  $C(-4, 1)$  and  $D(X, 2)$  are four points in the Cartesian coordinates plane and  $\overline{AB} // \overline{CD}$  , **find the value of :  $X$**

**Solution**

$$\therefore \overline{AB} // \overline{CD}$$

$\therefore$  The slope of the straight line passes through  $A(-1, 2)$  and  $B(2, 3)$  is equal to the slope of the straight line passes through  $C(-4, 1)$  and  $D(X, 2)$

$$\therefore \frac{3-2}{2-(-1)} = \frac{2-1}{X-(-4)}$$

$$\therefore X+4=3$$

$$\therefore \frac{1}{3} = \frac{1}{X+4}$$

$$\therefore X = -1$$

**Example 6** In the Cartesian coordinates plane, prove that the points A (-1, 6), B (3, -4) and C (2, -1.5) are collinear.

**Solution**

$$\therefore \text{The slope of } \overrightarrow{AB} = \frac{-4 - 6}{3 - (-1)} = \frac{-10}{4} = -\frac{5}{2}$$

$$\bullet \text{ the slope of } \overrightarrow{BC} = \frac{-1.5 - (-4)}{2 - 3} = \frac{2.5}{-1} = -2\frac{1}{2} = -\frac{5}{2}$$

$$\therefore \text{The slope of } \overrightarrow{AB} = \text{the slope of } \overrightarrow{BC} \quad \therefore \overrightarrow{AB} \parallel \overrightarrow{BC}$$

$\bullet \therefore$  B is a common point between  $\overrightarrow{AB}$  and  $\overrightarrow{BC}$

$\therefore$  A, B and C are collinear.

**Notice that :**

If the slope of  $\overrightarrow{AB}$  = the slope of  $\overrightarrow{BC}$ , then A, B and C are collinear points.



**1 Prove that :** The straight line  $L_1$  passing through the two points (1, 5) and (-2, -1) is parallel to the straight line  $L_2$  that passes through the two points (0, -1) and (5, 9)

**2** If the straight line  $\overrightarrow{AB} \parallel$  the X-axis where A (5, -4) and B (-2, y), find the value of : y

### The relation between the slopes of the two perpendicular (orthogonal) straight lines

If  $L_1$  and  $L_2$  are two straight lines of slopes  $m_1$  and  $m_2$

respectively and  $L_1 \perp L_2$ , then  $m_1 \times m_2 = -1$ , unless one of them is parallel to one of the coordinate axes.

i.e. The product of the slopes of the perpendicular straight lines = -1

and vice versa : If  $L_1$  and  $L_2$  are two straight lines of slopes  $m_1$  and  $m_2$

respectively and  $m_1 \times m_2 = -1$ , then  $L_1 \perp L_2$

i.e. If the product of the two slopes of two straight lines equals -1, then the two straight lines are perpendicular (orthogonal)

**Example 7** **Prove that :** The straight line  $L_1$  which passes through the two points (-1, 4) and (3, 7) is perpendicular to the straight line  $L_2$  which passes through the two points (1, 1) and (4, -3)

**Solution**

$$\therefore \text{The slope of } L_1 = \frac{7 - 4}{3 - (-1)} = \frac{3}{4}, \text{ the slope of } L_2 = \frac{-3 - 1}{4 - 1} = -\frac{4}{3}$$

$$\bullet \therefore \text{The slope of } L_1 \times \text{the slope of } L_2 = \frac{3}{4} \times -\frac{4}{3} = -1 \quad \therefore L_1 \perp L_2$$

**Example 8**

In the Cartesian coordinates plane, if the points A (1, 7), B (2, 4) and C (5, y) represent the vertices of a right-angled triangle at B, find the value of y

**Solution**

$\therefore$  The slope of  $\overrightarrow{AB} = \frac{4-7}{2-1} = -3$ , the slope of  $\overrightarrow{BC} = \frac{y-4}{5-2} = \frac{y-4}{3}$ ,

$\therefore \overrightarrow{AB} \perp \overrightarrow{BC}$

$\therefore$  The slope of  $\overrightarrow{AB} \times$  the slope of  $\overrightarrow{BC} = -1$

$$\therefore -3 \times \frac{y-4}{3} = -1$$

$$\therefore y - 4 = 1$$

$$\therefore y = 5$$

**Remark**

If  $L_1 \perp L_2$ , the slope of  $L_1$  is  $m_1$  and the slope of  $L_2$  is  $m_2$  where  $m_1 \in \mathbb{R}^*$ ,  $m_2 \in \mathbb{R}^*$ , then  $m_2 = \frac{-1}{m_1}$ ,  $m_1 = \frac{-1}{m_2}$

For example:

- If the slope of the straight line L is 2, then the slope of the perpendicular to it is  $-\frac{1}{2}$
- If the slope of the straight line L is  $-\frac{2}{3}$ , then the slope of the perpendicular to it is  $\frac{3}{2}$

**Example 9** In the opposite figure :

If  $L_1 \perp L_2$

**Find :** The value of k

**Solution**

$\therefore$  The straight line  $L_1$  passes

through the two points B (-1, 0) and C (0, 1)

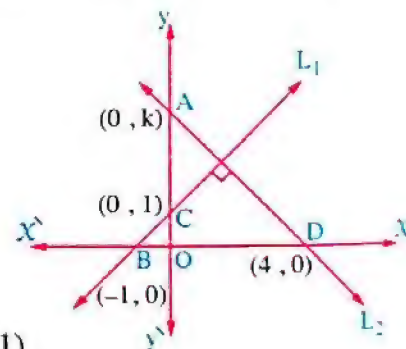
$$\therefore \text{The slope of } L_1 = \frac{1-0}{0-(-1)} = 1$$

$\therefore$  the straight line  $L_2$  passes through the two points A (0, k) and D (4, 0)

$$\therefore \text{The slope of } L_2 = \frac{0-k}{4-0} = -\frac{k}{4} \quad (1)$$

$$\therefore L_1 \perp L_2, \text{ the slope of } L_1 = 1 \quad \therefore \text{The slope of } L_2 = -1 \quad (2)$$

$$\text{From (1) and (2) : } \therefore -\frac{k}{4} = -1 \quad \therefore k = 4$$

**TRY 3**  
in yourself

- 1 If A (-2, 5), B (1, 2) and C (3, 4) are three points in a Cartesian coordinates plane, **prove that** :  $\overrightarrow{AB} \perp \overrightarrow{BC}$
- 2 **Prove that** : The straight line which passes through the two points (7, -1) and (5, -3) is perpendicular to the straight line which makes with the positive direction of X-axis an angle of measure  $135^\circ$

## Remarks to solve the problems on quadrilateral

To prove that a quadrilateral is a trapezium, we prove that :

Two opposite sides are parallel and the other two sides are not parallel.

To prove that a quadrilateral is a parallelogram, we prove only one of the following properties :

- ① Each two opposite sides are parallel.
- ② Each two opposite sides are equal in length.
- ③ Two opposite sides are parallel and equal in length.
- ④ The two diagonals bisect each other.

To prove that a quadrilateral is a rectangle, rhombus or square, we prove at first that the quadrilateral is a parallelogram, then :

• To prove that the parallelogram is a rectangle, we prove only one of the following two properties :

- ① Two adjacent sides are perpendicular.
- ② The two diagonals are equal in length.

• To prove that the parallelogram is a rhombus, we prove only one of the following two properties :

- ① Two adjacent sides are equal in length.
- ② The two diagonals are perpendicular.

• To prove that the parallelogram is a square, we prove only one of the following properties :

- ① Two adjacent sides are perpendicular and equal in length.
- ② Two adjacent sides are perpendicular and its diagonals are perpendicular.
- ③ Two diagonals are equal in length and perpendicular.
- ④ Two adjacent sides are equal in length and its two diagonals are equal in length.

**Example 10** On a Cartesian coordinates plane, represent the points A (3, -2), B (-5, 0), C (0, -7) and D (8, -9), then prove that the quadrilateral ABCD is a parallelogram.

**Solution**

$$\therefore \text{The slope of } \overrightarrow{AB} = \frac{0 - (-2)}{-5 - 3} = \frac{2}{-8} = -\frac{1}{4}$$

$$\text{, the slope of } \overrightarrow{CD} = \frac{-9 - (-7)}{8 - 0} = \frac{-2}{8} = -\frac{1}{4}$$

$$\therefore \text{The slope of } \overrightarrow{AB} = \text{the slope of } \overrightarrow{CD}$$

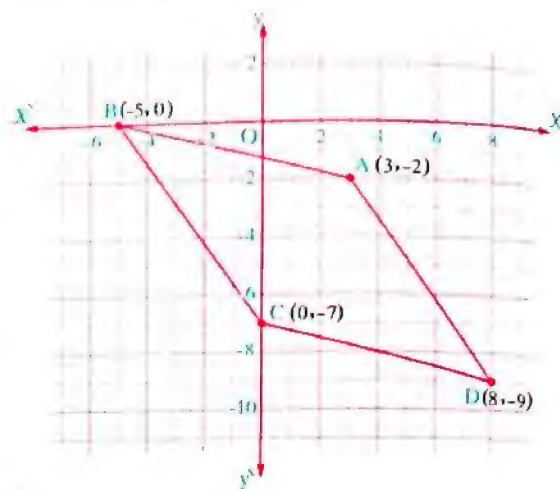
$$\therefore \overrightarrow{AB} \parallel \overrightarrow{CD} \quad (1)$$

$$\therefore \text{The slope of } \overrightarrow{AD} = \frac{-9 - (-2)}{8 - 3} = \frac{-7}{5}, \text{ the slope of } \overrightarrow{BC} = \frac{-7 - 0}{0 - (-5)} = \frac{-7}{5}$$

$$\therefore \text{The slope of } \overrightarrow{AD} = \text{the slope of } \overrightarrow{BC}$$

$$\therefore \overrightarrow{AD} \parallel \overrightarrow{BC} \quad (2)$$

From (1) and (2) :  $\therefore$  The quadrilateral ABCD is a parallelogram.



**Example 11** Prove that : The points A (2, -2), B (8, 4), C (5, 7) and D (-1, 1) are vertices of the rectangle ABCD

**Solution**

$$\therefore \text{The slope of } \overrightarrow{AB} = \frac{-2 - 4}{2 - 8} = \frac{-6}{-6} = 1$$

$$\text{, the slope of } \overrightarrow{CD} = \frac{7 - 1}{5 - (-1)} = \frac{6}{6} = 1$$

$$\therefore \text{The slope of } \overrightarrow{AB} = \text{the slope of } \overrightarrow{CD} \quad \therefore \overrightarrow{AB} \parallel \overrightarrow{CD} \quad (1)$$

$$\therefore \text{The slope of } \overrightarrow{AD} = \frac{-2 - 1}{2 - (-1)} = \frac{-3}{3} = -1$$

$$\text{, the slope of } \overrightarrow{BC} = \frac{4 - 7}{8 - 5} = \frac{-3}{3} = -1$$

$$\therefore \text{The slope of } \overrightarrow{AD} = \text{the slope of } \overrightarrow{BC} \quad \therefore \overrightarrow{AD} \parallel \overrightarrow{BC} \quad (2)$$

From (1) and (2) we deduce that the quadrilateral ABCD is a parallelogram.

$$\therefore \text{The slope of } \overrightarrow{AB} \times \text{the slope of } \overrightarrow{BC} = 1 \times -1 = -1$$

$$\therefore \overrightarrow{AB} \perp \overrightarrow{BC} \quad \therefore \text{The quadrilateral ABCD is a rectangle.}$$

**Example 12**

On a Cartesian coordinates plane, represent the points  $A(-3, -3)$ ,  $B(3, 1)$ ,  $C(1, 5)$  and  $D(-2, 3)$ , then prove that the quadrilateral ABCD is a trapezium.

**Solution**

$$\therefore \text{The slope of } \overline{CD} = \frac{5-3}{1-(-2)} = \frac{2}{3}$$

$$\text{, the slope of } \overline{AB} = \frac{1-(-3)}{3-(-3)} = \frac{4}{6} = \frac{2}{3}$$

$$\therefore \text{The slope of } \overline{CD} = \text{the slope of } \overline{AB}$$

$$\therefore \overline{CD} \parallel \overline{AB} \quad (1)$$

$$\text{The slope of } \overline{BC} = \frac{5-1}{1-3} = -2$$

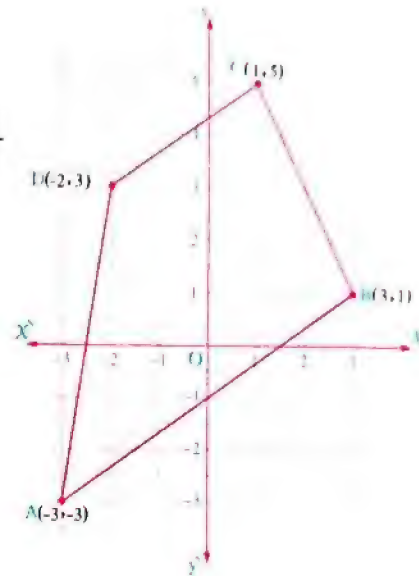
$$\text{, the slope of } \overline{AD} = \frac{3-(-3)}{-2-(-3)} = 6$$

$$\therefore \text{The slope of } \overline{BC} \neq \text{the slope of } \overline{AD}$$

$$\therefore \overline{BC} \text{ is not parallel to } \overline{AD} \quad (2)$$

From (1) and (2) :

$\therefore$  The quadrilateral ABCD is a trapezium.



- Answers**
- 1** (1) 0.58 (approximately) (2) 1.4 (approximately) (3)  $-1.73$  (approximately)
- 2**  $80^\circ$   $50^\circ$   $16^\circ$  (approximately)
- 3**  $116^\circ$   $33^\circ$   $54^\circ$  (approximately)
- 1** Prove by yourself [Hint : The slope of  $L_1$  = The slope of  $L_2$ ]
- 2**  $y = -4$
- 3** **1** Prove by yourself [Hint : The slope of  $\overline{AB} \times$  The slope of  $\overline{BC} = -1$ ]
- 2** Prove by yourself [Hint : The product of the slopes of the two straight lines =  $-1$ ]

of try by yourself

## LESSON

# 4

## The equation of the straight line given its slope and the intercepted part of y-axis

We studied before that the relation :  $aX + by + c = 0$  where  $a \neq 0$  ,  $b \neq 0$  together is a linear relation represented graphically by a straight line and we can find its slope ( $m$ ) by one of the following methods :

**1**  $m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{y_2 - y_1}{x_2 - x_1}$

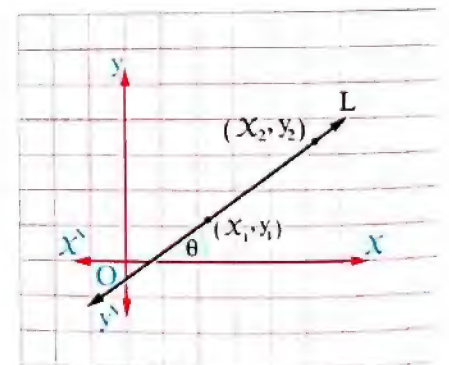
Where  $(x_1, y_1)$  and  $(x_2, y_2)$  are two points on the straight line

**2**  $m = \tan \theta$

Where  $\theta$  is the measure of the positive angle which the straight line makes with the positive direction of the  $X$ -axis.

• We will continue our study about this subject by studying how :

- To find the slope of the straight line and the length of the intercepted part from y-axis if we know the equation of the straight line.
- To find the equation of the straight line if we know its slope and the length of the intercepted part from the y-axis.



## First

## Finding the slope of the straight line and the length of the intercepted part from y-axis

## Prelude Example

Represent graphically the relation :  $2x - y + 3 = 0$  and from the graph, find the slope of the straight line which represents the relation and the intercepted part of the y-axis by the straight line.

## Solution

To graph the straight line which represents the relation, find two points of the points of the straight line at least, to facilitate that, put one of the variables  $x$  or  $y$  in a side of the equation

$$\therefore 2x - y + 3 = 0$$

$$\therefore -y = -2x - 3$$

$$\therefore y = 2x + 3$$

$$\text{At } x = 0$$

$$\therefore y = 0 + 3 = 3$$

$\therefore (0, 3)$  is one of the points of the straight line.

$$\text{At } x = -1$$

$$\therefore y = -2 + 3 = 1$$

$\therefore (-1, 1)$  is one of the points of the straight line.

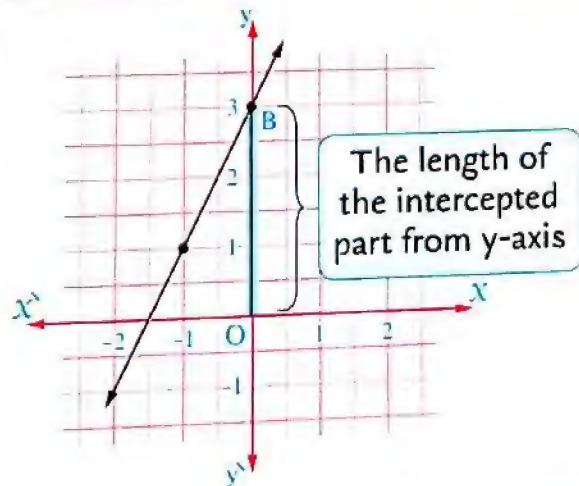
i.e. The straight line passes through the two points  $(0, 3)$  and  $(-1, 1)$

$$\therefore \text{The slope of the straight line} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{-1 - 0} = \frac{-2}{-1} = 2$$

• From the graph, we find that :

OB = 3 length units.

i.e. The straight line intercepts from the positive part from y-axis 3 length units



Observing the graph of the straight line :  $y = 2x + 3$

we find that :

- The slope of the straight line = the coefficient of  $x = 2$
- The length of the intercepted part from y-axis = | absolute term | = | 3 | = 3 length units.

The slope of the straight line

$$y = 2x + 3$$

The length of the intercepted part from y-axis

i.e.

If the equation of a straight line is in the form :  $y = mX + c$  , then :

- The slope of the straight line =  $m$
- The length of the intercepted part from y-axis =  $|c|$   
and it passes through the point  $(0, c)$



**Example 1** Find the slope of the straight line :  $2X + 5y - 15 = 0$   
 , then find the intercepted part of y-axis.

**Solution**

Write the equation of the straight line in the form :  $y = mX + c$

$$\therefore 5y = -2X + 15 \qquad \therefore y = \frac{-2}{5}X + 3$$

$\therefore$  The slope of the straight line =  $\frac{-2}{5}$  and the intercepted part of the positive part of y-axis is of length = 3 length units.

### “ Remark

In the previous example , observing the equation in the form :  $2X + 5y - 15 = 0$   
 , we find that :

- The slope of the straight line =  $\frac{-\text{coefficient of } X}{\text{coefficient of } y} = \frac{-2}{5}$
- The straight line cuts y-axis at the point  $\left(0, \frac{-\text{absolute term}}{\text{coefficient of } y}\right)$  i.e.  $(0, 3)$

i.e. The straight line intercepts a part of y-axis of length =  $\left| \frac{-\text{absolute term}}{\text{coefficient of } y} \right|$   
 =  $|3| = 3$  length units.

”

i.e.

If the equation of a straight line is in the form :  $aX + by + c = 0$  , then :

- The slope of the straight line =  $\frac{-\text{coefficient of } X}{\text{coefficient of } y} = \frac{-a}{b}$
- The straight line cuts y-axis at the point  $\left(0, \frac{-c}{b}\right)$

i.e. The length of the intercepted part from y-axis =  $\left| \frac{-c}{b} \right|$

For example:

- The straight line whose equation is :  $X - 2y + 3 = 0$

Its slope  $= \frac{-1}{-2} = \frac{1}{2}$  and cuts y-axis at the point  $(0, \frac{3}{2})$

i.e. The straight line intercepts a part of length  $\frac{3}{2}$  length unit from the positive part of y-axis.

- The straight line whose equation is :  $3X + y + 4 = 0$

Its slope  $= -3$  and cuts y-axis at the point  $(0, -4)$

i.e. The straight line intercepts a part of length 4 length units from the negative part of y-axis.

**Example 2** If the straight line that passes through the two points  $(-1, 7)$  and  $(9, 3)$  is perpendicular to the straight line whose equation is :  $X + ky - 13 = 0$ ,  
find the value of : k

**Solution**

Let the slope of the straight line that passes through the two points  $(-1, 7)$  and  $(9, 3)$  be  $m_1$

$$\therefore m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 7}{9 - (-1)} = \frac{-4}{10} = \frac{-2}{5}$$

Let the slope of the straight line whose equation is :  $X + ky - 13 = 0$  be  $m_2$

$$\therefore m_2 = \frac{-a}{b} = \frac{-1}{k}$$

$\therefore$  The two straight lines are perpendicular

$$\therefore m_1 \times m_2 = -1$$

$$\therefore \frac{-2}{5} \times \frac{-1}{k} = -1$$

$$\therefore \frac{2}{5k} = -1$$

$$\therefore -5k = 2$$

$$\therefore k = \frac{-2}{5}$$

**TRY**

**1**

yourself

- 1 If the two straight lines :  $3y + X - 7 = 0$  and  $y = kX + 5$  are perpendicular, then find the value of : k

- 2 Find the measure of the positive angle which is made by the straight line whose equation is :  $3X - 3y + 5 = 0$  with the positive direction of X-axis.

- 3 Find the length of the intercepted part from y-axis by the straight line whose equation is :  $2y = 3X + 12$

**Second****Finding the equation of the straight line given its slope and the length of intercepted part of y-axis**

The straight line whose slope =  $m$  and cuts y-axis at the point  $(0, c)$  its equation is in the form :

$$y = mX + c$$

**Example 3** Find the equation of the straight line :

- 1 Whose slope =  $-\frac{3}{4}$  and intercepts from the positive part of y-axis 3 length units.
- 2 Whose slope = 2 and intercepts from the negative part of y-axis 7 length units.

**Solution**

$$y = mX + c$$

$$1 \therefore m = -\frac{3}{4}, c = 3$$

$$\therefore \text{The equation is : } y = -\frac{3}{4}X + 3$$

$$2 \therefore m = 2, c = -7$$

$$\therefore \text{The equation is : } y = 2X - 7$$

**Example 4** Find the equation of the straight line which makes with the positive direction of X-axis a positive angle of measure  $135^\circ$  and intercepts from the positive part of y-axis a part of length 7 length units.

**Solution**

$$\therefore \text{The slope} = \tan 135^\circ = -1$$

$$\therefore \text{The equation of the straight line is : } y = -X + 7$$

**Remarks**

- 1 The equation of the straight line which passes through the origin point O  $(0, 0)$  is  $y = mX$ , where  $m$  is the slope of the straight line.
- 2 The equation of X-axis is  $y = 0$
- 3 The equation of y-axis is  $X = 0$
- 4 The equation of the straight line which is parallel to X-axis and passes through the point  $(0, l)$  is  $y = l$
- 5 The equation of the straight line which is parallel to y-axis and passes through the point  $(k, 0)$  is  $X = k$



**Example 5** Find the equation of the straight line which passes through the two points  $(1, -1)$  and  $(2, 2)$

**Solution**

Let the equation of the straight line be in the form  $y = mX + c$

$$\therefore \text{The slope (m)} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-1)}{2 - 1} = 3$$

$\therefore$  The equation of the straight line is in the form :  $y = 3X + c$

$\therefore (1, -1)$  belongs to the straight line.

$$\therefore -1 = 3 \times 1 + c$$

$$\therefore c = -1 - 3 = -4$$

$\therefore$  The equation of the straight line is :  $y = 3X - 4$

**Example 6** Find the equation of the straight line which passes through the point  $(1, 2)$  and parallels the straight line  $2X + 3y - 6 = 0$

**Solution**

$$\therefore \text{The slope of the given straight line} = \frac{-\text{coefficient of } X}{\text{coefficient of } y} = \frac{-2}{3}$$

$$\therefore \text{The slope of the required straight line} = \frac{-2}{3}$$

$$\therefore \text{The equation of the required straight line is : } y = -\frac{2}{3}X + c$$

$\therefore$  The straight line passes through the point  $(1, 2)$

$$\therefore 2 = -\frac{2}{3} \times 1 + c$$

$$\therefore c = \frac{8}{3}$$

$$\therefore \text{The equation of the required straight line is : } y = -\frac{2}{3}X + \frac{8}{3}$$

**Example 7** Find the equation of the straight line which passes through the point  $(2, 3)$  and perpendicular to the straight line passing through the two points  $A(3, -4)$  and  $B(5, -3)$

**Solution**

$$\therefore \text{The slope of the straight line which passes through the two points } (3, -4) \text{ and } (5, -3) \text{ equals } \frac{-3 - (-4)}{5 - 3} = \frac{1}{2}$$

$$\therefore \text{The slope of the required straight line} = -2$$

$$\therefore \text{The equation of the required straight line is } y = -2X + c$$

$\therefore$  The straight line passes through the point  $(2, 3)$

$\therefore$  It satisfies the equation.

$$\therefore 3 = -2 \times 2 + c$$

$$\therefore c = 7$$

$$\therefore \text{The equation of the required straight line is : } y = -2X + 7$$

**TRY 2**

to yourself

- 1 Find the equation of the straight line which intercepts from the positive part of y-axis 5 length units and it is parallel to the straight line passing through the two points  $(-2, 3)$  and  $(-1, -6)$
- 2 Find the equation of the straight line which passes through the point  $(3, 4)$  and perpendicular to  $\overline{AB}$ , where  $A(2, -3)$  and  $B(5, 4)$

**Example 8** ABC is a triangle whose vertices are  $A(1, 2)$ ,  $B(-2, 3)$ ,  $C(-4, -3)$ ,  $\overline{AD}$  is a median of it, find the equation of  $\overline{AD}$

**Solution**

$\therefore \overline{AD}$  is a median of  $\Delta ABC$

$\therefore D$  is the midpoint of  $\overline{BC}$

$$\therefore D = \left( \frac{-2 + (-4)}{2}, \frac{3 + (-3)}{2} \right) = (-3, 0)$$

$$\therefore \text{The slope of } \overline{AD} = \frac{2 - 0}{1 - (-3)} = \frac{1}{2}$$

$$\therefore \text{The equation of } \overline{AD} \text{ is : } y = \frac{1}{2}x + c$$

$\therefore \overline{AD}$  passes through the point  $A(1, 2)$

$\therefore$  It satisfies its equation

$$\therefore 2 = \frac{1}{2} \times 1 + c \qquad \therefore c = \frac{3}{2}$$

$$\therefore \text{The equation of } \overline{AD} \text{ is : } y = \frac{1}{2}x + \frac{3}{2}$$

**TRY 3**

to yourself

ABC is a triangle whose vertices are  $A(-1, 5)$ ,  $B(4, -2)$  and  $C(-3, 0)$   
Find the equation of the straight line passing through A and perpendicular to  $\overline{BC}$

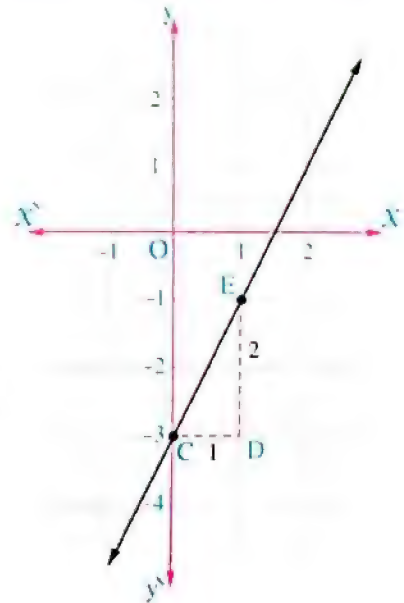
**Example 9** Using the slope and the intercepted part of y-axis, represent graphically the straight line whose equation is  $y = 2x - 3$

**Solution**

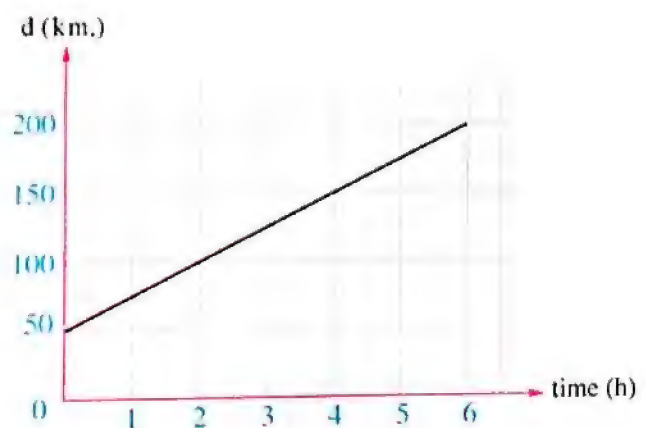
$$\text{The slope of the straight line} = 2 = \frac{2}{1} = \frac{\text{vertical change}}{\text{horizontal change}}$$

and the straight line passes through the point  $C(0, -3)$

From the point C, we move horizontally towards the right one unit (the horizontal change (+1)) to reach the point D, then we move vertically towards up two units (the vertical change (+2)) to reach the point E, then  $\overline{CE}$  is the graph of the equation of the straight line  $y = 2x - 3$



**Example 10** The opposite graph represents the motion of a car moving with a uniform velocity where the distance (d) is measured in km. and the time (t) in hours, find :



- 1 The distance (d) at the beginning of the motion.
- 2 The velocity of the car.
- 3 The equation of the straight line representing the motion of the car.

**Solution**

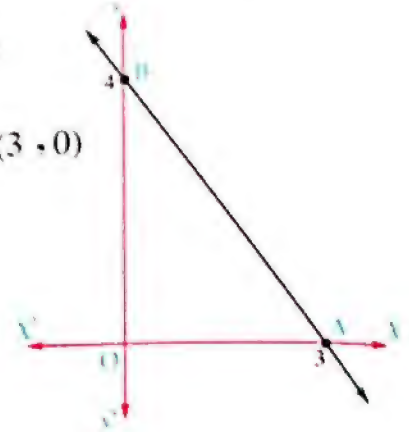
- 1 The distance (d) at the beginning of the motion = 50 kilometres.
- 2 The velocity of the car = the slope of the straight line passing through the two points (0, 50) and (6, 200) =  $\frac{200 - 50}{6 - 0} = \frac{150}{6} = 25 \text{ km/hr.}$
- 3 The equation of the straight line is :  $d = mt + c$

i.e.  $d = 25t + 50$

**Example 11** Find the equation of the straight line which intercepts from the coordinate axes (X-axis and y-axis) two positive parts with lengths 3 and 4 length units respectively, then find the area of the triangle included between the straight line and the two axes.

**Solution**

- ∴ The straight line intercepts from the positive part of X-axis 3 length units.
- ∴ The straight line passes through the point A (3, 0)
- ∴ The straight line intercepts from the positive part of y-axis 4 length units.
- ∴ The straight line passes through the point B (0, 4)
- ∴ The straight line passes through the two points A (3, 0) and B (0, 4)



Let the equation of the required straight line be  $y = mX + c$

, where the slope  $(m) = \frac{4-0}{0-3} = -\frac{4}{3}$

∴  $y = -\frac{4}{3}X + c$

, ∴  $c = 4$

∴ The equation is :  $y = -\frac{4}{3}X + 4$

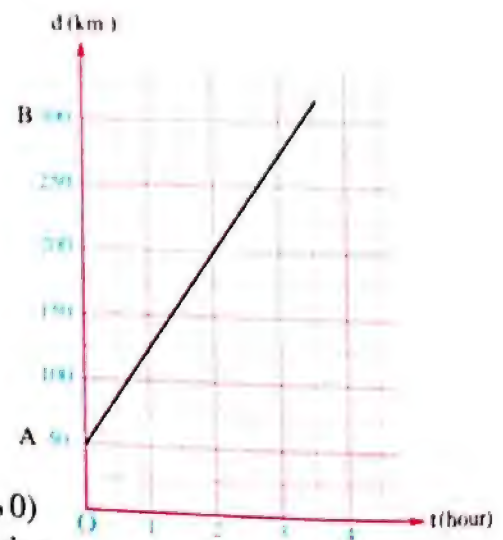
, the area of  $\triangle ABO = \frac{1}{2} \times AO \times BO = \frac{1}{2} \times 3 \times 4 = 6$  square units.

**TRY 4**  
by yourself

A person moved between the cities A and B using his car with a uniform velocity and the opposite graph represents the relation between the distance (d) in kilometres and the time (t) in hours.

**Answer the following :**

- 1 What is the uniform velocity of the car ?
- 2 Find the equation of the straight line representing the motion of the car.
- 3 Find the distance between the car and O (0, 0) after 3 hours from the beginning of the motion.



- Answers**
- 1 13
  - 2  $y = -9x + 5$
  - 3  $y = \frac{2}{7}x + \frac{2}{17}$
  - 4 1 75 km/hr.
  - 2  $d = 75t + 50$
  - 3 275 km.
  - 2  $y = -\frac{3}{37}x + \frac{7}{37}$
  - 3 6 length units
  - 2 45°

of try by yourself

# Maths

By a group of supervisors



EXERCISES

**3<sup>rd</sup>** PREP.  
2021  
FIRST TERM



EL-MOASSER

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A research  
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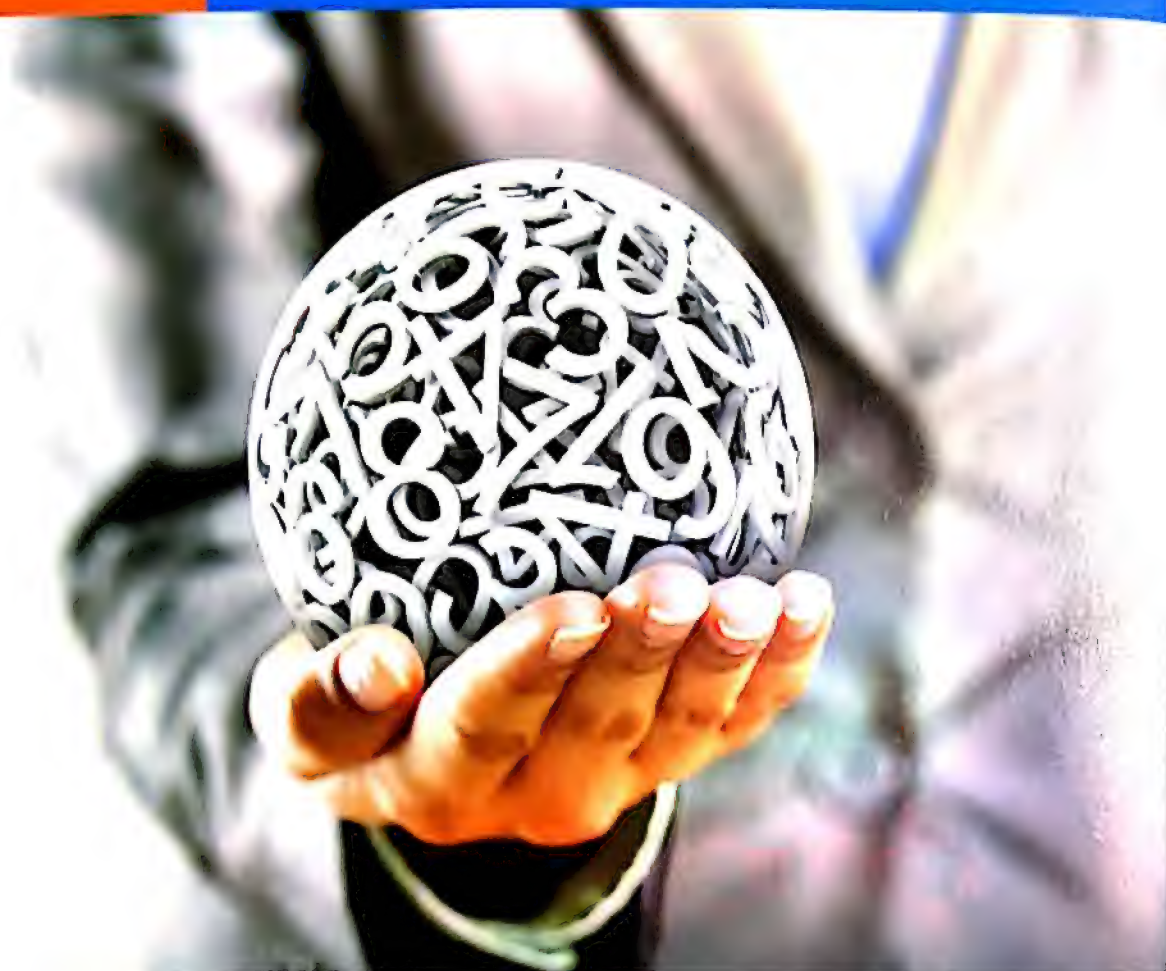


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# Algebra and Statistics



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# UNIT 1

## Relations and functions



### Exercises of the unit :

1. Cartesian product.
  2. Relation - function (mapping).
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🔍 A research project on unit one



Scan the  
**QR code**  
to solve an  
interactive  
test on each  
lesson



## EXERCISE

# 1

## Cartesian product



From the school book

Interactive test

### First Cartesian product of two finite sets

1 Find the values of  $a$  and  $b$  in each of the following if :

1  $(a, b) = (-5, 9)$

3  $(a - 2, b + 1) = (2, -3)$

5  $(a - 7, 26) = (-2, b^3 - 1)$

7  $(a^5, b^2 - 1) = (32, \sqrt[3]{27})$

9  $(2a, 7) = (2b + 1, a)$

2  $(a, b) = (\sqrt{25}, \sqrt[3]{27})$

4  $(6, b - 3) = (2 - a, -1)$

6  $(a, b) = (2 - a, 2b - 3)$

8  $(a, 7) = (b^2, b)$

10  $(3, b) = (5a - 1, 4a)$

2 If  $X = \{1, 2\}$  ,  $Y = \{3, 4, 5\}$  Find  $X \times Y$  and represent it by :

1 The arrow diagram.

2 The Cartesian diagram.

3 If  $X = \{3, 4, 8\}$  Find  $X^2$  and represent it :

1 By an arrow diagram.

2 By Cartesian diagram.

4 If  $X = \{1, 2, 3\}$  ,  $Y = \{4\}$  , find :

1  $X \times Y$

2  $Y \times X$

3  $Y^2$

4  $n(X^2)$

5 If  $X = \{2, -1\}$  ,  $Y = \{4, 0\}$  ,  $Z = \{4, 5, -2\}$  , find :

1  $X \times Y$

2  $Y \times Z$

3  $X^2$

4  $n(X \times Z)$

5  $n(Y^2)$

6  $n(Z^2)$

## 6 Choose the correct answer from those given :

- 1 If  $(X - 1, 11) = (8, y + 3)$ , then  $\sqrt{X + 2y} = \dots\dots\dots$  (Port Said 19)  
 (a) 5 (b)  $\pm 5$  (c)  $\sqrt{17}$  (d) 25
- 2 If  $(X + 2, y) = (2, 3)$ , then  $X^5 y + 1 = \dots\dots\dots$  (El-Sharkia 19)  
 (a) 3 (b) 2 (c) zero (d) 1
- 3 If  $(3^X, \sqrt{y}) = (1, 4)$ , then  $X + y = \dots\dots\dots$  (El-Gharbia 18)  
 (a) 2 (b) 3 (c) 16 (d) 17
- 4 If A and B are two sets, then the set  $\{(X, y) : X \in A, y \in B\}$  expresses  $\dots\dots\dots$  (El-Dakahlia 16)  
 (a)  $n(A \times B)$  (b)  $A \times B$  (c)  $n(B \times A)$  (d)  $B \times A$
- 5 If  $X = \{1, 2\}$ , then  $X \times \emptyset = \dots\dots\dots$   
 (a) X (b)  $\emptyset$  (c)  $\{0\}$  (d)  $\{(1, 0), (2, 0)\}$
- 6 If  $X = \{2\}$ ,  $Y = \{3\}$ , then  $X \times Y = \dots\dots\dots$  (Giza 17)  
 (a) 6 (b)  $\{6\}$  (c)  $(2, 3)$  (d)  $\{(2, 3)\}$
- 7 If  $X = \{3\}$ , then  $X^2 = \dots\dots\dots$  (Cairo 13 - El-Sharkia 17)  
 (a) 9 (b)  $(3, 3)$  (c)  $\{9\}$  (d)  $\{(3, 3)\}$
- 8 If  $X = \{3\}$ , then  $n(X^2) = \dots\dots\dots$  (Qena 20)  
 (a) 1 (b) 9 (c)  $\{3, 3\}$  (d) 3
- 9 If  $X = \{1, 2\}$  and  $Y = \{3, 4\}$ , then  $(3, 4) \in \dots\dots\dots$  (Qena 11 - Suez 19)  
 (a)  $X \times Y$  (b)  $Y \times X$  (c)  $X^2$  (d)  $Y^2$
- 10 If  $n(X) = 2$ ,  $Y = \{1, 2\}$ , then  $n(X \times Y) = \dots\dots\dots$  (Giza 15)  
 (a) 4 (b) 3 (c) 5 (d) 6
- 11 If  $n(X) = 3$ ,  $n(X \times Y) = 12$ , then  $n(Y) = \dots\dots\dots$   
 (Damietta 15 - Port Said 17 - Cairo 18 - El-Menia 19 - Port Said 20)  
 (a) 4 (b) 9 (c) 15 (d) 36
- 12 If  $n(X^2) = 9$ , then  $n(X) = \dots\dots\dots$   
 (a) 2 (b) 3 (c) 9 (d) 81
- 13 If  $n(X^2) = 4$ ,  $n(X \times Y) = 6$ , then  $n(Y^2) = \dots\dots\dots$  (Giza 20)  
 (a) 4 (b) 9 (c) 16 (d) 12
- 14 If  $n(X) = n(X \times Y)$ , then  $n(Y) = \dots\dots\dots$  (Damietta 18)  
 (a) 1 (b) 2 (c) 3 (d) 4
- 15 If  $a \in X^2$ , where  $X = \{X : 5 < X < 7, X \in \mathbb{N}\}$ , then  $a = \dots\dots\dots$  (El-Sharkia 20)  
 (a) 36 (b)  $\{36\}$  (c)  $(6, 6)$  (d)  $[5, 7]$

16 If  $(3, 5) \in \{3, 6\} \times \{X, 8\}$ , then  $X = \dots\dots\dots$

(Qena 15 – Kafr El-Sheikh 18 – Port Said 19 – Alex. 20)

- (a) 8 (b) 6 (c) 5 (d) 3

17 If  $\{2\} \times \{X, y\} = \{(2, 4), (2, 3)\}$ , then  $X - y = \dots\dots\dots$

(El-Sharkia 15 – Kafr El-Sheikh 20)

- (a) 1 (b) -1 (c)  $\pm 1$  (d) 0

7 If  $X \times Y = \{(2, 6), (2, 9), (3, 6), (3, 9), (5, 6), (5, 9)\}$ , find :  $X$  and  $Y$

8 If  $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$ , find :

1  $X$  and  $Y$

2  $Y \times X$

3  $Y^2$

(Giza 16 – Souhag 19 – El-Kalyoubia 20)

9 If  $X^2 = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$ , find :  $X$

10 If  $Y \times X = \{(1, 3), (2, 3), (3, 3)\}$ , find  $X^2$

11 If  $X = \{1, 2, 3, 4\}$ ,  $Y = \{3, 4, 5\}$ , represent  $X$  and  $Y$  by Venn diagram, then find

1  $(X \cap Y) \times Y$

2  $(X - Y) \times Y$

3  $(Y - X) \times X$

12 If  $X = \{3, 4\}$ ,  $Y = \{4, 5\}$  and  $Z = \{6, 5\}$ , then find :

1  $X \times (Y \cap Z)$

2  $(X - Y) \times Z$

3  $(X - Y) \times (Y - Z)$

(El-Dakahlia 13 – El-Monofia 18 – El-Menya 20)

13 If  $X = \{1\}$ ,  $Y = \{2, 3\}$ ,  $Z = \{2, 5, 6\}$

Represent each of  $X$ ,  $Y$  and  $Z$  by Venn diagram, then find :

First : 1  $X \times Y$

2  $Y \times Z$

3  $X \times Z$

4  $Y^2$

Second :  $(X \times Y) \cup (Y \times Z)$

Third :  $X \times (Y \cap Z)$

Fourth :  $(X \times Y) \cap (X \times Z)$

Fifth :  $(Z - Y) \times (X \cup Y)$

## Second Cartesian product of two infinite sets

14 Identify the following points on a perpendicular graphical net of the Cartesian product  $\mathbb{R} \times \mathbb{R}$  :

A (4, 5), B (6, -3), C (-2, 7), D (-1, 6), E (-4, -5)

, M (0, 6), K (9, 0)

Then mention the quadrant that each point is located on the perpendicular graphical net or the axis it belongs to.

## 15 Choose the correct answer from those given :

- 1 The point ( ..... , ..... ) lies on the second quadrant.  
 (a) (3 , 2) (b) (- 2 , 3) (c) (- 3 , - 2) (d) (2 , - 3)
- 2 If the point (a - b , 5) lies on y-axis , then ..... (Giza 18)  
 (a) a = b (b) a + b = 0 (c) a ≠ b (d) a - b = 5
- 3 If the point (5 , b - 7) is located on the X-axis ,  
 then b = ..... (Alex. 11 - North Sinai 16 - Qena 17 - Cairo 18 - El-Kalyoubia 20)  
 (a) 2 (b) 5 (c) 7 (d) 12
- 4 If the point (X , 7) lies on the y-axis , then 5 X + 1 = ..... (El-Beheira 17)  
 (a) zero (b) 1 (c) 5 (d) 6
- 5 If  $(X + 1, \sqrt[3]{27}) = (-1, y)$  , then the point (X , y) lies in the ..... quadrant  
 (El-Fayoum 20)  
 (a) first (b) second (c) third (d) fourth
- 6 If  $b < 3$  , then the point (5 , b - 3) lies in the ..... quadrant. (Cairo 16)  
 (a) first (b) second (c) third (d) fourth
- 7 If  $X \in \mathbb{R}_-$  , then the point  $(-X, \sqrt[3]{X})$  lies in the ..... quadrant. (El-Monofia 20)  
 (a) first (b) second (c) third (d) fourth
- 8 If the point (a , b) lies on the fourth quadrant , then  $a \times b$  ..... zero.  
 (a) = (b) > (c) < (d) ≥
- 9 If the point  $(2a, 3b) \in \overrightarrow{XX}$  , then  $\frac{b}{a} =$  ..... (where  $a \neq 0$ )  
 (a) zero (b)  $\frac{2}{3}$  (c) 2 (d) 3
- 10 If  $(|X|, 4) = (3, y^2)$  and the point (X , y) lies in the second quadrant ,  
 then  $X + y =$  ..... (El-Sharkia 14)  
 (a) 7 (b) 1 (c) - 1 (d) - 7
- 11 If  $a < \text{zero}$  ,  $b > \text{zero}$  , then the point which lies on the second quadrant is .....  
 (El-Fayoum 18)  
 (a) (a , b) (b) (- a , b) (c) (a , - b) (d) (- a , - b)
- 12 If the point (X - 2 , X - 4) lies in the fourth quadrant , then X = ..... where  $X \in \mathbb{Z}$   
 (a) zero (b) 2 (c) 3 (d) 4
- 13 If the point (X - 4 , 2 - X) where  $X \in \mathbb{Z}$  is located in the third quadrant  
 , then X = ..... (Alexandria 15 - Kafr El-Sheikh 16 - El-Monofia 17 - Port Said 19 - El-Beheira 20)  
 (a) 2 (b) 3 (c) 4 (d) 6

- 14 If the point  $(k^2 - 4, k)$  lies on the negative part of  $y$ -axis, then  $k = \dots\dots\dots$  (El-Sharkia 18)  
 (a)  $\pm 2$  (b) 4 (c)  $-2$  (d) 2

- 16 If  $A(-2, 0)$ ,  $B(-2, 3)$ ,  $C(2, 3)$ , identify on the perpendicular square net  $\mathbb{R}^2$  the points  $A, B, C$  and find the area of  $\Delta ABC$  « 6 square units »

- 17 If  $X = [-2, 3]$ , find the location which represents  $X \times X$   
 Show which of the following points belongs to the Cartesian product of  $X \times X$   
 $A(1, 2)$ ,  $B(3, -1)$ ,  $C(-1, 4)$  and  $D(-2, 0)$

- 18 If  $X = [-2, 3]$ ,  $Y = [0, 4]$ , find the region which represents each of :

- 1  $X \times Y$                       2  $Y \times X$                       3  $Y^2$



### For excellent pupils

- 19 Choose the correct answer from those given :

- 1 If  $(X - Y) \times Y = \{(1, 2), (1, 3)\}$ ,  $n(X \times Y) = 6$ , then  $X = \dots\dots\dots$  (El-Sharkia 1)

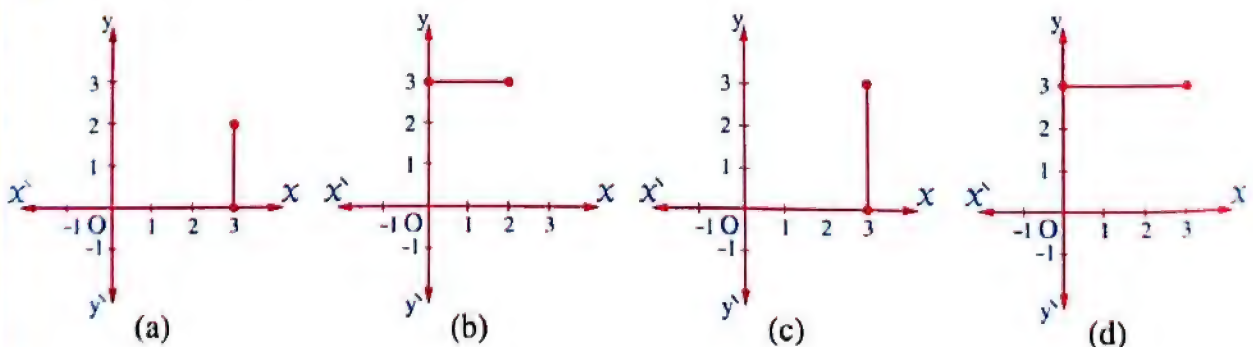
- (a)  $\{1\}$  (b)  $\{1, 2\}$  (c)  $\{1, 3, 6\}$  (d)  $\{1, 3, 2\}$

- 2 If  $X - Y = \{7\}$ ,  $Y - X = \{2, 4\}$ ,  $X \cap Y = \{6\}$ , then  $(X \times Y) \cap (Y \times X) = \dots\dots\dots$

- (a)  $\{(6, 6)\}$  (b)  $\{(7, 2), (7, 4)\}$

- (c)  $\{(2, 7), (4, 7)\}$  (d)  $\{(7, 6)\}$

- 3  $\{3\} \times [0, 2]$  is represented graphically in the figure .....



- 20 If  $X \subset Y$ ,  $X \times Y = \{(a, 1), (a, 2), (a, 3), (2, 1), (2, 2), (2, 3)\}$

Find the values of  $a$

- 21 If  $X \subset Y$ ,  $n(X \times Y) = 6$ ,  $4 \in X$  and  $(1, 7) \in X \times Y$ ,

then find  $X, Y$  and  $X \times Y$

(Damietta 17)



## EXERCISE

# 2

## Relation - Function (mapping)



From the school book

Interactive test

1 Choose the correct answer from those given :

1 If  $f$  is a function from the set  $X$  to the set  $Y$ , then  $X$  is called .....

- (a) the range of the function  $f$  (b) the domain of the function  $f$   
(c) the codomain of the function  $f$  (d) the rule of the function  $f$

2 If  $f$  is a function from the set  $X$  to the set  $Y$ , then  $Y$  is called .....

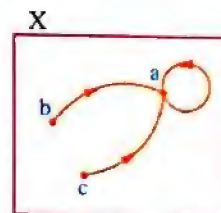
- (a) the domain of the function. (b) the codomain of the function.  
(c) the range of the function. (d) the rule of the function.

3 If the relation  $R = \{(4, 3), (1, 3), (2, 5)\}$ , then  $R$  represents a function where its range is ..... (El-Kalyoubia 17)

- (a)  $\{1, 2, 4\}$  (b)  $\{4, 1, 2, 3, 5\}$  (c)  $\{3, 5\}$  (d)  $\mathbb{N}$

4 The opposite diagram represents a function on  $X$ , its range is .....

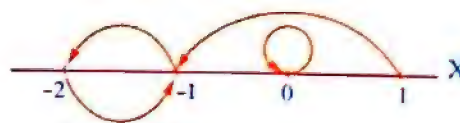
- (a)  $\{a\}$  (b)  $\{a, b, c\}$   
(c)  $\{a, b\}$  (d)  $\{b, c\}$



(Cairo 11)

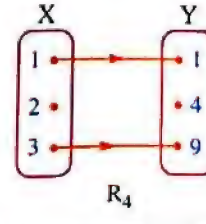
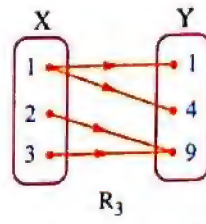
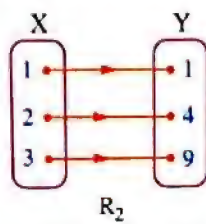
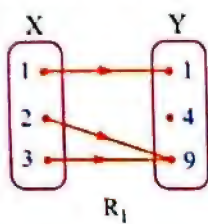
5 The opposite figure represents a function on  $X$ , its range is .....

- (a)  $\{1, 0, -1, -2\}$  (b)  $\{1, 0, -1\}$   
(c)  $\{0, -1, -2\}$  (d)  $\{1, -1, -2\}$

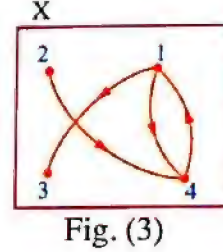
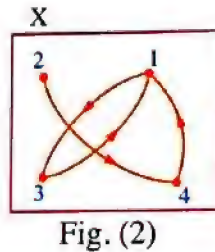
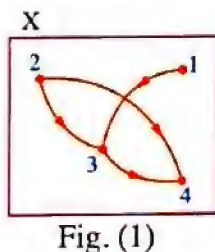


- 6 If  $R$  is a function from  $X$  to  $Y$  where  $X = \{2, 4, 5\}$ ,  $Y = \{6, 7\}$  and  $R = \{(2, 6), (a, 6), (5, 6)\}$ , then  $a = \dots\dots\dots$
- (a) 4 (b) 5 (c) 12 (d) 6

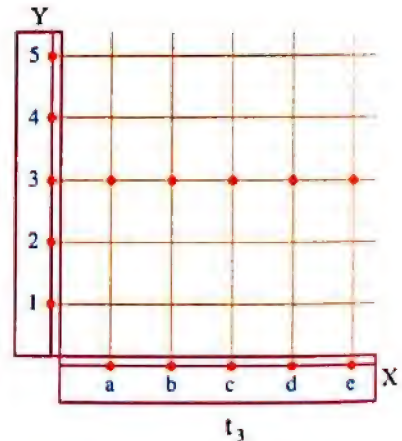
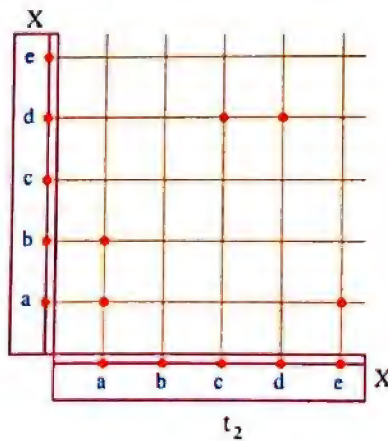
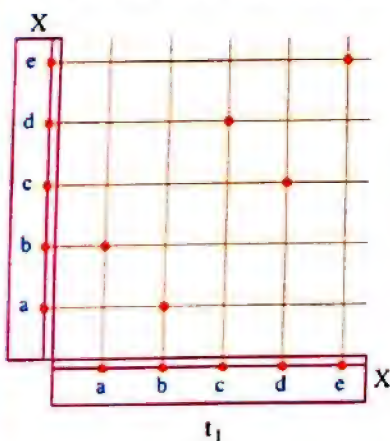
- 2 Which of the following relations represents a function from  $X$  to  $Y$  ?  
If the relation represents a function, then find the function range :



- 3 If  $X = \{1, 2, 3, 4\}$ , which of the following arrow diagrams represents a function on the set  $X$  ?

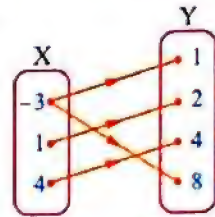


- 4 Show which of the following Cartesian diagrams represents a function, then mention the set of each function and its range :



- 5 If  $X = \{a, b, c\}$ ,  $Y = \{2, 4, 6, 8, 10\}$ , which of the following relations is a function from  $X$  to  $Y$  and which is not with giving reasons, if the relation is a function, state its range :
- 1  $R_1 = \{(a, 2), (b, 4)\}$  2  $R_2 = \{(a, 2), (b, 4), (b, 6), (c, 8)\}$
- 3  $R_3 = \{(a, 2), (b, 8), (c, 10)\}$

- 6 The opposite arrow diagram represents the relation  $R$  from the set  $X$  to the set  $Y$ , where :  $X = \{-3, 1, 4\}$  ,  $Y = \{1, 2, 4, 8\}$



1 Write  $R$

2 Is  $R$  a function ? why ?

3 Find the value of  $x$  if  $(x, 2) \in R$

(Souhag 16 – Beni Suef 17)

- 7 If  $X = \{1, 2, 3\}$  ,  $Y = \{1, 3, 6, 9, 12\}$  and  $R$  is a relation from  $X$  to  $Y$  , where " $a R b$ " means " $a = \frac{1}{3} b$ " for each  $a \in X$  ,  $b \in Y$

Write  $R$  and show that it is a function and write its range. (El-Monofia 15 – Souhag 17 – Matrouh 19)

- 8 If  $X = \{4, 6, 8, 10\}$  ,  $Y = \{2, 3, 4, 5\}$  and  $R$  is a relation from  $X$  to  $Y$  , where " $a R b$ " means " $a = 2b$ " for each  $a \in X$  ,  $b \in Y$

Write  $R$  and represent it by an arrow diagram.

(Aswan 11)

- 9 If  $X = \{1, 3, 4, 5\}$  ,  $Y = \{1, 2, 3, 4, 5, 6\}$  and  $R$  is a relation from  $X$  to  $Y$  , where " $a R b$ " means " $a + b = 7$ " for each of  $a \in X$  ,  $b \in Y$

Write  $R$  and represent it by an arrow diagram and also by a Cartesian diagram.

(El-Menia 11 – Beni Suef 15 – Port Said 17)

- 10 If  $X = \{0, 1, 4, 7\}$  ,  $Y = \{1, 3, 5, 6\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + b < 8$ " for each  $a \in X$  ,  $b \in Y$  Write  $R$  and represent it by an arrow diagram and also by a Cartesian diagram. Is  $R$  a function ? And why ?

(El-Kalyoubia 11 – Alex. 18)

- 11 If  $X = \{2, 4, 5, 7\}$  ,  $Y = \{4, 5, 6, 7, 9\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a \leq b$ " for each  $a \in X$  and  $b \in Y$

Write  $R$  and represent it by an arrow diagram and also by a Cartesian diagram.

- 12 If  $X = \{1, 2, 3, 4\}$  ,  $Y = \{y : y \in \mathbb{N}, y \text{ is an even number } \leq 10\}$  where  $\mathbb{N}$  is the set of natural number and  $R$  is relation from  $X$  to  $Y$  where " $a R b$ " means " $a = \frac{1}{2} b$ " for each  $a \in X$  ,  $b \in Y$

1 Write  $R$  and represent it by an arrow diagram.

2 Show that  $R$  is a function from  $X$  to  $Y$  and find its range.

(El-Monofia 17)

- 13 If  $X = \{1, 2, 3\}$  ,  $Y = \{2, 3, 7\}$  and  $R$  is a relation from  $X$  to  $Y$  , where " $a R b$ " means " $a + b = \text{a prime number}$ " for each  $a \in X$  ,  $b \in Y$

1 Write  $R$  and represent it by an arrow diagram. Is  $R$  a function ?

2 If  $2 a R 3$  , then find the value of  $a$

- 14 If  $X = \{-1, 0, 1, 2, 3\}$ ,  $Y = \{0, 1, 4, 6, 9\}$  and  $R$  is a relation from  $X$  to  $Y$ , where " $a R b$ " means " $a^2 = b$ " for each  $a \in X, b \in Y$

1 Write  $R$  and represent it by a Cartesian diagram.

2 Is  $R$  a function? and why?

(Red Sea 16 – Qena 18)

- 15 If  $X = \{-2, -1, 1, 2\}$ ,  $Y = \{\frac{1}{8}, \frac{1}{3}, 1, 3, 8\}$  and  $R$  is a relation from  $X$  to  $Y$ , where " $a R b$ " means " $a^3 = b$ " for each  $a \in X, b \in Y$

Write  $R$  and represent it by an arrow diagram and also Cartesian diagram.

- 16 If  $X = \{-2, -1, 0, 1, 2\}$  and  $Y = \{4, 2, \frac{3}{2}, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $b = 2^a$ " for each  $a \in X, b \in Y$  Write  $R$  and represent it by an arrow diagram. Prove that  $R$  represents a function and mention its range.

- 17 If  $X = \{2, 5, 8\}$  and  $Y = \{10, 16, 24, 30\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a$  is a factor of  $b$ " for each  $a \in X, b \in Y$  Write  $R$  and represent it by an arrow diagram and by Cartesian diagram. Is  $R$  a function? And why?

- 18 If  $X = \{2, 3, 4\}$ ,  $Y = \{6, 8, 10, 11, 15\}$  and  $R$  is a relation from  $X$  to  $Y$ , where " $a R b$ " means " $a$  divides  $b$ " for each  $a \in X, b \in Y$  Write the relation  $R$

- 19 If  $X = \{6, 4, 2, 0, -2, -4, -6\}$  and  $R$  is a relation on  $X$  where " $a R b$ " means " $a$  is the additive inverse of  $b$ " for each  $a \in X, b \in X$  Write  $R$  and represent it by an arrow diagram and show with reason if  $R$  is a function or not? And if  $R$  is a function, mention its range.

- 20 If  $X = \{0, 1, 2, \frac{1}{2}\}$  and  $R$  is a relation on  $X$  where " $a R b$ " means " $a$  is the multiplicative inverse of  $b$ " for each  $a \in X, b \in X$  Write  $R$  and represent it by an arrow diagram and show if  $R$  is a function or not.

- 21 If  $X = \{1, 2, 3, 6, 11\}$  and  $R$  is a relation on  $X$  where " $a R b$ " means " $a + 2b$  is an odd number" for each  $a \in X, b \in X$  Write  $R$  and represent it by an arrow diagram. Is  $R$  a function? And why?

- 22 If  $X = \{x: x \in \mathbb{N}, 1 \leq x \leq 3\}$  and  $R$  is a relation on  $X$  where " $a R b$ " means " $a + b$  is divisible by 3" for each  $a \in X, b \in X$

Write  $R$  and represent it by an arrow diagram, then mention if  $R$  is a function or not?

And if  $R$  is a function, mention its range.

(Luxor 16)

- 23** If  $X = \{1, 2, 4, 6, 10\}$  and  $R$  is a relation on  $X$  where " $a R b$ " means " $a$  is a multiple of  $b$ " for each  $a \in X, b \in X$ .  
Write  $R$  and represent it by an arrow diagram and also by a Cartesian diagram.  
Is  $R$  a function? And why?
- 24** If  $X = \{34, 72, 25, 4, 5, 32, 47\}$  and  $R$  is a relation on  $X$  where " $a R b$ " means " $a$  and  $b$  have the same units digit" for each  $a \in X, b \in X$ .  
Write  $R$  and represent it by an arrow diagram.
- 25** If  $X = \{-2, 2, 5\}$ ,  $Y = \{3, 7, \ell\}$  and  $R$  is a function from  $X$  to  $Y$  where " $a R b$ " means " $b = a^2 - 1$ " for each  $a \in X$  and  $b \in Y$ .  
1 Find the value of  $\ell$   
2 Represent  $R$  by an arrow diagram.
- 26** If  $X = \{0, 4, 16\}$ ,  $Y = \{0, 2, 4\}$  show which of the following relations represents a function from  $X$  to  $Y$ .  
1  $R_1$  where " $a R_1 b$ " means " $a = b^2$ " for each  $a \in X, b \in Y$   
2  $R_2$  where " $a R_2 b$ " means " $a = \sqrt{b}$ " for each  $a \in X, b \in Y$   
3  $R_3$  where " $a R_3 b$ " means " $\frac{1}{2}a = b$ " for each  $a \in X, b \in Y$
- 27** If  $R$  is a relation on the set of natural numbers ( $\mathbb{N}$ ) where " $a R b$ " means " $a \times b = 12$ " for each  $a \in \mathbb{N}, b \in \mathbb{N}$ :  
1 If  $x R 4$ , then find the value of  $x$       2 If  $y R 3$ , then find the value of  $y$
- 28** If  $R$  is a relation on the set of the positive real numbers ( $\mathbb{R}_+$ ) where " $x R y$ " means " $y^2 = 2x$ " for each  $x \in \mathbb{R}_+, y \in \mathbb{R}_+$ , and each of the following ordered pairs belongs to  $R$ :  
 $(a, 2), (\frac{2}{9}, b), (c, 3)$  and  $(\frac{9}{32}, d)$  Find the value of each of:  $a, b, c$  and  $d$
- 29** If  $X = \{1, 0, -1\}$ ,  $R_1$  is a relation of the additive inverse on  $X$  and  $R_2$  is a relation of the multiplicative inverse on  $X$ .  
Find  $R = R_1 \cap R_2$  Is  $R$  a function on  $X$ ?

- 30 If  $X = \{1, 2, 3\}$ ,  $Y = \{13, 31, 65, 23\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a$  is a digit of the number  $b$ " for each  $a \in X, b \in Y$

- 1 Write  $R$  and represent it by an arrow diagram.
- 2 Show which of the following is true. Give reasons  $2 R 65, 1 R 31, 3 R 13$
- 3 Write by listing method :  $M = \{(y, 23) : (y, 23) \in R\}$

- 31 If  $A = \{-1, 1, 2\}$ ,  $B = \{d : d \in \mathbb{N}\}$  and  $R$  is a relation from  $A$  to  $B$  where  $x R y$  means " $y = 2x + 3$ " for each  $x \in A, y \in B$

Write  $R$  and represent it by an arrow diagram.

- 32 If  $X = \{1, 2, 3\}$ ,  $Y = \{3, 4, 5\}$  show with reasons which of the following represents a relation from  $X$  to  $Y$

1  $L = \{(1, 3), (3, 3), (5, 3)\}$

2  $M = \{(2, 4), (1, 3), (3, 3), (3, 4)\}$

- 33 If  $X = \{1, 3, 5\}$ ,  $R$  is a function on  $X$  and  $R = \{(a, 3), (b, 1), (1, 5)\}$

Find : 1 The range of the function

2 The numerical value of the expression :  $a + b$

(El-Kalyoubia 20 – Aswan 21)



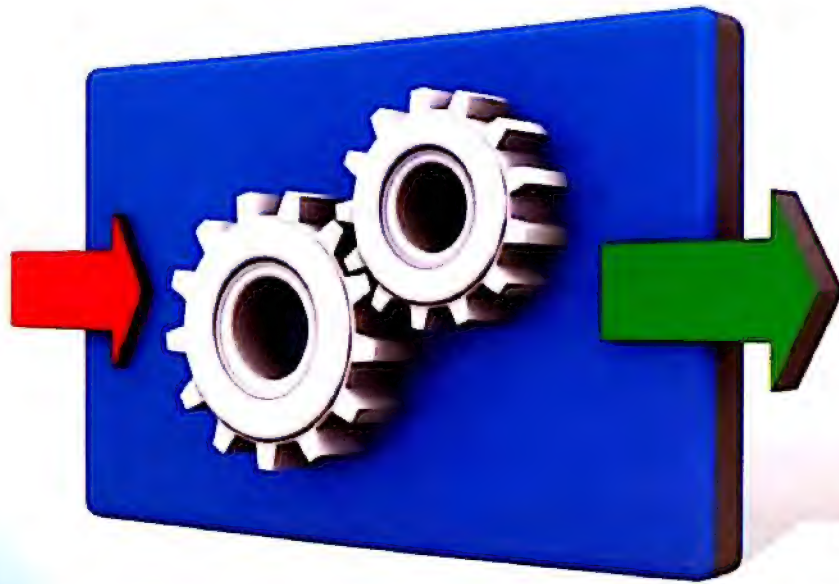
**For excellent pupils**

- 34 If  $X = \{-2, -1, 0, 1, 2\}$ ,  $Y = [0, 4[$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a^2 = b$ " for each  $a \in X, b \in Y$

Write  $R$  and mention whether  $R$  is a function from  $X$  to  $Y$  or not ? Give a reason.

- 35 If  $f$  is a function from  $X$  to  $Y$  where " $a R b$ " means " $a$  divides  $b$ " for each  $a \in X, b \in Y$   
If  $X \cup Y = \{2, 3, 5, 11, 14, 9, 35\}$ ,  $n(X) = 3$  and  $n(X \times Y) = 12$   
Find each of  $X$  and  $Y$  and write  $R$  of the function  $f$  and find its range.

- 36 If  $f$  is a function from  $X$  to  $Y$  where " $a R b$ " means " $a$  is a multiple of  $b$ " for each  $a \in X, b \in Y$  If  $n(X) = 4, n(Y) = 2$  and  $X \cup Y = \{4, 8, 9, 27\}$   
Find each of  $X$  and  $Y$  and write  $R$  of the function  $f$  and find its range.



## EXERCISE

# 3

## The symbolic representation of the function - Polynomial functions



From the school book

Interactive test

### 1 Choose the correct answer from those given :

- 1 The set of images of the elements of the domain of the function is called .....  
(Damietta 15 – Matrouh 16)  
(a) the rule (b) the domain (c) the range (d) the codomain
- 2 If the function  $f : X \longrightarrow Y$ , then the range of the function  $f \subset$  .....  
(Cairo 17)  
(a)  $X \times Y$  (b)  $X$  (c)  $Y \times X$  (d)  $Y$
- 3 The function  $f$  where  $f(x) = x^4 - 2x^3 + 7$  is a polynomial function of the ..... degree.  
(South Sinai 19 – Suez 15)  
(a) first (b) second (c) third (d) fourth
- 4 The function  $f : f(x) = x(x - 2x^2)$  is a polynomial of the ..... degree.  
(a) first (b) second (c) third (d) fourth
- 5 The function  $f$  where  $f(x) = x^2 - (x^2 - 3x)$  is a polynomial of the ..... degree.  
(a) first (b) second (c) third (d) fourth  
(Port Said 16)
- 6 The function  $f : f(x) = x^2(x - 3)^2$  is a polynomial of the ..... degree.  
(a) first (b) second (c) third (d) fourth
- 7 The function  $f : f(x) = (x - 5)^3$  is a polynomial function of the ..... degree. (Qena 11)  
(a) zero (b) second (c) third (d) fourth
- 8 If  $f(x) = x^2 - x + 3$ , then  $f(-2) =$  .....  
(a) -2 (b) -1 (c) 5 (d) 9

- 9 If  $f(x) = x^2 - \sqrt{2}x$ , then  $f(\sqrt{2}) = \dots\dots\dots$  (El-Dakahlia 11)  
 (a) 4 (b) 2 (c) 6 (d) zero
- 10 If the function  $f: \mathbb{Z} \longrightarrow \mathbb{Z}$  where  $f(x) = x^2$ , then  $f(2) + f(-2) = \dots\dots\dots$   
 (a) 0 (b) 4 (c) 8 (d) -8
- 11 If  $f(x) = kx + 8$ ,  $f(2) = 0$ , then  $k = \dots\dots\dots$  (El-Sharkia 15 - El-Dakhlia 20)  
 (a) 8 (b) 6 (c) 4 (d) -4
- 12 If  $f(x) = x - 5$  and  $\frac{1}{2}f(a) = 3$ , then  $a = \dots\dots\dots$   
 (a) 2 (b) 8 (c) 11 (d) 16
- 13 If  $f: \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = x^{k-2} + 3$ ,  $f(2) = 11$ , then  $k = \dots\dots\dots$  (El-Sharkia 20)  
 (a) 5 (b) 3 (c) 2 (d) -3
- 14 If  $(-1, 0) \in$  the set of the function  $f$  where  $f(x) = mx + 2$ , then  $m = \dots\dots\dots$   
 (a) 0 (b) -1 (c) 2 (d) -2
- 15 If  $(3, y) \in$  the set of the function  $f$  where  $f(x) = x + 2$ , then  $y = \dots\dots\dots$   
 (a) 5 (b) 3 (c) 2 (d) 1
- 16 If  $(a, a) \in$  the set of the function  $f$  where  $f(x) = 2x + 3$ , then  $a = \dots\dots\dots$   
 (a) 2 (b) 3 (c) -3 (d) -2
- 17 If  $f(x+3) = x - 3$ , then  $f(7) = \dots\dots\dots$  (El-Dakahlia 19)  
 (a) 4 (b) 1 (c) 7 (d) 10
- 18 If  $X = \{2, 4, 6\}$ ,  $n(Y) = 4$  and the function  $f: X \longrightarrow Y$ ,  $f(x) = x^2 - 1$ , then  $Y$  may be  $\dots\dots\dots$   
 (a)  $\{3, 7, 13\}$  (b)  $\{3, 15, 25, 45\}$   
 (c)  $\{3, 15, 35\}$  (d)  $\{3, 15, 25, 35\}$
- 19 If  $f(x) = nx^2 + 2x^n - 3$ , then the possible values of  $n$  such that  $f$  is a function of second degree is  $\dots\dots\dots$  (El-Dakahlia 16)  
 (a)  $\{2, 3\}$  (b)  $\{1, -1\}$  (c)  $\{2, 1, 0\}$  (d)  $\{2, 1\}$

**2 Which of the following functions represents a polynomial function :**

- |                                  |   |
|----------------------------------|---|
| 1 $f: f(x) = 2x - 5$             | 2 $f: f(x) = 3$                                 |
| 3 $f: f(x) = x + \frac{1}{x}$    | 4 $f: f(x) = x^3 + x^2 + 3$                     |
| 5 $f: f(x) = x^2 + \sqrt{x} + 8$ | 6 $f: f(x) = x\left(x + \frac{1}{x} - 2\right)$ |
| 7 $f: f(x) = \sqrt[3]{x} + 8$    | 8 $f: f(x) = x(x^2 + x^{-2} - 4)$               |

3 If  $f : \mathbb{R} \longrightarrow \mathbb{R}$ , mention the degree of  $f$ , then find  $f(-2)$ ,  $f(0)$ ,  $f\left(\frac{1}{2}\right)$  when :

1  $f(x) = 3 - 2x$

2  $f(x) = x^2 - 4$

4 If  $f(x) = 2x^2 - 5x + 2$ , then prove that :  $f(2) = f\left(\frac{1}{2}\right)$  (Luxor 14)

5 If  $f(x) = 2x - 1$ , then prove that :  $f(2) - 3f(1) = \text{zero}$  (El-Gharbia 11)

6 If  $f(x) = x^2 - 3x$ ,  $g(x) = x - 3$  (El-Menia 17 - Alex.18 - Qena 19 - Port Said 20)

1 Find :  $f(\sqrt{2}) + 3g(\sqrt{2})$

2 Prove that :  $f(3) = g(3) = 0$

7 If  $f(x) = x^2 - 2x - 5$ , then prove that :  $f(1 + \sqrt{6}) = f(1 - \sqrt{6}) = 0$

8 The function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = ax^2 + bx + 5$ ,  $a = \text{zero}$  and  $b$  is a real number does not equal to zero.

1 Find the degree of the function  $f$

2 If  $f(3) = 11$ , then find the value of  $b$  (El-Menia 18) « 2 »

9 If  $f(x) = 5x - b$  and  $h(x) = x - 2b$  and  $f(1) + h(3) = -7$ , then find :  $f(3) + h(1)$  « 1 »

10 If the function  $f : \mathbb{Z} \longrightarrow \mathbb{N}$  where  $f(x) = (x - 3)^2$  and the function  $t : \mathbb{Z} \longrightarrow \mathbb{N}$  where  $t(x) = x - 3$ , then find the value of  $x$  which makes :  $f(x) = t(x)$  « 3 or 4 »

11 If  $f$  is a function on  $X$  where  $X = \{3, 4, 5, 6\}$  and  $f(3) = 3$ ,  $f(4) = 5$ ,  $f(5) = 5$ ,  $f(6) = 5$

1 Represent  $f$  by an arrow diagram.

2 Write the set of  $f$  and mention its range. (Ismailia 15)

12 If  $X = \{0, 1, 3\}$ ,  $Y = \{1, 2, 3, 4, 5, 7\}$  and the function  $f : X \longrightarrow Y$  where  $f(x) = 5 - x$

1 Find the range of  $f$  (New Valley 17)

2 Draw a Cartesian diagram for the function  $f$

13 If the function  $t : \mathbb{N} \longrightarrow \mathbb{N}$  where  $t : x \longrightarrow 2x + 3$

1 Find :  $t(0)$ ,  $t(1)$ ,  $t(2)$ ,  $t(3)$ ,  $t(4)$ ,  $t(5)$

2 Represent five elements of the elements of  $t$  on a part of a square net of the Cartesian product  $\mathbb{N} \times \mathbb{N}$

3 What is the range of  $t$ ?

**14** If the function  $f : \mathbb{Z} \longrightarrow \mathbb{Z}$  where  $\mathbb{Z}$  is the set of integers ,  $f(x) = x^2 - 2x - 3$

**1** Find :  $f(4)$  ,  $f(3)$  ,  $f(2)$  ,  $f(1)$  ,  $f(0)$  ,  $f(-1)$  ,  $f(-2)$

**2** Draw a part of the perpendicular square net of the Cartesian product  $\mathbb{Z} \times \mathbb{Z}$  and represent on it seven elements of the elements of  $f$

**3** If  $f(x) = 5$  , find the value of  $x$

« 4 or - 2 »

**15** If  $f(x) = ax + b$  ,  $f(a) = b$  Find the value of :  $a^2 + 5$

(El-Sharkia 19) « 5 »

**16** If the set of the function  $f = \{(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)\}$  write :

**1** The domain of the function  $f$

**2** The range of the function  $f$

**3** The rule of the function  $f$

(Damietta 16 – North Sinai 17 – Luxor 19)



**For excellent pupils**

**17** If  $f(x) = 2x^2 + bx + c$  and  $f(x) = 0$  , when  $x \in \{0, 3\}$

Find the value of each of  $b$  and  $c$

« - 6 »

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## EXERCISE

# 4

## The study of some polynomial functions



From the school book

Interactive test

1 Choose the correct answer from those given :

1 If  $f(x) = 7$  , then  $f(-3) = \dots\dots\dots$  (Giza 17)  
 (a) 7 (b) -7 (c) 21 (d) -21

2 If  $f(x) = 2$  , then  $3f(\sqrt{2}) = \dots\dots\dots$   
 (a)  $f(3\sqrt{2})$  (b) 6 (c) 3 (d) 2

3 If  $f(x) = 2$  , then  $f(3) - f(1) = \dots\dots\dots$  (El-Dakahlia 13)  
 (a)  $f(2)$  (b) 2 (c) zero (d) 10

4 If  $f(x) = 5$  , then  $\frac{f(5)}{f(10)} = \dots\dots\dots$   
 (a) 5 (b)  $\frac{1}{2}$  (c) 1 (d) 10

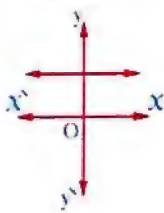
5 If  $f$  is a function such that  $f: \mathbb{R} \longrightarrow \mathbb{R}$  ,  $f(x) = 3$  , then  $\frac{f(6)}{f(\text{zero})} = \dots\dots\dots$  (El-Dakahlia 17)  
 (a) 6 (b) 1 (c) 3 (d) undefined

6 If  $f(x) = 3$  , then  $\frac{2f(3)}{3f(2)} = \dots\dots\dots$  (Alex, 05)  
 (a)  $\frac{2}{3}$  (b)  $\frac{3}{2}$  (c) 1 (d)  $\frac{32}{23}$

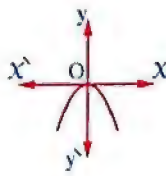
7 If  $f(2x) = 4$  , then  $f(-x) = \dots\dots\dots$  (El-Dakahlia 09)  
 (a) -2 (b) -4 (c) 4 (d) 2

8 The function  $f$  where  $f(x) = 3x$  is represented graphically by a straight line which passes through the point  $\dots\dots\dots$  (Beni Suef 17)  
 (a) (3 , 3) (b) (3 , 0) (c) (0 , 0) (d) (0 , 3)

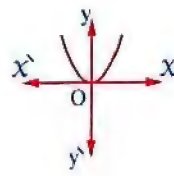
- 9 If the straight line which represents the function  $f : f(x) = 2x - a$  passes through the origin point, then  $a = \dots\dots\dots$  (El-Fayoum 17)  
 (a)  $-2$  (b)  $2$  (c) zero (d)  $3$
- 10 If the point  $(a, 4)$  one of the points of function  $g : \mathbb{R} \longrightarrow \mathbb{R}$  where  $g(x) = 2x + b$ , then  $6a + 3b = \dots\dots\dots$  (El-Dakahlia 17)  
 (a)  $12$  (b)  $9$  (c)  $6$  (d)  $3$
- 11 If the point  $(3, 2)$  is the vertex of the curve of the quadratic function  $f$ , then the equation of the line of symmetry is  $\dots\dots\dots$   
 (a)  $x = 3$  (b)  $x = 2$  (c)  $y = 3$  (d)  $y = -3$
- 12 The vertex of the curve of the function  $f : f(x) = 2x^2 - 4x + 5$  is  $\dots\dots\dots$   
 (a)  $(-1, 11)$  (b)  $(1, 3)$  (c)  $(2, 5)$  (d)  $(3, 11)$
- 13 The equation of the axis of symmetry of the curve of the function  $f : f(x) = (x - 2)^2$  is  $\dots\dots\dots$   
 (a)  $x = 0$  (b)  $x = 2$  (c)  $x = -2$  (d)  $x = -4$
- 14 The graph of the function  $f : f(x) = x^2 - 2x + 1$  is the graph number  $\dots\dots\dots$  (Giza 08)



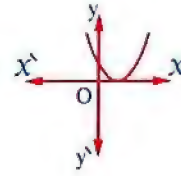
(a)



(b)



(c)



(d)

15 In the opposite figure :

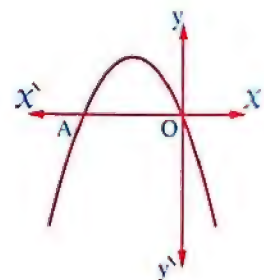
The curve of a quadratic function,  $A(-4, 0)$

, then the equation of the axis

of symmetry is  $x = \dots\dots\dots$

(El-Dakahlia 19)

- (a)  $1$  (b)  $-1$   
 (c)  $-2$  (d)  $0$



- 16 If  $f(x) = x^2$ ,  $x \in [-2, 2]$ , then  $f(x) \in \dots\dots\dots$   
 (a)  $]0, 4]$  (b)  $]0, 4[$  (c)  $[0, 4]$  (d)  $[-4, 4[$

2 Complete the following :

- 1 The function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = 5$  is represented by a straight line parallel to  $\dots\dots\dots$  and intersects y-axis at the point  $\dots\dots\dots$

- 2 The linear function given by the rule  $y = 2x - 1$  is represented graphically by a straight line intersecting the y-axis at the point .....
- 3 The linear function given by the rule  $y = 3x + 6$  is represented graphically by a straight line intersecting the x-axis at the point .....
- 4 If the point  $(a, 3)$  is located on the straight line which represents the function  $f: \mathbb{R} \longrightarrow \mathbb{R}$ , where  $f(x) = 4x - 5$ , then  $a = \dots\dots\dots$  (El-Beheira 11)
- 5 The straight line which represents the function  $f: \mathbb{R} \longrightarrow \mathbb{R}$ , where  $f(x) = 5x - a$  intersects the y-axis at the point  $(b, 2)$ , then  $a = \dots\dots\dots$  and  $b = \dots\dots\dots$
- 6 The equation of the axis of symmetry of the function  $f: f(x) = x^2$  is .....
- (Souhag 09)
- 7 If the curve of the function  $f$  such that  $f(x) = x^2 + c$  passes through the point  $(0, 2)$ , then  $c = \dots\dots\dots$  (Damietta 11)
- 8 If  $(-2, y)$  belongs to the curve of the function  $f: f(x) = x^2 + 1$ , then  $y = \dots\dots\dots$  (South Sinai 09)

3 Represent the following functions graphically, where  $x \in \mathbb{R}$ :

1  $f(x) = 5$

2  $f(x) = -4$

3  $f(x) = 0$

4  $f(x) = 2 \frac{1}{2}$

4 Represent each of the following linear functions graphically and find the points of intersection of the straight line which represents each of them with the coordinate axes, where  $x \in \mathbb{R}$ :

1  $f: f(x) = x$

2  $f: f(x) = -x$

3  $f: f(x) = 3x$

4  $f: f(x) = -2x$

5  $f: f(x) = x + 2$

6  $f: f(x) = 2 - x$

7  $f: f(x) = 3x - 1$

8  $f: f(x) = -2x + 3$

9  $f: f(x) = \frac{1}{2}x$

10  $f: f(x) = 5 - \frac{1}{2}x$

5 Represent each of the following functions graphically and from the graph, deduce the coordinates of the vertex of the curve and the equation of the line of symmetry and the maximum or minimum value of the function, where  $x \in \mathbb{R}$ :

1  $f: f(x) = 2x^2$  taking  $x \in [-2, 2]$

2  $f: f(x) = x^2 + 1$  taking  $x \in [-3, 3]$

(Beni Suef 14)

3  $f: f(x) = x^2 - 2$  taking  $x \in [-3, 3]$  (El-Beheira 17 - El-Port Said 18 - Damietta 20)

4  $f : f(x) = 2 - x^2$  taking  $x \in [-3, 3]$

(Alex. 18 – El-Gharbia 19 – Souhag 20)

5  $f : f(x) = x^2 - 2x$  taking  $x \in [-2, 4]$

(Qena 11 – Cairo 18 – Kafr El-Sheikh 20)

6  $f : f(x) = x^2 + 2x + 1$  taking  $x \in [-4, 2]$

(El-Sharkia 17 – El-Gharbia 18)

7  $f : f(x) = (x - 2)^2$  taking  $x \in [-1, 5]$

(Kafr El-Sheikh 19 – El-Gharbia 20)

8  $f : f(x) = x(x - 2) - 3$  taking  $x \in [-2, 4]$

(El-Dakahlia 17)

9  $f : f(x) = 3 - 2x - x^2$  taking  $x \in [-4, 2]$

10  $f : f(x) = 4x + 3 - 2x^2$  taking  $x \in [-2, 3]$

11  $f : f(x) = x^2 - 4x + 5$  taking  $x \in [0, 5]$

12  $f : f(x) = 1 - 3x + x^2$  taking  $x \in [-1, 4]$

- 6 If the function  $f : f(x) = 3x - 6$  is represented by a straight line passing through the point  $(a, 2a)$ , find the value of  $a$ , then find the intersection point of the straight line with the y-axis

(El-Gharbia 20)  $\alpha 6, (0, -6)$

- 7 If the straight line representing the function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = ax + b$  cuts a positive part of the y-axis of length 3 units and passes through the point  $(1, 5)$ . Find the value of each of :  $a, b$

(Kafr El-Sheikh 20)  $\alpha 2, 3$

- 8 If the straight line which represents the function  $f : \mathbb{R} \longrightarrow \mathbb{R}$ ,  $f(x) = ax + b$  intersects the x-axis at the point  $(3, 0)$  and intersects the y-axis at the point  $(0, -3)$ , then find the value of the two constants :  $a$  and  $b$  and find the value of  $f(1)$

(El-Sharkia 17)

- 9 If the curve of the function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = m - x^2$  intersects x-axis at the point  $(-2, b)$ . Find the value of :  $m^b + 2m$

(El-Sharkia 15)

- 10 If  $X = \{2, 3, 6\}$ ,  $Y = \{3, 4, 5, 6, 7, 8\}$  and  $r : X \longrightarrow Y$  where  $r(x) = 9 - x$

1 Find the set of images of the elements of the set  $X$  by the function  $r$

2 Is  $r$  a linear function ? "state the reason"

(El-Dakahlia 14)

- 11 If  $f(x) = a + x^2$ ,  $l(x) = c$  are two polynomial functions where  $3f(2) + 3l(x) = 6$ , find the numerical value of :  $2f(0) + 2l(7)$  where  $a$  and  $c$  are constants.

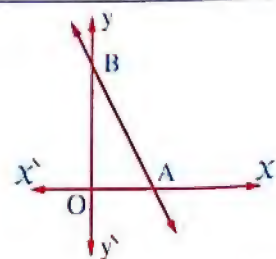
(El-Dakahlia 19)  $\alpha -4$

- 12 The opposite figure represents the function  $f$  where  $f(x) = 4 - 2x$

Find :

1 The coordinates of  $A, B$

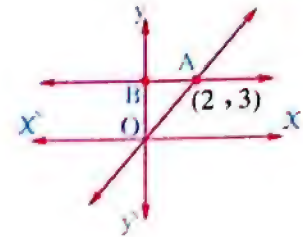
2 The area of  $\triangle AOB$



(Ismailia 16 – Luxor 19)

## 13 In the opposite figure :

The constant function  $f$  is represented graphically by straight line  $\overline{BA}$  and the linear function  $g$  is represented graphically by straight line  $\overline{OA}$  where  $A = (2, 3)$



1 Write the rule of the function  $f$  and the rule of the function  $g$

2 Find the value of :  $f(-10) + g(6)$

(El-Sharkia 14) ~ 12 ~

14 The opposite figure shows the straight line  $\overline{AB}$ 

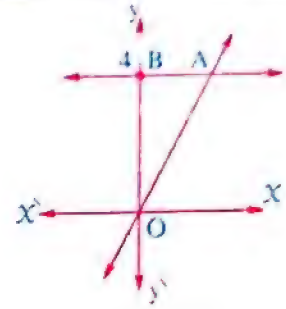
which represents the function  $f : f(x) = 4$

, if  $\overline{AO}$  represents the linear function

$g : g(x) = nx + k$  and the area of the

triangle ABO equals 4 square units

, then find the values of  $n$  and  $k$ , where  $O$  is the origin point.



(El-Dakahlia 17)

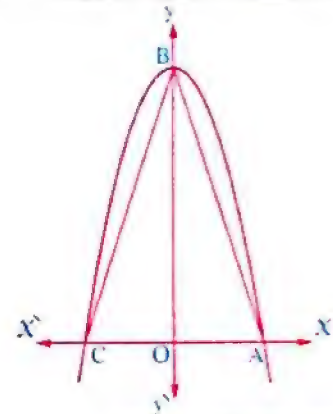
15 The opposite figure represents the curve of the function  $f$ 

where :  $f(x) = 9 - x^2$

Find :

1 The coordinates of A and C

2 The area of the triangle ABC



(Kafr El-Sheikh 18)

## 16 The opposite figure represents the curve of the function

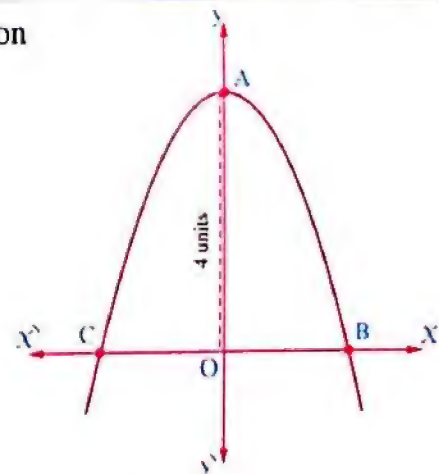
$f$  where  $f(x) = m - x^2$ ,

if  $OA = 4$  units

Find : 1 The value of  $m$

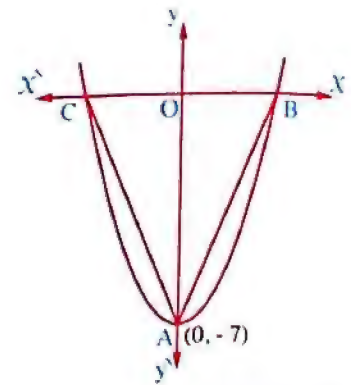
2 The coordinates of B and C

3 The area of the triangle with vertices A, B and C



(North Sinai 16 - Luxor 18 - Giza 20)

- 17** The opposite figure represents the curve of the function  $f : f(x) = \ell x^2 - 7$ , the area of triangle ABC = 21 square units, A (0, -7). Find the coordinates of the point B, then find the value of  $\ell$ .



(El-Dakahlia 18)

**18 In the opposite figure :**

The curve represents a function of second degree  $f$  :

- 1** Write the domain of  $f$

Use the graph to find :

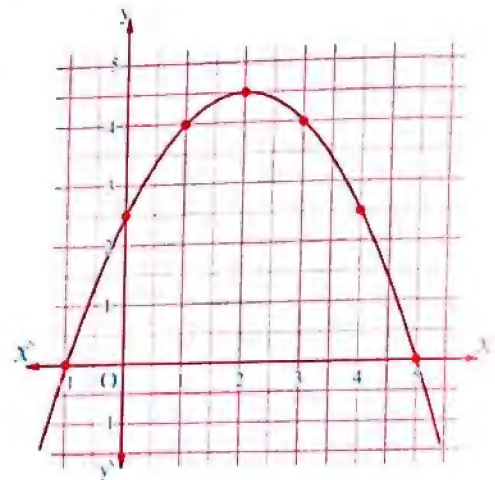
- 2** The range of the function  $f$

- 3** The equation of the line symmetry of the curve of function  $f$

- 4** The maximum value of  $f$

- 5** The value of  $f(1)$

- 6** If  $f(x) = a(x - 2)^2 + k$ , then find the numerical value of :  $a + k$



(El-Dakahlia 16)

**Life Application**

- 19** While Karim was reading a book, he found that after 3 hours, 50 pages remained and after 6 hours, 20 pages remained. If the relation between time (t) and the number of pages (b) is a linear relation :

- 1** Represent graphically the relation between t and b then find the algebraic relation between the two variables.

- 2** What is the time that should be taken to finish the book ?

- 3** What is the number of pages remaining when Karim began to read ?

(Ismailia 20)



## For excellent pupils

20 In the opposite figure :

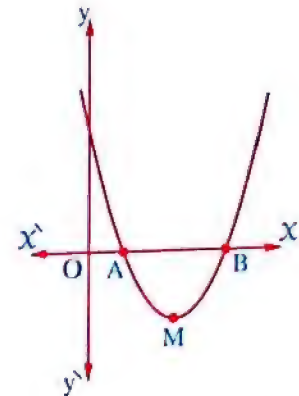
If the curve of the function

$f$  intersects the  $X$ -axis at two points :

$A(1, 0)$ ,  $B(4, 0)$  and  $M$  is the point  
of the vertex of the curve

If  $f(-2) + f(7) = 8$

Find :  $f(-2)$



« 4 »

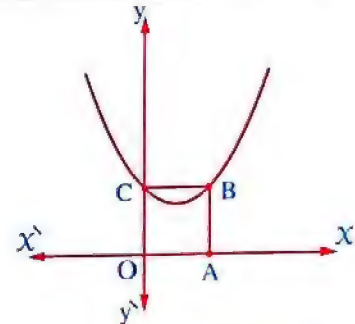
21 In the opposite figure :

The drawn curve represents the quadratic function :

$$f : f(x) = x^2 - (k-2)x - k + 4$$

If ABCO is a square

Find the value of :  $k$



(El-Dakahlia 19) « 3 »

22 In the opposite figure :

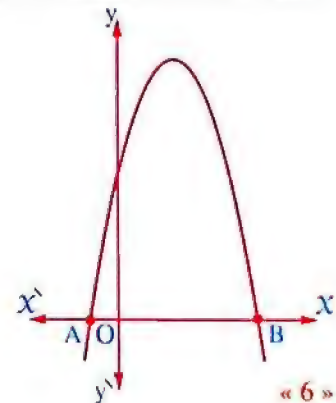
The curve represents the function :

$$f : f(x) = -x^2 + 4x + k - 1 \text{ and intersects}$$

the  $X$ -axis at two points :  $A$  and  $B$

If  $OB = 5 OA$

Find the value of :  $k$



« 6 »

# Summary of Unit I



## The Cartesian product :

- ★ For any two finite non empty sets  $X$  and  $Y$

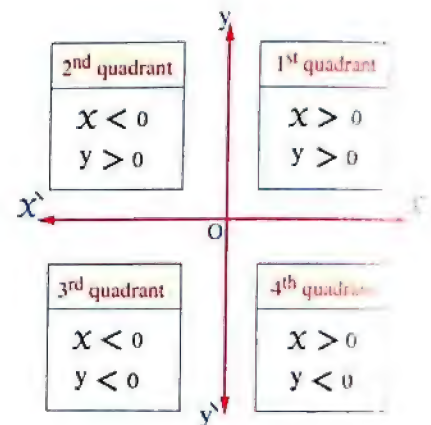
The Cartesian product of  $X \times Y$

is the set of all ordered pairs whose first projection of each belongs to  $X$  and the second projection of each belongs to  $Y$

*i.e.*  $X \times Y = \{(a, b) : a \in X, b \in Y\}$

**Note that :**  $X \times Y \neq Y \times X$  where  $X \neq Y$

- ★ For any set  $X$  :  $X \times \emptyset = \emptyset \times X = \emptyset$
- ★ The two axes  $\overrightarrow{XX}$  and  $\overrightarrow{yy}$  divide the plane into four quadrants as shown in the opposite figure and you can determine the quadrant in which any point lies by knowing the signs of its two coordinates.
- ★ If the  $X$ -coordinate of the point = 0  
 , then the point lies on  $y$ -axis
- ★ If the  $y$ -coordinate of the point = 0  
 , then the point lies on  $X$ -axis



## The relation :

- ★ The relation from the set  $X$  to the set  $Y$  is a connection joining some or all the elements of  $X$  with some or all the elements of  $Y$
- ★ If  $R$  is a relation from the set  $X$  to the set  $Y$  , then :
  - $R$  is a set of ordered pairs such that the first projection belongs to  $X$  and the second projection belongs to  $Y$
  - $R \subset X \times Y$
- ★ If  $R$  is a relation from  $X$  to  $X$  , then  $R$  is a relation on  $X$  and  $R \subset X \times X$

### The function :

★ A relation from  $X$  to  $Y$  is said to be a function if one of the following cases is satisfied :

- 1 Each element of the set  $X$  appears only once as a first projection in one of the ordered pairs of the relation.
- 2 Each element of the set  $X$  has one and only one arrow going out of it to an element of  $Y$  in the arrow diagram which represents the relation.
- 3 Each vertical line has only one point lying on it of the points which represent the relation, in the Cartesian diagram which represents the relation.

★ If  $f$  is a function from the set  $X$  to the set  $Y$  is written as  $f : X \longrightarrow Y$ , then :

- 1  $X$  is called the domain of the function  $f$
- 2  $Y$  is called the codomain of the function  $f$
- 3 The set of images of the elements of the set  $X$  by the function  $f$  is called the range of the function  $f$  which is a subset of the codomain  $Y$

### The polynomial functions :

★ The polynomial function is a function whose rule is a term or an algebraic expression satisfying the following conditions :

- 1 Each of the domain and the codomain of the function is the set of real numbers  $\mathbb{R}$
- 2 The power (the index) of the variable  $X$  in any of its terms is a natural number

**Notice that :** the degree of the function is the highest power of the variable  $X$

★ **The constant function :** The function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(X) = b$ ,  $b \in \mathbb{R}$  is called a constant function and is represented by a straight line parallel to  $X$ -axis and intersects  $y$ -axis at the point  $(0, b)$

★ **The linear function :** The function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(X) = aX + b$ ,  $a \in \mathbb{R} - \{0\}$ ,  $b \in \mathbb{R}$  is called linear function (function of first degree) and is represented by a straight line intersects  $y$ -axis at  $(0, b)$  and  $X$ -axis at  $\left(\frac{-b}{a}, 0\right)$

★ **The quadratic function :** The function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(X) = aX^2 + bX + c$  where  $a, b$  and  $c$  are real numbers,  $a \neq 0$  is called a quadratic function and it is a polynomial function of the second degree and it is represented by a curve whose vertex is  $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

# Exams on Unit One



## Model 1

Answer the following questions :

1 Choose the correct answer from those given :

- 1 If  $X \times Y = \{(2, 3), (2, 5)\}$ , then  $n(X) = \dots\dots\dots$ 
  - (a) 1
  - (b) 2
  - (c) 3
  - (d) 5
- 2 If  $b < 3$ , then the point  $(6, b - 3)$  lies in the  $\dots\dots\dots$  quadrant.
  - (a) first
  - (b) second
  - (c) third
  - (d) fourth
- 3 If the point  $(X, 3)$  lies on y-axis, then  $7X - 1 = \dots\dots\dots$ 
  - (a) 20
  - (b) -1
  - (c) 6
  - (d) 8
- 4 If  $f(X) = 4X + b$ ,  $\frac{1}{3}f(3) = 5$ , then  $b = \dots\dots\dots$ 
  - (a) -57
  - (b) 3
  - (c) 4
  - (d) -3
- 5 The function  $f$  where  $f(X) = 3X$  is represented graphically by a straight line which passes through the point  $\dots\dots\dots$ 
  - (a)  $(3, 3)$
  - (b)  $(3, 0)$
  - (c)  $(0, 0)$
  - (d)  $(0, 3)$
- 6 The maximum value of the function  $f : f(X) = -2X^2 + 4X + 3$  is  $\dots\dots\dots$ 
  - (a) 5
  - (b) 1
  - (c) 3
  - (d) -1

2 [a] If  $X = \{1, 2, 3\}$ ,  $Y = \{0, 1, 2, 3, 4\}$  and  $R$  is a relation from  $X$  to  $Y$  where "a R b" means " $b - a = 1$ " for each  $a \in X, b \in Y$   
Write  $R$  and show that it is a function and write its range.

[b] If  $X = \{2, 3\}$ ,  $Y = \{3, 5\}$  and  $Z = \{7, 5\}$ , then find :

- 1  $X \times (Y \cap Z)$
- 2  $(X - Y) \times Z$
- 3  $(X - Y) \times (Y - Z)$

3 [a] If the set of the function  $f = \{(3, 9), (5, 15), (7, 21)\}$  write :

- 1 The domain of the function  $f$
- 2 The range of the function  $f$
- 3 The rule of the function  $f$

[b] Represent graphically the function  $f : f(X) = -2X + 3$  and find the two intersection points of the straight line which represents the function with the coordinates axes, where  $X \in \mathbb{R}$

- 4 [a] If  $X = \{-1, 1, 0, 2\}$  and  $R$  is a relation on  $X$  where " $a R b$ " means " $b = a^2$ " for each  $(a, b) \in X^2$

1 Write  $R$  and represent it by an arrow diagram.

2 Is  $R$  a function? And why?

- [b] Represent graphically the function  $f : f(x) = (x-2)^2 + 1$  taking  $x \in [0, 4]$  and from the graph deduce :

1 The coordinates of the vertex of the curve.

2 The equation of the line of symmetry.

3 The maximum or minimum value of the function.

- 5 [a] If  $f$  and  $g$  are two functions where  $f(x) = -2x + 3$  and  $g(x) = -7$

1 Find the degree of the function  $f$

2 Calculate the value of :  $f(0) + g(0)$

- [b] The opposite figure represents the curve of the quadratic function  $f$  where :

$$f(x) = 4 - kx^2, (k \text{ constant } \neq 0)$$

,  $A(0, 4)$  is the vertex of the curve

,  $O$  is the origin point,  $B, C \in x$ -axis

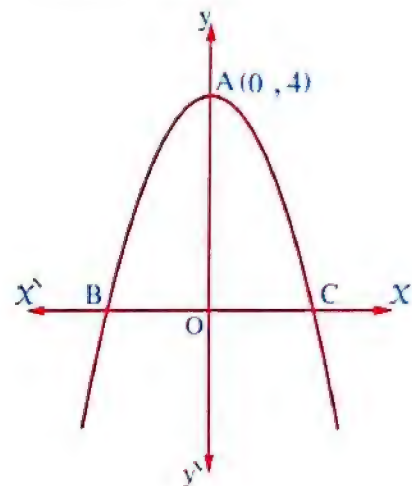
The area of the triangle with vertices  $A, B$  and  $C$  equals 8 square units.

Find :

1 The equation of the symmetry axis, and the maximum value of the function  $f$

2 The coordinates of  $B$

3 The value of  $k$



### Model 2

Answer the following questions :

- 1 Choose the correct answer from those given :

1 If the point  $(a, 3 - a)$  is located on the  $x$ -axis, then  $a = \dots\dots\dots$

(a) zero

(b) 3

(c) -3

(d) 5

2 If  $f(x) = x^3$ , then  $f(2) + f(-2) = \dots\dots\dots$

(a) 16

(b) zero

(c) -16

(d) 4

- 3 If  $X \times Y = \{(2, 3)\}$ , then  $X^2 = \dots\dots\dots$   
 (a)  $\{(4, 9)\}$  (b)  $\{(4, 3)\}$  (c)  $\{(2, 2)\}$  (d)  $\{(2, 9)\}$
- 4 If  $f : f(X) = 5$  is represented by a straight line parallel to the  $X$ -axis, then it passes through the point  $\dots\dots\dots$   
 (a)  $(0, 5)$  (b)  $(5, 0)$  (c)  $(5, -5)$  (d)  $(0, 0)$
- 5 If  $R$  is a function from  $X$  to  $Y$  where  $X = \{3, 5, 7\}$ ,  $Y = \{4, 9\}$  and  $R = \{(3, 4), (b, 9), (5, 9)\}$ , then  $b = \dots\dots\dots$   
 (a) 3 (b) 5 (c) 7 (d) 9
- 6 The function  $f : f(X) = (X - 5)^3$  is a polynomial of the  $\dots\dots\dots$  degree.  
 (a) first (b) second (c) third (d) fourth

- 2 [a] If  $X = \{1, 4, 7\}$ ,  $Y = \{-1, 1, 4, 7\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + |b| = 8$ " for each  $a \in X, b \in Y$   
 1 Write  $R$  and represent it by an arrow diagram.  
 2 Show if  $R$  is a function or not, with reason.
- [b] If  $X \times Y = \{(1, 1), (1, 5), (1, 3), (4, 1), (4, 5), (4, 3)\}$   
 Find : 1  $Y \times X$  2  $X$  and  $X^2$

- 3 [a] Represent graphically the function  $f : f(X) = X^2 + 2X - 4$  taking  $X \in [-4, 2]$  and from the graph deduce :  
 1 The coordinates of the vertex of the curve  
 2 The equation of the line of symmetry.  
 3 The maximum or minimum value of the function.
- [b] If the straight line which represents the function  $f$  where  $f : \mathbb{R} \longrightarrow \mathbb{R}$ ,  $f(X) = 2X - 3k$  intersects the  $X$ -axis at the point  $(6, m - 2)$   
 Find the value of each of :  $m, k$

- 4 [a] If  $(X - 2, 9) = (5, X + y)$  Find the value of :  $\sqrt{3X + 2y}$   
 [b] If  $X = \{1, 2\}$ ,  $Y = \{2, 5\}$  and  $Z = \{4, 5\}$   
 Find : 1  $(X - Y) \times Z$  2  $n(X \times Y) + n(Z^2)$

- 5 [a] If  $X = \{4, 5, 7\}$  and  $R$  is a function on  $X$ ,  $R = \{(a, 5), (b, 5), (4, 7)\}$   
 Find : 1 The numerical value of the expression :  $3a + 3b$   
 2 The range of the function.
- [b] If  $f : f(X) = 2X^2 - 5X + 2$   
 Find the value of :  $f(2) - f\left(\frac{1}{2}\right)$

# A Research Project

## On Unit One



### Project aims :

- Graphing quadratic function.
- Associating mathematics with computer science.

### Do a research project on the following topic :

*"The computer has become one of the important tools in different sciences, one of which is math".*

**Discuss the following points using available resources :**

- Write a short note on the Visual Basic language.
- Use one of the Dynamic Mathematics Software (DMS) like GeoGebra ([www.geogebra.org](http://www.geogebra.org)) to :
  1. Graph the function  $f : f(x) = x^2$
  2. Graph the function  $r : r(x) = (x - 1)^2$  , on the same figure.
  3. Graph the function  $n : n(x) = (x + 1)^2$  , on the same figure.
  4. Compare the curve of function  $r$  with that of function  $f$  , then compare the curve of function  $n$  with that of function  $f$  , and write what do you observe.
  5. Describe how  $g : g(x) = (x - 3)^2$  and  $k : k(x) = (x + 4)^2$  will appear after graphing them.

# UNIT 2

## Ratio, proportion, direct variation and inverse variation



### Exercises of the unit :

5. Ratio and proportion.
  6. Follow properties of proportion.
  7. Continued proportion.
  8. Direct variation and inverse variation.
- ✦ Unit Exams.
  - 🔍 A research project on unit two



Scan the  
QR code  
to solve an  
interactive  
test on each  
lesson

## EXERCISE

# 5

## Ratio and proportion



From the school book

Interactive test

### 1 Complete the following :

- 1 If  $a$  ,  $b$  ,  $c$  and  $d$  are proportional quantities , then  $c$  is called ..... *(Damietta 08)*
- 2 If the quantities  $a$  ,  $b$  ,  $c$  and  $d$  are proportional , then  $\frac{a}{b} = \dots\dots\dots$  *(El-Beheira 11)*
- 3 The fourth proportional for the numbers 4 , 12 , 16 and ... is ..... *(Kafr El-Sheikh 11)*
- 4 The second proportional for the numbers 2 , ... , 4 and 6 and is .....
- 5 The third proportional for the numbers 8 , 6 , ... and 12 is .....
- 6 The first proportional for the numbers ... , 5 , 27 and 45 is .....
- 7 If 3 ,  $a - 1$  ,  $a + 1$  and 5 are proportional , then  $a = \dots\dots\dots$
- 8 A sum of money is divided between two persons with the ratio 2 : 3  
If the share of the first is L.E. 30 , then the share of the other is L.E. ....  
*(South Sinai 09)*
- 9 If  $7x = 3y$  , then  $\frac{x}{y} = \dots\dots\dots$  *(New Valley 11)*
- 10 If  $5a - 4b = 0$  , then  $\frac{a}{b} = \dots\dots\dots$  *(Souhag 11)*
- 11 If  $\frac{5a - 7b}{8a + 11b} = 0$  , then  $\frac{b}{a} = \dots\dots\dots$  *(El-Fayoum 12)*
- 12 If  $\frac{a}{2} = \frac{b}{3}$  , then  $\frac{2a}{3b} = \dots\dots\dots$

**2 Choose the correct answer from those given :**

- 1** The ratio between the area of a square shaped region of side length  $l$  to the area of another square shaped region of side length  $2l$  is ..... *(El-Monofia 13)*

(a)  $1 : 2$  (b)  $l : 4$  (c)  $1 : 4$  (d)  $4 : 1$

- 2** If  $\frac{3a}{5b} = \frac{1}{2}$ , then  $\frac{a}{b} = \dots\dots\dots$  *(Red Sea 11 – Alex. 20)*

(a)  $\frac{6}{5}$  (b)  $\frac{5}{6}$  (c)  $\frac{2}{3}$  (d)  $\frac{3}{2}$

- 3** If  $4X = 5y$ , then  $\frac{5y}{4X} = \dots\dots\dots$  *(Qena 11)*

(a) 1 (b) 2 (c) 3 (d) 4

- 4** If  $3a = 5b$ , then  $\frac{3a}{b} = \dots\dots\dots$  *(El-Fayoum 17)*

(a) 3 (b) 5 (c)  $\frac{3}{5}$  (d)  $\frac{5}{8}$

- 5** If  $2X = 7y$ , then  $\left(\frac{X}{y}\right)^{-1} = \dots\dots\dots$  *(El-Fayoum 09)*

(a)  $\frac{2}{7}$  (b)  $\frac{7}{2}$  (c)  $\frac{49}{4}$  (d)  $\frac{4}{49}$

- 6** If  $a, b, 2$  and  $3$  are proportional, then  $\frac{b}{a} = \dots\dots\dots$  *(El-Kalyoubia 17)*

(a)  $\frac{3}{2}$  (b)  $\frac{2}{3}$  (c) 3 (d) 2

- 7** If  $a, X, b$  and  $2X$  are proportional quantities, then  $\frac{a}{b} = \dots\dots\dots$  *(Aswan 17)*

(a) 2 (b)  $\frac{1}{2}$  (c)  $\frac{1}{3}$  (d)  $\frac{1}{4}$

- 8** If  $5a, 2, 3b, 7$  are four proportional quantities, then  $\frac{a}{b} = \dots\dots\dots$  *(Souhag 13)*

(a)  $\frac{3}{7}$  (b)  $\frac{6}{35}$  (c)  $\frac{3}{5}$  (d)  $\frac{3}{2}$

- 9** If  $4X^2 = 9y^2$ , then  $\frac{X}{y} = \dots\dots\dots$  *(Beni Suef 16)*

(a)  $\frac{9}{4}$  (b)  $\frac{3}{2}$  (c)  $\pm \frac{2}{3}$  (d)  $\pm \frac{3}{2}$

- 10** If  $\frac{a+2b}{a-b} = \frac{2}{3}$ , then  $\frac{b}{a} = \dots\dots\dots$  *(Alexandria 11 – El-Monofia 20)*

(a)  $\frac{1}{8}$  (b) 8 (c)  $-\frac{1}{8}$  (d)  $-8$

- 11** If  $a, b, c$  and  $d$  are proportional quantities, then ..... *(Alexandria 11 – El-Monofia 20)*

(a)  $\frac{b}{d} = \frac{a}{c}$  (b)  $\frac{a}{c} = \frac{d}{b}$  (c)  $\frac{b}{c} = \frac{a}{d}$  (d)  $ab = cd$

- 12**  $\frac{18X^2y}{\dots\dots\dots} = \frac{3X}{5y}$

(a)  $20Xy^2$  (b)  $30Xy$  (c)  $30Xy^2$  (d)  $15Xy$

- 13**  $\frac{a+b}{a} = \frac{\dots\dots\dots}{a^2}$

(a)  $a^2 + b^2$  (b)  $a^2 + ab$  (c)  $a^2 + a^2b$  (d)  $2a + 2b$

14 If  $4x^2 + 9y^2 = 12xy$ , then  $\frac{x}{y} = \dots\dots\dots$

(El-Kalyoubia 09)

(a)  $\frac{3}{2}$

(b)  $\frac{2}{3}$

(c)  $-\frac{2}{3}$

(d)  $-\frac{3}{2}$

3 Find each of the following :

1 The first proportional for the numbers : ... ,  $\sqrt{8}$  , 7 and  $14\sqrt{2}$

2 The third proportional for the quantities : a , (a + b) , ... and  $(a^2 - b^2)$

3 The fourth proportional for the quantities : (a + b) , (a - b) , (a - b) and ...

4 Find the value of x in each of the following , if :

1  $(2x - 3) : (x - 5) = 1 : 4$

« 1 »

2  $(x - 5) : (5x + 3) = 2 : 3$

« -3 »

3  $(x^2 - 8) : (2x^2 + 1) = 1 : 3$

«  $\pm 5$  »

4  $(x^2 + 10x) : (2x^2 - 3) = 24 : 5$  where x is an integer.

« 2 »

5 If  $\frac{x - 2y}{x + 3y} = \frac{1}{3}$ , find :  $\frac{y}{x}$

(Aswan 15) «  $\frac{2}{9}$  »

6 If  $\frac{2x + 3}{2x - 3} = \frac{2y + 5}{2y - 5}$ , prove that :  $\frac{x}{y} = \frac{3}{5}$

7 If  $x^2 - 4y^2 = 3xy$ , find x : y

« -1 : 1 or 4 : 1 »

8 If  $3x^2 - 10xy + 7y^2 = 0$ ,  $x \neq y$ , find the ratio x : y

« 7 : 3 »

9 If  $\frac{x}{y} = \frac{2}{3}$ , find the value of the ratio :  $\frac{3x + 2y}{6y - x}$

(Souhag 19 - El-Menia 20) «  $\frac{3}{4}$  »

10 If  $\frac{a}{b} = \frac{3}{5}$ , find the value of :  $7a + 9b : 4a + 2b$

(Qena 15 - Cairo 20) « 3 »

11 If  $4a = 3b$ , then find the value of :

1  $\frac{4a + b}{2a - b}$

« 8 »

2  $\frac{b^2 - a^2}{a^2 - b^2}$

« -1 »

12 If  $\frac{a}{b} = \frac{1}{3}$ ,  $\frac{c}{d} = \frac{7}{2}$ , find the ratio :  $\frac{2ac + bd}{bc - 3ad}$

«  $\frac{4}{3}$  »

- 13 If  $7x - 3y : x + y = 3 : 1$ , find the ratio :  $12x + 9y : 11x - 3y$  « 2 : 1 »

- 14 If  $\frac{21x + a}{7x + b} = \frac{a}{b}$ , where  $x \neq 0$ , then find the value of :  $\frac{a + 2b}{2a}$  (Ismailia 13) «  $\frac{5}{6}$  »

- 15 Find the number that if it is added to each of the numbers 3, 5, 8 and 12, it becomes proportional. (Southern Sinai 17 – Assiut 18) « 2 »

- 16 Find the number which is subtracted from each of the following numbers to be proportional 16, 21, 14 and 18 « 6 »

- 17 Prove that :  $a, b, c$  and  $d$  are proportional quantities if :

1  $\frac{a+b}{b} = \frac{c+d}{d}$  (El-Fayoum 09)

2  $\frac{a}{b-a} = \frac{c}{d-c}$  (El-Sharkia 15 – El-Fayoum 15 – Aswan 20)

3  $\frac{a-b}{a+b} = \frac{c-d}{c+d}$

4  $\frac{a^2 - 2c^2}{b^2 - 2d^2} = \frac{a^2}{b^2}$  where  $a, b, c$  and  $d$  are positive quantities.

- 18 If  $a : b : c = 5 : 7 : 3$  and  $a + b = 27.6$ , find the value of each of :  $a, b$  and  $c$  « 11.5, 16.1, 6.9 »

- 19 If  $a : b : c = 3 : 4 : 5$ , find the numerical value of the expression :  $\frac{a^2 + b^2 + c^2}{a(b+c)}$  «  $\frac{50}{27}$  »

- 20 If  $2a = 3b = 4c$ , find  $a : b : c$  « 6 : 4 : 3 »

- 21 Answer the following :

- 1 Find the number which if it is added to the two terms of the ratio 7 : 11 it will be 2 : 3 (Alex. 14 – Cairo 17 – El-Fayoum 18 – Giza 19) « 1 »

- 2 Find the number that if we subtract thrice of it from each of the two terms of the ratio  $\frac{49}{69}$ , the ratio becomes  $\frac{2}{3}$  (Giza 12 – El-Beheira 20) « 3 »

- 3 Find the number which if its square is added to each of the two terms of ratio 7 : 11 it becomes 4 : 5 (Suez 17 – El-Monofia 20) « 3 or -3 »

- 4 Find the positive number which if we add its square to each of the two terms of ratio 5 : 11 it becomes 3 : 5 (Kafr El-Sheikh 17 – Giza 19 – Beni Suef 20) « 2 »

- 5 What is the number which is subtracted from the antecedent of the ratio  $15 : 13$  and added to its consequent to become  $3 : 4$  ?  
(Luxor 20) « 3 »
- 6 Two integers , the ratio between them is  $3 : 7$  and if we subtracted 5 from each term , the ratio between each of them becomes  $1 : 3$  , find the two numbers.  
(Alex. 18 – Ismailia 20) « 15 , 35 »
- 7 Two integers , the ratio between them is  $2 : 3$  , if you add to the first 7 and subtract from the second 12 , the ratio between them becomes  $5 : 3$   
Find the two numbers. (El-Beheira 15 – Beni Suef 17 – Matrouh 18) « 18 , 27 »
- 8 Two positive real numbers , the ratio between them is  $4 : 7$  and the square of the small number exceeds 5 times the great number by 39 , find the two numbers. « 12 , 21 »

### Geometric Applications

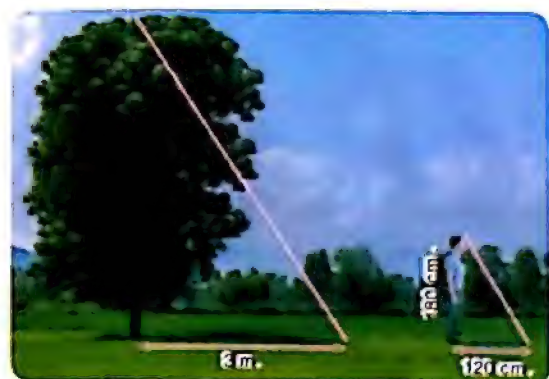
- 22 The ratio between the two dimensions of a rectangle is  $4 : 7$  and its perimeter is 88 cm.  
Find its area. « 448 cm<sup>2</sup> »
- 23 The ratio between the base length and the height of a triangle is  $3 : 2$  and its area = 48 cm<sup>2</sup>  
Find the length of the base and the height. « 12 cm. , 8 cm. »
- 24 In the opposite figure :  
Alaa shaded  $\frac{5}{6}$  the area of the circle ,  $\frac{2}{3}$  the area of the triangle ,  
find the ratio between the area of the circle and the area of the triangle.



(Giza 08) « 2 : 1 »

### Life Applications

- 25 If the shadow length of a tree is 3 m.  
and in the same moment the shadow  
length of Islam is 120 cm. , if the  
height of Islam is 180 cm.  
Find the height of the tree.



« 4  $\frac{1}{2}$  m. »

- 26 Through the interest of the Egyptian authorities in the villages , a budget of  $1.85 \times 10^6$  pounds was set for one of the villages to build a school , a medical unit and a youth centre. If the cost of the school is  $\frac{3}{2}$  of the cost of the medical unit and the cost of the medical unit is  $\frac{5}{6}$  of the cost of the youth centre , what is the cost of each of them ?



«  $7.5 \times 10^5$  ,  $5 \times 10^5$  ,  $6 \times 10^5$  »

- 27 If the rate of success in one of the governorates of the third preparatory is 83% and the rate of success for boys is 79% and the rate of success of girls is 89% , find the ratio between the number of boys and the number of girls in this governorate.



« 3 : 2 »

- 28 The length of a piece of wire is 152 cm. , it is divided into two parts of ratio 11 : 8 , a circular shape is made from the long part and a square shap is made from the short part. Find the ratio between the area of the square and the area of the circle. ( $\pi = \frac{22}{7}$ )



« 32 : 77 »



### For excellent pupils

- 29 Four proportional numbers , the fourth proportion equals the square of the second proportion , the first proportion decreases the second proportion by 2 , the third proportion = 8 , find the four numbers.

« 2 , 4 , 8 , 16 or - 4 , - 2 , 8 , 4 »

- 30 If  $X$  ,  $y$  ,  $z$  and  $l$  are four proportional numbers and  $X + y = 8$  ,  $y + z = 14$  ,  $z + l = 24$  , find the value of each of  $X$  ,  $y$  ,  $z$  and  $l$

« 3 , 5 , 9 , 15 »

- 31 Find the positive number which if its multiplicative inverse is added to the consequent of the ratio  $\frac{2}{3}$  , it will become  $\frac{3}{5}$

« 3 »

## EXERCISE

# 6

## Follow properties of proportion



From the school book

Interactive test

### 1 Complete the following :

1 If  $\frac{a}{b} = \frac{c}{d} = \frac{3}{5}$ , then  $\frac{a+c}{b+d} = \dots\dots\dots$

(El-Fayoum 09)

2 If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{3}{5}$ , then  $\frac{a-2c+e}{b-2d+f} = \dots\dots\dots$

3  $\frac{x}{6} = \frac{y}{5} = \frac{z}{4} = \frac{\dots\dots\dots}{11} = \frac{2y-z}{\dots\dots\dots} = \frac{3x-5y+4z}{\dots\dots\dots}$

4 If  $\frac{x+y}{5} = \frac{y+e}{3} = \frac{e+x}{6}$ , then  $\frac{x-e}{2} = \frac{y-x}{\dots\dots\dots}$

(Cairo 09)

5 If  $\frac{4}{x} = \frac{7}{y} = \frac{a}{y-x}$ , then  $a = \dots\dots\dots$

6  $\frac{l}{3} = \frac{m}{8} = \frac{l + \frac{1}{2}m}{\dots\dots\dots}$

(El-Dakahlia 08)

7 If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ , then  $\frac{5a+3c+\dots\dots\dots}{5b+\dots\dots\dots+4f} = \frac{a}{b}$

(Damietta 08)

### 2 Choose the correct answer from those given :

1 If  $\frac{a}{b} = \frac{c}{d} = m$ , where  $m \neq 0$ , then  $\frac{a \times c}{b \times d} = \dots\dots\dots$

(Cairo 17)

- (a)  $2m^2$  (b)  $m^2$  (c)  $m$  (d)  $2m$

2 If  $\frac{a}{5} = \frac{b}{7}$ , then  $7a - 5b + 3 = \dots\dots\dots$

(Port Said 09)

- (a) 3 (b) 7 (c) 5 (d) 2

3 If  $\frac{x}{5} = \frac{y}{4} = \frac{x+2y}{k}$ , then  $k = \dots\dots\dots$

(Kafr El-Sheikh 11 - El-Gharbia 17)

- (a) 9 (b) 13 (c) 14 (d) 8

4 If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ , then  $\frac{a+2c+3e}{b+2d+3f} = \frac{\dots\dots}{5f}$

(a) 5 a

(b) 5 c

(c) 5 e

(d) 5 a + 5 c + 5 e

5 If  $\frac{a}{4} = \frac{b}{5}$ ,  $2a + 3b = 46$ , then a = .....

(a) 2

(b) 4

(c) 5

(d) 8

6 If  $\frac{a}{b} = \frac{2}{3}$ ,  $\frac{a}{c} = \frac{4}{5}$ , then b : c = ..... : .....

(El-Gharbia 17)

(a) 3 : 4

(b) 5 : 6

(c) 6 : 5

(d) 4 : 3

3 If a, b, c and d are proportional quantities, prove that :

1  $\frac{3a+c}{5a-2c} = \frac{3b+d}{5b-2d}$

(Assiut 17)

2  $\frac{3a-2c}{5a+3c} = \frac{3b-2d}{5b+3d}$

(Suez 16 – Kafr El-Sheikh 18)

3  $\frac{a^2+c^2}{ab+cd} = \frac{a}{b}$

(El-Monofia 11)

4  $\frac{a^2+c^2}{b^2+d^2} = \frac{ac}{bd}$

(El-Monofia 16 – El-Kalyoubia 17 – El-Gharbia 18)

5  $\frac{ac}{bd} = \left(\frac{a-c}{b-d}\right)^2$

(Alex. 14 – Suez 18)

6  $\left(\frac{a+b}{c+d}\right)^2 = \frac{2a^2-3b^2}{2c^2-3d^2}$

7  $\sqrt{\frac{3a^2-5c^2}{3b^2-5d^2}} = \frac{a}{b}$  where : a, b, c and d are positive quantities.

8  $\sqrt[3]{\frac{5a^3-3c^3}{5b^3-3d^3}} = \frac{a+c}{b+d}$

(El-Kalyoubia 19)

9  $\frac{a^2-2ac+c^2}{ac} = \frac{b^2-2bd+d^2}{bd}$

(Ismailia 18)

4 If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ , prove that :

1  $\frac{a+5c}{b+5d} = \frac{c-3e}{d-3f}$

2  $\frac{2a+7c-4e}{2b+7d-4f} = \frac{a-8e}{b-8f}$

3  $\frac{2a^4b^2+3a^2e^2-5e^4f}{2b^6+3b^2f^2-5f^5} = \frac{a^4}{b^4}$

4  $\sqrt{\frac{5a^2-7ce}{5b^2-7df}} = \frac{2a+c}{2b+d}$

5 If  $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ , prove that :

1  $\frac{2y-z}{3x-2y+z} = \frac{1}{2}$

(Giza 15 - North Sinai 18 - Port Said 19 - Beni Suef 20)

2  $\sqrt{3x^2 + 3y^2 + z^2} = 2x + y$

(Souhag 16 - El-Menia 12 - Damietta 19)

6 If  $x = \frac{y}{2} = \frac{z}{3}$ , then prove that :  $\frac{x+y-2z}{x-3z} = \frac{3}{8}$

(Assiut 17)

7 If  $\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$ , prove that :  $2a - 5b + 3c =$  one of the given ratios.

8 If  $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a-b+5c}{3x}$ , then find the value of :  $x$

(El-Gharbia 16 - Qena 17 - Luxor 18 - Aswan 19 - El-Kalyoubia 20) = 7

9 If  $\frac{a}{4x+y} = \frac{b}{x-4y}$ , prove that :  $\frac{a+b}{5x-3y} = \frac{a-b}{3x+5y}$

(Damietta 12 - El-Dakahlia 19)

10 If  $\frac{x+y}{19} = \frac{y+z}{7}$ , prove that :  $\frac{x+2y+z}{13} = \frac{x-z}{6}$

11 If  $\frac{y}{x-z} = \frac{x}{y} = \frac{x+y}{z}$ , prove that each ratio is equal to 2 (unless  $x+y=0$ ),

then find  $x : y : z$

(El-Beheira 18) = 4 : 2 : 3

12 If  $\frac{x}{a-b+c} = \frac{y}{b-c+a} = \frac{z}{c-a+b}$ , prove that :  $\frac{x+y}{a} = \frac{y+z}{b}$

(Port Said 09)

13 If  $\frac{x}{2a+b} = \frac{y}{2b-c} = \frac{z}{2c-a}$ , then prove that :  $\frac{2x+y}{4a+4b-c} = \frac{2x+2y+z}{3a+6b}$

(El-Beheira 17 - El-Kalyoubia 18 - Matrouh 19)

14 If  $\frac{a}{2x-y} = \frac{b}{2y-x}$ , prove that :  $\frac{2a+b}{a+2b} = \frac{x}{y}$

15 If  $\frac{a}{2x+y} = \frac{b}{3y-x} = \frac{c}{4x+5y}$ , prove that :  $\frac{a+2b}{4b+c} = \frac{7}{17}$

16 If  $\frac{a}{2} = \frac{b}{7} = \frac{c}{3}$ , find the value of :  $\frac{a+2b}{b-c}$

(North Sinai 09) = 4

17 If  $\frac{x+y}{7} = \frac{y+z}{5} = \frac{z+x}{8}$ , prove that :  $\frac{x+y+z}{x-z} = 5$

(El-Monofia 16)

18 If  $\frac{a+b}{4} = \frac{b+c}{5} = \frac{c+a}{7}$ , prove that :  $\frac{a+b+c}{8} = \frac{a}{3}$

(Kafr El-Sheikh 15)

19 If  $\frac{X+y}{3} = \frac{y+z}{8} = \frac{z+X}{6}$ , prove that :  $\frac{X+y+z}{2X+3y+3z} = \frac{17}{50}$  (Kafir El-Sheikh 20)

20 If  $\frac{X+y}{5} = \frac{y+z}{8} = \frac{z+X}{7}$ , prove that :  $\frac{X}{2} = \frac{y}{3} = \frac{z}{5}$  (New Valley 17)

21 If  $\frac{X+y}{25} = \frac{X-y}{11} = \frac{X+y-z}{8}$ , prove that :  $X : y : z = 18 : 7 : 17$

22 If  $\frac{a+3b}{X+6y} = \frac{3b+5c}{6y+10z} = \frac{5c+a}{10z+X}$ , prove that :  $\frac{a}{b} = \frac{X}{2y}$  and find  $a : b : c$  «  $X : 2y : 2z$  »

23 If  $\frac{a}{3X+4y} = \frac{b}{5X-2y} = \frac{c}{y+2X}$ , prove that :  $13X(3c-2a) + 5y(a+2b) = 0$

24 If  $\frac{X}{7} = \frac{y}{3}$ , prove that :  $(2X-3y), (X+2y), 10$  and  $26$  are proportional. (Luxor 19)

25 If  $\frac{a}{b} = \frac{3}{5}$  and  $\frac{a}{c} = \frac{3}{7}$ , find the value of the expression :  $a+b+c$  in terms of  $a$  «  $5a$  »

26 If  $\frac{a}{b} = \frac{2}{3}$ ,  $\frac{a}{c} = \frac{3}{5}$  and  $a+b+c = 75$  find the value of each of :  $a, b$  and  $c$   
(Red Sea 16) «  $18, 27, 30$  »

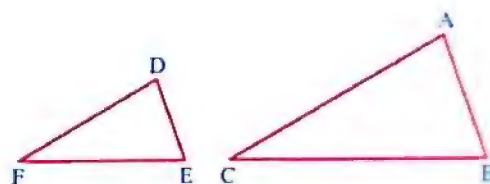
### Geometric Application

27 In the opposite figure :

If  $\Delta ABC \sim \Delta DEF$

where  $DF : AC = 2 : 3$  and the perimeter of  $\Delta DEF = 22$  cm.

Find the perimeter of :  $\Delta ABC$



«  $33$  cm »



### For excellent pupils

28 If  $\frac{a}{X-y+z} = \frac{b}{X+y-z} = \frac{c}{y+z-X}$ , prove that : each ratio =  $\frac{aX+bY+cZ}{X^2+Y^2+Z^2}$

29 If  $\frac{2X+y}{X} = \frac{4y+z}{y} = \frac{4z+3X}{z}$ , find the ratio  $X : y : z$

, then prove that :  $\frac{2X+y+z}{3X-y+2z} = \frac{4}{3}$

«  $1 : 3 : 3$  »

30 If  $\frac{a+2b}{5} = \frac{3b-c}{3} = \frac{c-a}{2}$ , prove that :

1  $a+b-c = \text{zero}$

2  $\frac{3b-a}{2b+c} = \frac{5}{7}$

## EXERCISE

# 7

## Continued proportion



1 Find the middle proportion between :

From the school book

Interactive test

1  $3, 27$

2  $9, 25$

3  $-2, -8$  (Giza 09)

4  $\frac{1}{5}, 125$

5  $2a, 8ab^2$

6  $(l+m)^2, (l-m)^2$

2 Find the third proportion of each of the following :

1  $6, 12$

2  $x^2, -5x$

3  $x^2, -3x^2$

3 If  $b$  is the middle proportion between  $a$  and  $c$ , prove that :

1  $\frac{a}{c} = \frac{b^2}{c^2}$

(Giza 14)

2  $\frac{2a+3b}{2b+3c} = \frac{a}{b}$

(Ismailia 17)

3  $\frac{a-b}{b-c} = \frac{a+3b}{3c+b}$

4  $\frac{a^2+b^2}{b^2+c^2} = \frac{a}{c}$  (El-Kalyoubia 18 - Cairo 20)

5  $\left(\frac{b-c}{a-b}\right)^2 = \frac{c}{a}$

6  $\frac{a^3+b^3}{b^3+c^3} = \frac{a^2}{cb}$  (El-Monofia 11)

7  $\frac{a^3-4b^3}{b^3-4c^3} = \frac{b^3}{c^3}$

8  $\frac{2c^2-3b^2}{2b^2-3a^2} = \frac{c}{a} = \frac{c^2}{b^2}$

9  $\frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{a^2-b^2}{b^2-c^2}$

10  $\frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{2a}{c}$

(Aswan 16 - Port Said 17 - El-Dakahlia 19)

11  $\frac{a+b+c}{a^{-1}+b^{-1}+c^{-1}} = b^2$

12  $\frac{ac}{b(b+c)} = \frac{a}{a+b}$

(El-Gharbia 17)

4 If  $a, b, c$  and  $d$  are in continued proportion, prove that :

$$1 \quad \frac{a-2b}{b-2c} = \frac{3b+4c}{3c+4d}$$

$$3 \quad \frac{3a-5c}{a-b+c} = \frac{3b-5d}{b-c+d}$$

$$5 \quad \frac{c^2-d^2}{a-c} = \frac{bd}{a}$$

$$6 \quad \frac{a^2-3c^2}{b^2-3d^2} = \frac{b}{d}$$

$$7 \quad \frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$$

$$8 \quad \frac{a}{b+d} = \frac{c^3}{c^2d+d^3}$$

$$9 \quad \frac{a^2+b^2+c^2}{b^2+c^2+d^2} = \frac{ac}{bd}$$

(El-Dakahlia 11)

$$11 \quad \frac{a+5b}{b+5c} = \sqrt{\frac{b}{d}}$$

$$13 \quad \left(\frac{a+b}{b+c}\right)^3 = \frac{a}{d}$$

$$2 \quad \frac{3a+5c}{3b+5d} = \frac{a-4c}{b-4d}$$

$$4 \quad \frac{a-d}{a+b+c} = \frac{a-2b+c}{a-b}$$

(Matrouh 17 - El-Beheira 18 - South Sinai 20)

(El-Beheira 15 - Alex. 17 - Beni Suef 18)

(Qena 16 - El-Monofia 17 - El-Monofia 20)

(Alex. 19 - El-Fayoum 20)

$$10 \quad \frac{2a+3d}{3a-4d} = \frac{2a^3+3b^3}{3a^3-4b^3}$$

$$12 \quad \sqrt[3]{\frac{5a^3-3c^3}{5b^3-3d^3}} = \frac{a+c}{b+d}$$

(Alexandria 11)

$$14 \quad \frac{a^2+d^2}{c(a+c)} = \frac{b}{d} + \frac{d}{b} - 1$$

(El-Sharkia 15)

5 Choose the correct answer from those given :

1 The third proportion of the two numbers 9 and -12 is .....

(El-Beheira 11)

(a) -16

(b) 8

(c) 16

(d) 108

2 The proportional mean between  $X$  and  $y$  is .....

(South Sinai 17)

(a)  $\sqrt{xy}$

(b)  $-\sqrt{xy}$

(c)  $\pm\sqrt{xy}$

(d)  $xy$

3 If  $l, m, z$  are in continued proportion, then  $l = \dots\dots\dots$

(a)  $\pm\sqrt{mz}$

(b)  $mz$

(c)  $\frac{m}{z}$

(d)  $\frac{m^2}{z}$

4 If the number 6 is the positive proportional mean of the two numbers 2 and  $m$ ,

then  $m = \dots\dots\dots$

(Aswan 13)

(a) 8

(b) 12

(c) 18

(d) 36

5 If  $\frac{a}{b} = \frac{b}{c} = \frac{c}{5} = 2$ , then  $a = \dots\dots\dots$

(El-Monofia 12)

(a)  $5 \times 2^2$

(b) 40

(c) 10

(d)  $2 \times 5^3$

6 If  $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = 2$ , then  $\frac{a}{d} = \dots\dots\dots$

(El-Sharkia 13)

(a) 2

(b) 4

(c) 8

(d) 16

7 If  $6a^2b^2, 3ab$  and  $c$  are proportional quantities, then  $c = \dots\dots\dots$

(a) -3

(b)  $3ab$

(c)  $\frac{3}{2}$

(d)  $\frac{2}{3}$

8 If  $a, 2, 4, b$  are in continued proportion, then  $a + b = \dots\dots\dots$  (El-Dakahlia 20)  
 (a) 2 (b) 4 (c) 6 (d) 9

9 The proportional mean between  $(X - 2)$  and  $(X + 2)$  is  $\dots\dots\dots$  (Cairo 09)  
 (a)  $\sqrt{X + 2}$  (b)  $X^2 - 4$  (c)  $\pm\sqrt{X^2 - 4}$  (d)  $\sqrt{X^2 - 4}$

10 The number which is added to each of the numbers 1, 3 and 6 to become in continued proportion is  $\dots\dots\dots$  (Damietta 13)  
 (a) 1 (b) 2 (c) 3 (d) 6

6 If  $a, 3, 9$  and  $b$  are in continued proportion, find the value of each  $a$  and  $b$   
 (Luxor 16) « 1, 27 »

7 If 3,  $\ell$ , 12 and  $m$  are in continued proportion, find the value of each of  $\ell$  and  $m$  «  $\pm 6, \pm 24$  »

8 Complete the following :

1 If 7,  $X$  and  $\frac{1}{y}$  are proportional, then  $X^2 y = \dots\dots\dots$

2 The third proportion of the two quantities  $9(r + 1)^2$  and  $6(r^2 - 1)$  is  $\dots\dots\dots$

3 The middle proportion of the two quantities  $9X^2 - 25y^2$  and  $\frac{3X + 5y}{3X - 5y}$  is  $\dots\dots\dots$

4 If  $\frac{y}{X} = \frac{X}{z} = \frac{2}{5}$ , then  $y = \dots\dots\dots z$

5 If  $a, b, c, d$  and  $e$  are in continued proportion and each ratio = the constant  $m$ , then  $\frac{a}{e} = \dots\dots\dots$

6 If  $a, b$  and  $c$  are proportional, then  $\frac{a^2 + b^2}{b^2 + c^2} = \frac{\dots\dots}{c}$

7 The real number  $X$  which makes  $X + 1, X + 5$  and  $X + 13$  proportional is  $\dots\dots\dots$

8 If 2,  $4 + X$  and 18 are proportional quantities,  $X \in \mathbb{R}_-$ , then  $X = \dots\dots\dots$

9 Find the number that if we subtract it from each of the numbers 3, 7, 19, then they become in continued proportional. (Luxor 17) « 1 »

10 If  $b$  is the middle proportion between  $a$  and  $c$  and  $a = 4c = 4$ , then find the value of :  $a^2 + b^2 + c^2$  (El-Fayoum 17) « 21 »

11 If  $b$  is the middle proportion between  $a$  and  $c$ ,  $c$  is the middle proportion between  $b$  and  $d$ , prove that :  $\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{d^2} = \frac{a}{c} + \frac{b}{d} + \frac{ac}{bd}$

12 If  $y^2 = xz$ , prove that :  $\frac{x(x-y)}{y(y-z)} = \frac{y^2}{z^2}$

13 If  $b^2 = ac$  and  $c^2 = bd$ , prove that :  $\frac{2a+3d}{3a-4d} = \frac{2a^3+3b^3}{3a^3-4b^3}$

14 If  $\frac{a^2+b^2}{b^2} = \frac{b^2+c^2}{c^2}$ , prove that  $b$  is the middle proportion between  $a$  and  $c$

where  $ac$  is a positive quantity.

(Alexandria 15 – Beni Suef 15)

15 If  $a, b, c$  and  $d$  are in continued proportion, prove that :  $(b+c)$  is the middle proportion between  $(a+b)$  and  $(c+d)$

16 If  $5a, 6b, 7c$  and  $8d$  are positive quantities in continued proportion,

prove that :  $\sqrt[3]{\frac{5a}{8d}} = \sqrt{\frac{5a+6b}{7c+8d}}$

17 If  $b$  is the middle proportion between  $a$  and  $c$ , prove that :  $\frac{a^4+b^4+c^4}{a^{-4}+b^{-4}+c^{-4}} = b^8$

### Geometric Applications

18  $x, y$  and  $z$  are three proportional side lengths in a triangle,  $x+y = 15$  cm.

and  $y+z = 22.5$  cm. find  $x:y$

« 2 : 3 »

19 ABC is a triangle in which  $m(\angle C) = 60^\circ$ , if the measures of its angles  $\angle A, \angle B$  and  $\angle C$  respectively are in continued proportion.

Find :  $m(\angle A)$  and  $m(\angle B)$

«  $60^\circ, 60^\circ$  »



### For excellent pupils

20 If  $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = 2$ , find the solution set of the equation :  $ax^2 - 2bx + c = 0$

«  $\{\frac{1}{2}\}$  »

21 If 5 is the middle proportion between  $x$  and  $y$ , find the middle proportion between

$(x + \frac{1}{y})$  and  $(y + \frac{1}{x})$

«  $\pm 5.2$  »



## EXERCISE

# 8

## Direct variation and inverse variation



From the school book

Interactive test

### 1 Complete the following :

- 1 If  $X \propto y$  , then  $X = \dots\dots\dots$
- 2 If  $z = \frac{m}{x^2}$  where  $m$  is a constant , then  $z \propto \dots\dots\dots$
- 3 If  $y \propto X$  , then  $\frac{x_1}{x_2} = \dots\dots\dots$  (New Valley 11)
- 4 If  $X$  varies inversely as  $y$  , then  $\frac{y_1}{y_2} = \dots\dots\dots$  (Qena 11)
- 5 If  $y = \frac{3}{5} X$  , then  $y \propto \dots\dots\dots$  (Aswan 08)
- 6 If  $y \propto \frac{5}{X}$  , then  $y$  varies inversely as  $\dots\dots\dots$  (Ismailia 09)
- 7 If  $X - 2y = 0$  , then  $X \propto \dots\dots\dots$  (El-Dukahlia 09)
- 8 If  $2Xy = 5$  , then  $X \propto \dots\dots\dots$  (El-Sharkia 09)
- 9 If  $y \propto X$  and  $y = 2$  as  $X = 8$  , then  $y = \dots\dots\dots$  when  $X = 12$  (El-Sharkia 11)
- 10 If  $y \propto \frac{1}{X}$  and  $y = 3$  as  $X = 20$  , then  $y = \dots\dots\dots$  when  $X = 12$  (El-Beheira 12)
- 11 If  $y \propto X$  and  $y = 2$  as  $X = 4$  , then  $y = \dots\dots\dots X$  (Cairo 11)
- 12 If  $y \propto X$  and  $y = 6$  as  $X = 4$  , then  $\frac{y}{X} = \dots\dots\dots$  (in the simplest form)
- 13 If  $t = \frac{mk}{h}$  where  $m$  is a constant  $\neq 0$  , then  $t$  varies  $\dots\dots\dots$  as  $k$  when  $h$  is constant  
,  $t$  varies  $\dots\dots\dots$  as  $h$  when  $k$  is constant.

### 2 If $y$ varies directly as $X$ and $y = 20$ as $X = 7$

Find :  $X$  when  $y = 40$

« 14 »

3 If  $a$  varies inversely as  $b$  and  $a = 12$  as  $b = 8$ , find :

1 The value of  $a$  as  $b = 1.5$

2 The value of  $b$  as  $a = 2$

« 64 , 48 »

4 If  $y \propto X$  and  $y = 14$  when  $X = 42$ , find :

(El-Monofia 15 – Port Said 18 – South Sinai 19 – Port Said 20)

1 The relation between  $X$  and  $y$

2 The value of  $y$  when  $X = 60$

«  $y = \frac{1}{3}X, 20$  »

5 If  $y \propto \frac{1}{X}$  and  $y = 3$  when  $X = 2$ , find :

(Alex. 17 – North Sinai 19 – Cairo 20)

1 The relation between  $X$  and  $y$

2 The value of  $y$  when  $X = 1.5$

«  $XY = 6, 4$  »

6 If  $y \propto \frac{1}{X}$  and  $X = 3$  as  $y = 10$ , find  $y$  when :

$X \in \{1, 2, 3, 4, 5\}$

« 30 , 15 , 10 , 7.5 , 6 »

7 If  $y \propto$  the multiplicative inverse of the expression  $\frac{1}{X^2}$ , then find the relation between  $X$  and  $y$ , if  $y = 4$  as  $X = 3$ , then find the value of  $y$  as  $X = 9$

(El-Sharkia 08) «  $y = \frac{4}{9}X^2, 36$  »

8 If  $y \propto X^3$  and  $y = 64$  as  $X = 2$ , find the relation between  $X$  and  $y$  and find the value of  $y$  as  $X = \frac{1}{2}$

(Luxor 20) «  $y = 8X^3, 1$  »

9 If  $y$  varies inversely as  $\sqrt{X}$ ,  $y = 2$  as  $X = 16$

Find the value of  $y$  as  $X = 32$

«  $\sqrt{2}$  »

10 If  $y^2 \propto X^3$ , find the relation between  $X$  and  $y$  where  $y = 3$  as  $X = 2$

(Qena 09) «  $y^2 = \frac{9}{8}X^3$  »

11 If  $y^2 \propto \frac{1}{\sqrt[3]{X}}$  and  $X = 8$  as  $y = 3$ , find  $X$  as  $y = 1.5$

« 512 »

12 If  $y \propto (X + 1)$  and  $X = 3$  when  $y = 2$ , then find the relation between  $X$  and  $y$

(Matrouh 09) «  $y = \frac{1}{2}(X + 1)$  »

13 If  $\frac{5X - 3y}{3X + 5y} = 1$  for all the values of  $X \in \mathbb{R}_+$ ,  $y \in \mathbb{R}_+$ , prove that :  $y \propto X$

14 If  $\frac{a+2b}{6} = \frac{b+3c}{3}$ , then prove that :  $a \propto c$

(El-Fayoum 17)

15 If  $\frac{21x-y}{7x-z} = \frac{y}{z}$ , prove that :  $y \propto z$

(Cairo 15 - El-Kalyoubia 18 - Damietta 19)

16 If  $x^2y^2 - 6xy + 9 = 0$ , then prove that :  $y$  varies inversely as  $x$

(Damietta 13 - South Sinai 14)

17 If  $4a^2 + 9b^2 = 12ab$ , prove that :  $a$  varies directly as  $b$

(Matrouh 17)

18 If  $x^4y^2 - 14x^2y + 49 = 0$ , prove that :  $y \propto \frac{1}{x^2}$

(Alex. 19)

19 If  $(4x + 7y) \propto (x + 2y)$  where  $x \in \mathbb{R}$  and  $y \in \mathbb{R}$ , then prove that :  $y \propto x$

20 If  $\left(\frac{a}{y} - \frac{a}{x}\right) \propto (x - y)$  where  $a$  is a constant,  $x \neq y \neq 0$ ,  
then prove that :  $x$  varies inversely as  $y$

21 Which of the following tables represents the direct variation and which of them represents the inverse variation and which does not represent the direct variation or inverse variation with mentioning the reason in each case :

x	y
3	20
5	12
4	15
6	10

x	y
2	9
4	18
12	54
16	72

x	y
5	9
10	18
15	27
25	45

x	y
3	6
-2	-9
-18	1
9	-2

22 From the data in the following table, answer the following questions :

- Show the type of variation between  $x$  and  $y$
- Find the constant of variation.
- Find the value of  $y$  at  $x = 3$
- Find the value of  $x$  at  $y = 2\frac{2}{5}$

x	2	4	6
y	6	3	2

(Damietta 16 - Ismailia 18) « 12, 4, 5 »

23 In the opposite table :

- Show the type of variation between  $x$  and  $y$
- Find the value of each of  $a$  and  $b$

x	1	2	b	4	6
y	12	a	36	48	72

«  $a = 24$ ,  $b = 3$  »

- 3 If  $a$  varies inversely as  $b$  and  $a = 12$  as  $b = 8$ , find :

1 The value of  $a$  as  $b = 1.5$

2 The value of  $b$  as  $a = 2$

« 64 , 48 »

- 4 If  $y \propto X$  and  $y = 14$  when  $X = 42$ , find :

(El-Monofia 15 – Port Said 18 – South Sinai 19 – Port Said 20)

1 The relation between  $X$  and  $y$

2 The value of  $y$  when  $X = 60$

«  $y = \frac{1}{3}X$  , 20 »

- 5 If  $y \propto \frac{1}{X}$  and  $y = 3$  when  $X = 2$ , find :

(Alex. 17 – North Sinai 19 – Cairo 20)

1 The relation between  $X$  and  $y$

2 The value of  $y$  when  $X = 1.5$

«  $Xy = 6$  , 4 »

- 6 If  $y \propto \frac{1}{X}$  and  $X = 3$  as  $y = 10$ , find  $y$  when :

$X \in \{1, 2, 3, 4, 5\}$

« 30 , 15 , 10 , 7.5 , 6 »

- 7 If  $y \propto$  the multiplicative inverse of the expression  $\frac{1}{X^2}$ , then find the relation between  $X$  and  $y$ , if  $y = 4$  as  $X = 3$ , then find the value of  $y$  as  $X = 9$

(El-Sharkia 08) «  $y = \frac{4}{9}X^2$  , 36 »

- 8 If  $y \propto X^3$  and  $y = 64$  as  $X = 2$ , find the relation between  $X$  and  $y$  and find the value of  $y$  as  $X = \frac{1}{2}$

(Luxor 20) «  $y = 8X^3$  , 1 »

- 9 If  $y$  varies inversely as  $\sqrt{X}$ ,  $y = 2$  as  $X = 16$

Find the value of  $y$  as  $X = 32$

«  $\sqrt{2}$  »

- 10 If  $y^2 \propto X^3$ , find the relation between  $X$  and  $y$  where  $y = 3$  as  $X = 2$

(Qena 09) «  $y^2 = \frac{9}{8}X^3$  »

- 11 If  $y^2 \propto \frac{1}{\sqrt[3]{X}}$  and  $X = 8$  as  $y = 3$ , find  $X$  as  $y = 1.5$

« 512 »

- 12 If  $y \propto (X + 1)$  and  $X = 3$  when  $y = 2$ , then find the relation between  $X$  and  $y$

(Matrouh 09) «  $y = \frac{1}{2}(X + 1)$  »

- 13 If  $\frac{5X - 3y}{3X + 5y} = 1$  for all the values of  $X \in \mathbb{R}_+$ ,  $y \in \mathbb{R}_+$ , prove that :  $y \propto X$

14 If  $\frac{a+2b}{6} = \frac{b+3c}{3}$ , then prove that :  $a \propto c$

(El-Fayoum 17)

15 If  $\frac{21X-y}{7X-z} = \frac{y}{z}$ , prove that :  $y \propto z$

(Cairo 15 – El-Kalyoubia 18 – Damietta 19)

16 If  $X^2y^2 - 6Xy + 9 = 0$ , then prove that :  $y$  varies inversely as  $X$

(Damietta 13 – South Sinai 14)

17 If  $4a^2 + 9b^2 = 12ab$ , prove that :  $a$  varies directly as  $b$

(Matrouh 17)

18 If  $X^4y^2 - 14X^2y + 49 = 0$ , prove that :  $y \propto \frac{1}{X^2}$

(Alex. 19)

19 If  $(4X + 7y) \propto (X + 2y)$  where  $X \in \mathbb{R}$  and  $y \in \mathbb{R}$ , then prove that :  $y \propto X$

20 If  $\left(\frac{a}{y} - \frac{a}{X}\right) \propto (X - y)$  where  $a$  is a constant,  $X \neq y \neq 0$ ,  
then prove that :  $X$  varies inversely as  $y$

21 Which of the following tables represents the direct variation and which of them represents the inverse variation and which does not represent the direct variation or inverse variation with mentioning the reason in each case :

X	y
3	20
5	12
4	15
6	10

X	y
2	9
4	18
12	54
16	72

X	y
5	9
10	18
15	27
25	45

X	y
3	6
-2	-9
-18	1
9	-2

22 From the data in the following table, answer the following questions :

1 Show the type of variation between  $X$  and  $y$

2 Find the constant of variation.

3 Find the value of  $y$  at  $X = 3$

4 Find the value of  $X$  at  $y = 2\frac{2}{5}$

X	2	4	6
y	6	3	2

(Damietta 16 – Ismailia 18) « 12, 4, 5 »

23 In the opposite table :

1 Show the type of variation between  $X$  and  $y$

2 Find the value of each of  $a$  and  $b$

X	1	2	b	4	6
y	12	a	36	48	72

«  $a = 24$ ,  $b = 3$  »

- 24 If  $y = z + 5$ ,  $z$  changes inversely with  $X$  and  $y = 6$  when  $X = 2$ , then find the relation between  $y$  and  $X$ , then find the value of  $y$  when  $X = 1$  (El Monofia 17)  $\therefore y = \frac{2}{X} + 5, 7$

- 25 If  $y = a + b$  where  $a$  is constant,  $b$  varies directly with  $X$ ,  $y = 3$  when  $X = 0$  and  $y = 5$  when  $X = 3$ , find the relation between  $X$  and  $y$  then find the value of  $y$  when  $X = 7$   
 $\therefore y = 3 + \frac{2}{3}X, 7\frac{2}{3}$

- 26 If  $y = a - 9$  and  $y \propto \frac{1}{X^2}$  and  $a = 18$  when  $X = \frac{2}{3}$ , find the relation between  $y$  and  $X$ , then deduce the value of  $y$  when  $X = 1$  (Kafri El-Sheikh 18 - Suez 18 - Luxor 19)  $\therefore y = \frac{4}{X^2}, 4$

- 27 If  $y = 2 + a$ ,  $a$  varies inversely as  $X$  and  $a = 5$  when  $X = 2$ , find :

1 The relation between  $y$  and  $X$

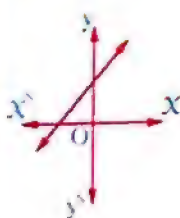
2 The value of  $y$  when  $X = 5$

(El-Sharkia 17)  $\therefore y = 2 + \frac{10}{X}, 4$

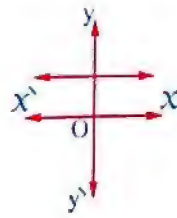
- 28 If  $X = l + 9$  and  $l \propto y$ , then find the relation between  $l$  and  $y$  known that :  $X = 24$  and  $y = 5$ , then find the value of  $y$  when  $l = 12$   $\therefore l = 3y, 4$

- 29 Choose the correct answer from those given :

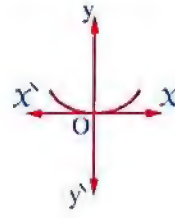
1 The graphical form representing the direct variation between  $X$  and  $y$  is .....



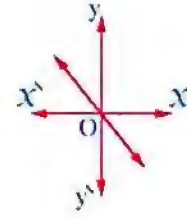
(a)



(b)



(c)



(d)

(El-Sharkia 16)

2 The relation which represents a direct variation between the two variables  $X$  and  $y$  is .....

(Souhag 20)

- (a)  $Xy = 5$  (b)  $y = X + 3$  (c)  $\frac{X}{3} = \frac{4}{y}$  (d)  $\frac{X}{5} = \frac{y}{2}$

3 Which of the following relations represents an inverse variation between the two variables  $X$  and  $y$  ?

(El-Beheira 15)

- (a)  $y = X + 5$  (b)  $y = 4X$  (c)  $\frac{X}{y} = \frac{5}{7}$  (d)  $Xy = 11$

4 If  $y$  varies inversely as  $X^2$ ,  $k$  is a constant, then .....


- (a)  $y = kX^2$  (b)  $y = k - X^2$  (c)  $y = \frac{k}{X^2}$  (d)  $y = \frac{k}{X}$

5 If  $X$  and  $y$  are two variables,  $\frac{X_1 y_1}{X_2 y_2} = 1$ , then  $y \propto$  .....

- (a)  $X$  (b)  $\frac{1}{X}$  (c)  $X^2$  (d)  $\frac{1}{X^2}$

6 If  $y \propto X$  and  $y = 5$  when  $X = 3$ , then the constant proportional equals .....

- (a) 15 (b) 5 (c) 3 (d)  $\frac{5}{3}$

7  If  $y$  varies inversely with  $X$  and  $X = \sqrt{3}$  when  $y = \frac{2}{\sqrt{3}}$ , then the constant proportional equals .....  
(El-Beheira 16 – Beni Suef 15 – New Valley 20)

- (a)  $\frac{1}{2}$  (b)  $\frac{2}{3}$  (c) 2 (d) 6

8 If  $X y^5 = \text{constant}$ , then  $X$  varies inversely as ..... (Ismailia 08)

- (a)  $\frac{1}{5}$  (b)  $y^5$  (c)  $y$  (d)  $y^2$

9 If  $y \propto \frac{1}{\sqrt{X}}$ , then  $X$  varies ..... (Matrouh 09)

- (a) directly as  $y^2$  (b) inversely as  $y^2$  (c) inversely as  $y$  (d) inversely as  $\sqrt{y}$

10 If  $y^2 + 4 X^2 = 4 X y$ , then ..... (South Sinai 19 – Alexandria 15)

- (a)  $y \propto X$  (b)  $y \propto X^2$  (c)  $y \propto \frac{1}{X}$  (d)  $y \propto \frac{1}{X^2}$

11 If  $X^2 y^2 + \frac{1}{4} = X y$ , then .....

- (a)  $X \propto y$  (b)  $y \propto X$  (c)  $2 X \propto 5 y$  (d)  $y \propto \frac{1}{X}$

12 If  $y = 3 X - 6$ , then  $y \propto$  ..... (El-Sharkia 14)


- (a)  $X$  (b)  $3 X$  (c)  $X - 2$  (d)  $3 X + 6$

13 If  $\frac{y+3}{y} = \frac{X+2}{X}$  where  $X \neq y \neq \text{zero}$ , then  $y \propto$  ..... (Ismailia 14)

- (a)  $X$  (b)  $\frac{1}{X}$  (c)  $X + 2$  (d)  $X + 5$


14 If  $y - X = \frac{2}{X} - \frac{2}{y}$  where  $X \neq y \neq 0$ , then .....

- (a)  $y \propto X + 1$  (b)  $y \propto X$  (c)  $y \propto \frac{1}{X}$  (d)  $y \propto \frac{1}{X^2}$

15  If the total cost of a trip is ( $y$ ), some of it is constant (a) and the other is directly proportional with the number of participants ( $X$ ), then ..... (Ismailia 11)


- (a)  $y = a X$  (b)  $y = \frac{a}{X}$   
(c)  $y = a + \frac{m}{X}$  ( $m$  is constant  $\neq 0$ ) (d)  $y = a + m X$  ( $m$  is a constant  $\neq 0$ )

### Geometric Application

- 30  If (h) the height of a right circular cylinder (its volume is constant) varies inversely as the square of radius length (r) and  $h = 27$  cm, when  $r = 10.5$  cm.  
Find h when  $r = 15.75$  cm.


(Port Said 20) « 12 cm. »

### Life Applications

- 31  A car moves with a uniform velocity where the distance varies directly with the time (t). If the car covered a distance of 150 km. in 6 hours , find the distance covered by that car in 10 hours ?




(El-Kalyoubia 13) « 250 km. »

- 32  If the weight of a body on the moon (W) is directly proportional with its weight on the ground (R)  
If the body weighs 84 kg., on the ground and its weight on the moon is 14 kg. What will its weight be on the moon if its weight on the ground is 144 kg.?



« 24 kg. »

- 33  If the number of hours (n) needed for carrying out a work varies inversely as the number of workers (X) who carry out this work.  
If the work is carried out by 6 workers within 4 hours , what is the needed time for carrying out the work by 8 workers ?



(El-Sharkia 11) « 3 hours »

- 34 If the distance covered by a bicycle ( $d$ ) varied directly with the square of the time ( $t$ ), if  $d = \frac{81}{16}$  km. when  $t = \frac{1}{4}$  hour, find the value of  $t$  when  $d = 144$  km.



(Assist 12)  $\approx 1\frac{1}{3}$  hour

- 35 If the value of speed  $v$  that water passes through a hose nozzle inversely changes with the square of the hose nozzle radius length  $r$  and  $v = 5$  cm./s. when  $r = 3$  cm., find  $v$  when  $r = 2.5$  cm.



$\approx 7.2$  cm./s.

- 36 If the weight of a body varies inversely as the square of its distance from the centre of the earth. If a satellite of weight 500 w. kg. is projected up to the space, what will its weight be when it becomes at a distance of 640 km. far from the surface of the earth to the nearest one (kg.) (Consider the radius length of the earth 6390 km.)



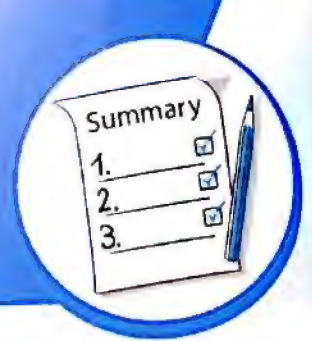
$\approx 413$  w.kg.



### For excellent pupils

- 37 If  $X \propto y$  and  $z \propto l$ , then prove that :  $(X + y)(z + l) \propto (X - y)(z - l)$
- 38 If  $(a + b) \propto \frac{a}{b}$ ,  $(a^2 - a b + b^2) \propto \frac{b}{a}$ , then prove that :  $a^3 + b^3 = \text{constant}$

# Summary of Unit 2



## The ratio :

- ★ The value of the ratio does not change if each of its terms is multiplied or divided by the same non-zero real number.
- ★ The value of the ratio ( $\neq 1$ ) changes if we add or subtract (to or from) each of its two terms the same non-zero real number.
- ★ If the ratio between two numbers is  $a : b$ , then :

The first number =  $a m$  , The second number =  $b m$  ,  $m \neq 0$

## The proportion :

- ★ If  $\frac{a}{b} = \frac{c}{d}$ , then  $a, b, c$  and  $d$  are proportional quantities.
- ★ If  $a, b, c$  and  $d$  are proportional quantities, then  $\frac{a}{b} = \frac{c}{d}$
- ★ If  $\frac{a}{b} = \frac{c}{d}$ , then  $a \times d = b \times c$

*i.e.* the product of the extremes = the product of the means.

- ★ If  $a \times d = b \times c$ , then  $\frac{a}{b} = \frac{c}{d}$ ,  $\frac{a}{c} = \frac{b}{d}$ ,  $\frac{d}{b} = \frac{c}{a}$  and  $\frac{b}{a} = \frac{d}{c}$
- ★ If  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{a}{c} = \frac{b}{d}$

*i.e.*  $\frac{\text{The antecedent of the first ratio}}{\text{The antecedent of the second ratio}} = \frac{\text{The consequent of first ratio}}{\text{The consequent of second ratio}}$

- ★ If  $\frac{a}{b} = \frac{c}{d}$ , then  $a = c m$ ,  $b = d m$  where  $m$  is a constant  $\neq 0$
- ★ If  $a, b, c$  and  $d$  are proportional quantities and we assume that :  $\frac{a}{b} = \frac{c}{d} = m$ , then  $a = b m$ ,  $c = d m$
- ★ If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$  and  $m_1, m_2, m_3, \dots$  are non-zero real numbers, then  $\frac{m_1 a + m_2 c + m_3 e + \dots}{m_1 b + m_2 d + m_3 f + \dots} = \text{one of the given ratios.}$

- ★ The quantities  $a$ ,  $b$  and  $c$  are said to be in continued proportion if  $\frac{a}{b} = \frac{b}{c}$   
 $a$  is called the **first proportion**,  $c$  is called the **third proportion**  
and  $b$  is called the **middle proportion (proportional mean)**

$$b^2 = ac$$

$$i.e. \quad b = \pm \sqrt{ac}$$

- ★ If  $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$ , then  $c = dm$ ,  $b = dm^2$  and  $a = dm^3$

- ★ The direct variation and inverse variation :

#### Direct variation

- If  $y$  varies directly as  $X$   
and is written as  $y \propto X$ , then :
  - 1  $y = mX$  (i.e.  $\frac{y}{X} = m$ )  
where  $m$  is a constant  $\neq 0$
  - 2  $\frac{y_1}{y_2} = \frac{X_1}{X_2}$
  - 3 The relation between  $X$  and  $y$  is represented graphically by a straight line passing through the origin point.
- To prove that  $y \propto X$ ,  
we prove that :  $y = mX$   
where  $m$  is a constant  $\neq 0$

#### Inverse variation

- If  $y$  varies inversely as  $X$   
and is written as  $y \propto \frac{1}{X}$ , then :
  - 1  $y = \frac{m}{X}$  (i.e.  $Xy = m$ )  
where  $m$  is a constant  $\neq 0$
  - 2  $\frac{y_1}{y_2} = \frac{X_2}{X_1}$
  - 3 The relation between  $X$  and  $y$  is not linear relation.
- To prove that  $y \propto \frac{1}{X}$ ,  
we prove that :  $Xy = m$   
where  $m$  is a constant  $\neq 0$

# Exams on Unit Two



## ? Model 1

Answer the following questions :

**1** Choose the correct answer from those given :

**1** If  $4x^2 = 9y^2$ , then  $\frac{x}{y} = \dots\dots\dots$

(a)  $\frac{9}{4}$

(b)  $\frac{3}{2}$

(c)  $\pm \frac{3}{2}$

(d)  $\pm \frac{2}{3}$

**2** If  $x^2 y = 5$ , then  $\dots\dots\dots$

(a)  $y \propto x$

(b)  $y \propto x^2$

(c)  $y \propto \frac{1}{x}$

(d)  $y \propto \frac{1}{x^2}$

**3** If  $\frac{a}{2} = \frac{b}{3}$ , then  $\frac{b-a}{b+a} = \dots\dots\dots$

(a)  $\frac{1}{5}$

(b)  $\frac{1}{3}$

(c)  $\frac{2}{5}$

(d)  $\frac{3}{5}$

**4** If  $\frac{x}{2} = \frac{y}{3} = \frac{4x-2y}{z}$ , then  $z = \dots\dots\dots$

(a)  $-2$

(b)  $-\frac{1}{2}$

(c)  $\frac{1}{2}$

(d)  $2$

**5** The second proportional for the quantities  $12ab^2, \dots, 21ab, 14b^2$  is  $\dots\dots\dots$

(a)  $8ab^2$

(b)  $8b^3$

(c)  $24ab$

(d)  $24b^4$

**6** If  $y \propto x$  and  $x = 1$  when  $y = 4$ , then the constant proportional equals  $\dots\dots\dots$

(a)  $1$

(b)  $-4$

(c)  $4$

(d)  $-\frac{1}{4}$

**2** [a] If  $\frac{a}{4} = \frac{b}{3}$ , find the value of :  $\frac{ab+a^2}{ab-b^2}$

[b] If  $b$  is the middle proportion between  $a$  and  $c$ , prove that :  $\sqrt{\frac{a^2+b^2}{b^2+c^2}} = \frac{a}{b}$   
where  $a, b$  and  $c$  are positive quantities.

**3** [a] If  $y$  varies inversely with  $x^2$  and  $x = 3$  when  $y = 4$ , find :

**1** The relation between  $y$  and  $x$

**2** The value of  $x$  when  $y = 9$

[b] If  $a, b, c$  and  $d$  are proportional quantities, then prove that :  $\frac{a+2c}{b+2d} = \frac{c-a}{d-b}$

4 [a] If  $\frac{a}{x+y} = \frac{b}{y-2x} = \frac{c}{x+4y}$ , find the value of :  $\frac{b+2c}{2a+b}$

[b] If  $y = 3 + a$  and  $a \propto \frac{1}{x}$  and  $y = 5$  when  $x = 1$ , find the relation between  $x$  and  $y$ , then find the value of  $y$  when  $x = 2$

5 [a] If  $a, b, c, d$  are in continued proportion,

prove that :  $\frac{2a^3 + 3b^3}{2a + 3d} = a^2$

[b] Find the positive number if its square is added to the antecedent of the ratio  $29 : 46$  and its square is subtracted from its consequent it becomes  $3 : 2$

### Model 2

Answer the following questions :

1 Choose the correct answer from those given :

1 If  $a, b, 2$  and  $3$  are proportional quantities, then  $\frac{b}{a} = \dots\dots\dots$

- (a)  $\frac{3}{2}$  (b)  $\frac{2}{3}$  (c)  $3$  (d)  $2$

2 If  $2, 6$  and  $x + 15$  are proportional, then  $x = \dots\dots\dots$

- (a)  $1$  (b)  $2$  (c)  $3$  (d)  $4$

3 If  $xy = 12$ , then  $y$  varies directly with  $\dots\dots\dots$

- (a)  $\frac{1}{x}$  (b)  $x - 12$  (c)  $x$  (d)  $x + 12$

4 If  $y$  varies inversely with  $x$  and  $x = \sqrt{3}$  when  $y = \frac{2}{\sqrt{3}}$ , then the constant of proportion equals  $\dots\dots\dots$

- (a)  $\frac{1}{2}$  (b)  $\frac{2}{3}$  (c)  $2$  (d)  $6$

5 If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = m$  (where  $m \in \mathbb{R}^*$ ), then  $\frac{ace}{bdf} = \dots\dots\dots$

- (a)  $m$  (b)  $3m$  (c)  $m^3$  (d)  $3m^3$

6 If  $\frac{a}{3} = \frac{b}{5}$ , then  $5a - 3b + 7 = \dots\dots\dots$

- (a)  $3$  (b)  $9$  (c)  $7$  (d)  $5ab$

**2** [a] If  $\frac{a}{b-a} = \frac{c}{d-c}$  Prove that : a , b , c and d are proportional.

[b] If  $y = a - 1$  , a varies inversely as  $X^2$  and  $a = 4$  when  $X = 2$  , find :

**1** The relation between y and X

**2** The value of X when  $y = 8$

**3** [a] If  $\frac{X+y}{5} = \frac{y+z}{3} = \frac{z+X}{6}$

Prove that :  $\frac{X-z}{2} = \frac{X+y+z}{7}$

[b] If a , b , c and d are proportional quantities

Prove that :  $\frac{2a+3b}{2c+3d} = \frac{a}{c}$

**4** [a] If  $5a = 3b$  , then find the value of :  $\frac{7a+9b}{4a+2b}$

[b] If b is the middle proportion between a and c

Prove that :  $\frac{a-b}{a-c} = \frac{b}{b+c}$

**5** [a] If  $4X^2 + y^2 = 4Xy$

**1** Prove that :  $y \propto X$

**2** Find the value of : X when  $y = 8$

[b] If  $a : b : c = 4 : 5 : 3$  Prove that :  $\frac{a-b+c}{a+b-c} = \frac{1}{3}$

# A Research Project

## On Unit Two



### Project aims :

- Using ratio and proportion to solve problems.
- Differentiation between direct variation and inverse variation.
- Using mathematics to solve life problems.
- Associating mathematics with social studies.
- Associating mathematics with science.

### Do a research project on the following topic :

*"Drawing scale is an application of ratio and proportion. It is considered one of the most important elements of a map".*

**Discuss the following points using available resources :**

- Define drawing scale and mention its importance.
- Stick a map of Egypt with a given drawing scale.
- Determine two governorates on this map and measure the distance between them. Then use the drawing scale on this map to calculate the real distance between these two governorates.
- Search for the law of uniform velocity and then calculate the time that a car , moving at uniform velocity of  $120 \text{ km./hr.}$  , takes between these two governorates.
- What is the type of variation between the time and the distance when the velocity is constant ?

# UNIT 3

## Statistics



### Exercises of the unit :

- 9. Collecting data.
- 10. Dispersion.
- Unit Exam.

• A research project on unit three




Scan the  
QR code  
to solve an  
interactive  
test on each  
lesson



## EXERCISE

# 9

## Collecting data

 From the school book

### 1 Complete the following :

- 1 The resources of collecting data are ..... and ..... (Port Said 12)
- 2 The personal interview is a ..... resource of collecting data.
- 3 The data of the students that are registered in the students affair office is a ..... resource of collecting data.
- 4 Central agency for public mobilization and statistics is a ..... resource of collecting data.
- 5 Direct observing is a ..... resource of collecting data.
- 6 The suitable method for examining blood of a patient is a .....
- 7 The suitable method for checking the production of a factory is .....
- 8 The suitable method to know the population is .....
- 9 The suitable method to know the ratio of absence in one of the schools is .....
- 10 If the society is divided into illiterates and literates , carries of mediate , intermediate and high qualifications , the choosen sample for making a research is called .....

### 2 Choose the correct answer from those given :

- 1 ..... is a secondary resource of collecting data. (El-Fayoum 12)
  - (a) Personal interview
  - (b) Questionnaires
  - (c) Data base of the employees
  - (d) Observing and measuring
- 2 ..... is a primary resource of collecting data.
  - (a) Central agency for statistics
  - (b) Data of the school pupils in the previous year
  - (c) Questionnaires
  - (d) Data of the employees in one of the companies

- 3 The method of mass population is suitable for .....
- (a) searching the formation of the sand of the Western Desert.
  - (b) examining the sweetness of water for one of the wells.
  - (c) finding out the ratio of finding a metal in one of the mines.
  - (d) getting the number of the students who had the full mark in maths exam in a class.
- 4 Selecting a sample of layers of a statistical society is called ..... sample. (El-Bcheira 17)
- (a) random
  - (b) class (layer)
  - (c) deliberate
  - (d) bunch
- 5 A factory has 125 workers , 75 technicians and 50 engineers , it is wanted to take a sample of layers of size 50 individuals such that it represents each layer according to its size , then the number of engineers of the sample equals ..... (El-Monofia 16)
- (a) 30
  - (b) 20
  - (c) 25
  - (d) 15

**3 Which of the following statistical data is primary and which of them is secondary ?**

- 1 A survey for the pupils in your class about the place to which will be the next trip.
- 2 If you count the number of seats existing in each class of your school.
- 3 If you make an investigation about the number of the successful pupils in each school subject in your school in the first session last year from the registered notebooks in the school.
- 4 If you go to a government authority in your governorate to collect data about the babies registered in each health office through March last year.
- 5 Searching the internet sites for the results of one team of sports teams in the league in Egypt in the year 2008 – 2009

**4**  Compare between the methods of mass population and samples , showing the advantages and disadvantages of each of them.

**5 Mention the suitable method (mass population or samples) for collecting data in each of the following statistical societies :**

- 1 The educational level of a class formed from 25 students.
- 2 The range of validity of drinking water in a well for drinking.
- 3 The ratio of oil existing in an exploratory location.
- 4 The range of spread of a disease in one of the crops.
- 5 The counting of factories in one of the industrial cities.

- 6 200 employees were surveyed about their favourite food break time. Every one was given a digit number from 1 to 200 , then a sample representing 10% was selected to be interviewed about their favourite food :

(a) Hot drinks. (b) Light meals. (c) Soft drinks.  
Determine using your calculator the digits of target employees in this sample.



- 7 A school makes a study about how the pupils come to school , if the number of pupils in the school is 320 , each pupil is given a number from 1 to 320  
A sample of 10% from this number is selected to ask them how they come to school.

• on foot



• public transport



• taxi



• bike



• private car



Determine the digits of the sample using the calculator.

- 8 The administration of a hotel wants to conduct a survey to 300 customers on the service level produced. Every customer got a digit from 201 to 500 , 10% of them were selected as a random sample to ask them about the service level.  
Determine using the calculator the digits of the marked customers in this sample.



- 9 At a faculty , there are 4000 university students in the first grade , 3000 in the second grade , 2000 in the third grade and 1000 in the fourth grade. If we want to draw a layer sample of 500 students , where each layer is represented in this sample according to its size. Calculate the number of students in each layer in the sample.



« 200 , 150 , 100 , 50 »

- 10 One of the factories of cars produces 3 models of cars in the year , their numbers are :

300 cars from the first model.

500 cars from the second model.

200 cars from the third model.

The directorate of the factory wanted to select a sample of 5% of production to represent each model according to its size.

- Determine the number of the selected sample.
- Determine the number of each model in the sample.



« 50 , 15 , 25 , 10 »

- 11 It is wanted to select a random sample to represent each layer due to its size from a society consisting of 5000 individuals and it is divided into two layers.

The number of the first layer is 1500 individuals.

If the number of the second layer in the sample is 140 individuals

- calculate the number of individuals in the sample.

« 200 »

- 12 There is a need to draw a random layer sample to represent all the layers according to their sizes from a society of a total 40000 values divided into three layers as follows :

Number of the layer	1	2	3
Number of values in the layer	12000	20000	8000

If the number of values in the first layer is 240 , calculate the size of the whole sample. « 800 »

- 13 A society has 2000 individuals divided into 4 layers. It is wanted to select a random sample such that each layer is represented due to its size.

The investigator designed the following table :

Number of the layer	1	2	3	4	Total
Number of individuals of the layer	500	700	.....	450	2000
Number of the layer in the sample	.....	.....	7	.....	.....

Complete the table.

« 350 , 10 , 14 , 9 , 40 »




## EXERCISE

# 10

## Dispersion



 From the school book

Interactive test

### 1 Complete the following :

- 1 Dispersion measurements are ..... and ..... *(Souhag 11)*
- 2 The simplest measure of the dispersion is ..... *(Damietta 11)*
- 3 The dispersion to any set equally values equals ..... *(Damietta 11)*
- 4 If the standard deviation for nine of the values is 3 ,  
then  $\sum (X - \bar{X})^2$  for these values is .....

### 2 Choose the correct answer from those given :

- 1 The difference between the greatest value and the smallest value in a set of individuals is called ..... *(El-Sharkia 18 – Souhag 18 – Port Said 19)*  
(a) the range. (b) the arithmetic mean.  
(c) the median. (d) the standard deviation.
- 2 The positive square root of the average of squares of deviations of the values from their mean is called ..... *(Port Said 18 – Kafr El-Sheikh 18 – El-Fayoum 19 – El-Kalyoubia 20)*  
(a) the range. (b) the arithmetic mean.  
(c) the standard deviation. (d) the median.
- 3 The mean of the values : 7 , 3 , 6 , 9 and 5 equals .....  
*(Alex. 17 – North Sinai 17 – El-Fayoum 18)*  
(a) 3 (b) 6 (c) 4 (d) 12

- 4 The range of the set of values : 23 , 22 , 15 , 18 and 17 is ..... (Cairo 15)  
 (a) 8 (b) 18 (c) 19 (d) 23
- 5 If 67 is the greatest value of a set and if the range equals 27 , then the smallest value of this set equals ..... (El-Menia 16)  
 (a) 67 (b) 40 (c) 27 (d) 94
- 6 The most repeated value in a set of values represents ..... (Damietta 13 – Luxor 16)  
 (a) the median. (b) the range.  
 (c) the mode. (d) the mean.
- 7 If the mean of numbers :  $3k - 3$  ,  $3k - 1$  ,  $2k + 1$  ,  $2k + 3$  and  $2k + 5$  is 13 , then  $k =$  ..... (Alexandria 11)  
 (a) - 5 (b) 10 (c) 5 (d)  $\frac{1}{5}$
- 8 If the range of the values :  $6 + k$  ,  $6 - k$  ,  $6 + 5k$  ,  $6 - 2k$  is 14 where  $k \in \mathbb{R}$  , then  $k =$  ..... (El-Sharkia 20)  
 (a) 1 (b) 2 (c) 3 (d) 4
- 9 If the range of the values 2 , 7 ,  $a$  , 6 is 8 where  $a > 0$  , then  $a =$  ..... (El-Sharkia 14)  
 (a) 4 (b) 9 (c) - 1 (d) 10
- 10 Which of the following values of  $a$  makes the range of the numbers : 53 ,  $a$  , 58 , 57 , 60 and 55 equal to 9 ? (El-Dakahlia 16)  
 (a) 63 (b) 61 (c) 51 (d) 50
- 11  $\frac{\text{Sum of values}}{\text{Number of these values}} =$  ..... (Suez 19 – Alex. 20)  
 (a) range (b) standard deviation (c) mean (d) mode
- 12 If  $2x + 2y = 10$  ,  $x, y \in \mathbb{R}^+$  , then the arithmetic mean of the values  $x$  and  $y$  is ..... (Suez 16)  
 (a)  $\frac{2}{5}$  (b)  $\frac{5}{2}$  (c) 5 (d) 2
- 13 The set which has more disperion of the following sets is the set ..... (El-Kalyoubia 15)  
 (a) 28 , 17 , 30 , 36 , 20 (b) 20 , 19 , 29 , 37 , 43  
 (c) 31 , 35 , 26 , 37 , 41 (d) 25 , 39 , 19 , 5 , 27
- 14 The commonest measure of dispersion and the most accurate is the ..... (Damietta 14 – El-Menia 18)  
 (a) range. (b) mean. (c) standard deviation. (d) median.
- 15 If all individuals are equal in values , then ..... (Southern Sinai 17 – Luxor 20)  
 (a)  $x - \bar{x} > 0$  (b)  $x - \bar{x} < 0$  (c)  $\sigma = 0$  (d)  $\bar{x} = 0$

16 If  $\sum (X - \bar{X})^2 = 48$  of a set of values and the number of these values = 12, then  $\sigma = \dots\dots\dots$  (Cairo 17 – El-Monofia 19)

- (a) - 4                      (b) - 2                      (c) 2                      (d) 4

3 Complete the following table, then answer :

	Set A	Set B
The values	40 , 45 , 50 , 52 , 55 , 58	20 , 35 , 50 , 55 , 60 , 80
The mean	$\frac{40 + 45 + \dots + \dots + \dots + \dots}{6} = \dots\dots\dots$	$\frac{20 + 35 + \dots + \dots + \dots + \dots}{6} = \dots\dots\dots$
The median	$\dots\dots\dots$	$\dots\dots\dots$
The range	$\dots\dots\dots$	$\dots\dots\dots$
Standard deviation	$\dots\dots\dots$	$\dots\dots\dots$

Which of the two sets is more homogeneous ?

4 Calculate the standard deviation for the next data :

- 1 16 , 32 , 5 , 20 , 27 (El-Monofia 17 – El-Gharbia 18 – El-Monofia 19 – Port Said 20) « 9.3 »  
 2 72 , 53 , 61 , 70 , 59 (Luxor 19 – Damietta 20) « 7.1 »  
 3 15 , - 12 , - 9 , 27 , - 6 « 15.3 »  
 4 22 , 20 , 20 , 20 , 18 « 1.3 »

5 Which of the following sets has more dispersion , using the standard deviation ?

Set (A) : 7 , 8 , 9 , 10 , 11      Set (B) : 21 , 20 , 11 , 19      Set (C) : 29 , 30 , 30 , 35

6 Calculate the mean and standard deviation of the following data :

- 1 73 , 54 , 62 , 71 , 60 (Assiut 17 – Qena 20) « 64 , 7.07 »  
 2 13 , 14 , 17 , 19 , 22 (to the nearest 3 decimals digits) (El-Sharkia 17) « 17 , 3.286 »  
 3 65 , 61 , 70 , 64 , 70 , 76 , 70 « 68 , 4.6 »  
 4 23 , 12 , 17 , 13 , 15 , 16 , 8 , 9 , 37 , 10 « 16 , 8.2 »

7 The following values represent marks of five pupils in a test : 8 , 9 , 6 , 12 , 10 Calculate : (El-Dakahlia 17) « 9 »

- 1 The mean of the marks. « 2 »  
 2 The standard deviation of the marks.

- 8 The opposite table shows the temperature in some cities :

- 1 Calculate the mean and standard deviation of the maximum temperature.
- 2 Calculate the mean and standard deviation of the minimum temperature.

City	Max.	Min.
Ismailia	25	11
Suez	26	12
El-Arish	24	10
Nakhl	24	6
Taba	22	7
El-Tur	26	16
Hurghada	27	15
Rafah	26	11

« 25 , 15 , 11 , 3.2 »

- 9 The following frequency distribution shows the number of children of some families in a new city :

(El-Beheira 16 – Alex. 19 – El-Monofia 20)

Number of children	zero	1	2	3	4
Number of families	8	16	50	20	6

Calculate the mean and the standard deviation of the number of children.

« 2 , 1 »

- 10 The following are the frequency distribution for a number of defective units found in 100 boxes of manufactured units :

(El-Beheira 14 – El-Beheira 17 – Souhag 18)

Number of defective units	zero	1	2	3	4	5
Number of boxes	3	16	17	25	20	19

Find the standard deviation to the defective units.

« 1.4 »

- 11 The following frequency distribution shows the number of goals which have been scored by 30 players from 5 penalty kicks during a training :

Number of scored goals	0	1	2	3	4	5
Number of players	2	4	5	8	7	4

Find the mean and standard deviation of the number of scored goals.

« 2.9 , 1.4 »

- 12 The following frequency distribution shows the ages of 10 children :

Age in years	5	8	9	10	12	Total
Number of children	1	2	3	3	1	10

Calculate the standard deviation of the ages in years.

(Alex. 17 – Cairo 18 – Qena 19 – Cairo 20) « 1.7 years »

- 13 The following table shows the frequency distribution of the number of students who won in an art competition from a school having 20 classes :

Number of students	0	1	2	3	4	5	Total
Number of classes	1	3	5	6	3	2	20

Find the mean and the standard deviation of the number of students.


« 2.65 , 1.3 »

- 14 The following table represents the frequency distribution of sets of temperature degrees in some of world cities :

Sets of temperature degrees	5 –	15 –	25 –	35 –	45 –
Frequency	7	9	11	15	8

Find the mean and the standard deviation of the temperature degrees.

« 31.6 , 12.9 »

- 15  Calculate the mean and the standard deviation for the following frequency distribution :

(Qena 16 – El-Gharbia 17)

Set	0 –	4 –	8 –	12 –	16 – 20	Total
Frequency	3	4	7	2	9	25

« 11.6 , 5.7 »


- 16 The following table represents the daily wages of a set of workers in a factory :

(Kafr El-Sheikh 20)

Sets of wages	20 –	30 –	40 –	50 –	60 –	70 –
Number of workers	10	12	8	6	3	1

Find the mean and standard deviation of the wages.

« 40.75 , 13.4 »

- 17  The following distribution table shows the amount of gasoline that a set of cars consumes :

Number of kilometres per litre	5 –	7 –	9 –	11 –	13 –	15 –	Total
Number of cars	3	6	10	12	5	4	40

Find the standard deviation of the number of kilometres per litre.

« 2.7 »

- 18 The following frequency distribution shows the value of electricity bill for 200 participants :

The value of bill in pounds	5 –	15 –	25 –	35 –	45 –	55 –	Total
Number of participants (frequency)	19	50	85	25	15	6	200

Find the mean and the standard deviation of the values of these bills.

« 29.25 , 11.5 »



**For excellent pupils**

- 19 The two frequency tables represent the marks of the students of two classes A and B of third preparatory in an exam :

Class A	Sets of marks	0 –	10 –	20 –	30 –	40 – 50	Total
	Number of students	2	5	11	15	7	40

Class B	Sets of marks	0 –	10 –	20 –	30 –	40 – 50	Total
	Number of students	2	3	18	7	10	40

- 1 Represent both of distributions using the frequency polygon in one figure.
- 2 Find the mean and standard deviation for both frequency distributions.
- 3 Which class is more homogeneous in getting marks ?

« 30 , 10.7 , 30 , 11 »

# Summary of Unit 3



## Resources of collecting data :

### ★ Primary resources (field resources)

These are the resources from which the researcher gets data directly.

### ★ Secondary resources (historical resources) :

These are the resources from which the researcher gets data that previously collected and registered by some authorities , formal organisations or persons.

## Methods of collecting data :

### ★ Method of mass population :

It is based on collecting the data related to the phenomenon under study from all individuals of the statistical society and it is used to inventory all individuals of society.

### ★ Method of samples :

It is based on collecting data related to the phenomenon under study from a representative sample of the society , and applying the research on it , then generalizing the results on the whole society.

## The samples :

### ★ The sample is a small part from a large society that looks like the society and represents it well.

### ★ Not a random sample (deliberate sample) :

It is a sample in which we select its individuals of the statistical society by a certain way to satisfy the objectives of the research.

### ★ Simple random sample :

It is a sample used for the homogeneous societies which are not naturally divided into groups or classes.

### ★ Layer random sample :

It is a sample used in the statistical societies which are heterogeneous or made up of qualitative sets that are different in characteristics.

### ★ The number of individuals of the layer in the sample

$$= \frac{\text{the total number of individuals in the layer}}{\text{the total number of individuals in the society}} \times \text{the number of individuals of the sample}$$

«approximated the result to the nearest unit»

### The dispersion :

- ★ The dispersion is a measure that expresses how much the sets are homogeneous.
- ★ The range of a set of values is the difference between the greatest value and the smallest value in the set.
- ★ Standard deviation is the most important , common and accurate measure of dispersion. We can calculate it by calculating the positive square root of the average of squares of deviations of the values from their mean.

★ The standard deviation ( $\sigma$ ) of a set of values  $= \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$

Where :

$x$  denotes a value of the values ,

$\bar{x}$  denotes the mean of the values ,

$n$  denotes the number of values

★ The standard deviation ( $\sigma$ ) of a frequency distribution  $= \sqrt{\frac{\sum (x - \bar{x})^2 k}{\sum k}}$

Where :

$x$  represents the value or the centre of the set ,  $k$  represents the frequency of the value or the set ,

$\sum k$  is the sum of frequencies and  $\bar{x}$  (the mean)  $= \frac{\sum (x \times k)}{\sum k}$

## Exam on Unit Three



**Answer the following questions :**

**1** Choose the correct answer from those given :

**1** The difference between the greatest value and the smallest value of a set of individuals is called .....

- (a) the range. (b) the arithmetic mean.  
(c) the median. (d) the standard deviation.

**2** If  $\sum (X - \bar{X})^2 = 36$  of a set of values and the number of these values = 9 , then  $\sigma = \dots\dots\dots$

- (a) 2 (b) 4 (c) 3 (d) 27

**3** Selecting a sample of layers of a statistical society is called ..... sample.

- (a) random (b) class (layer) (c) deliberate (d) bunch

**4** The range of the set of values : 5 , 14 , 4 , 37 , 15 , 16 and 7 is .....

- (a) 30 (b) 33 (c) 32 (d) 22

**5** The positive square root of the average of squares of deviations of the values from its arithmetic mean is called .....

- (a) the range. (b) the arithmetic mean.  
(c) the median. (d) the standard deviation.

**6** The standard deviation of the values : 5 , 5 , 5 , 5 equals .....

- (a) zero (b) 5 (c) 6 (d) 2

**2** Calculate the mean and standard deviation of the following data : 16 , 32 , 5 , 20 , 27

**3** If the number of persons in 50 families is as follows :

Number of persons	2	3	4	5	6	7	8
Number of families	5	7	8	12	9	5	4

Calculate the mean and the standard deviation of the number of persons.

**4** The following frequency distribution shows the marks of 20 students in an exam :

Sets	zero -	2 -	4 -	6 -	8 -
Frequency	1	3	6	5	5

Calculate the standard deviation.

# A Research Project

## On Unit Three



### Project aims :

- Organizing data in frequency tables with sets.
- Calculating the range of a set of individuals.
- Calculating the mean and the standard deviation of a frequency distribution of sets.
- Appreciating the role of statistics in everyday life.

### Do a research project on the following topic :

*"Standard deviation is considered the most accurate, important and common measure of dispersion measures".*

**Discuss the following points using available resources :**

- Choose two of dispersion measures and point out their advantages and disadvantages.
- Record your classmates' marks at one of the maths exams and at a social studies' exam , then do the following :
  1. Find the range of your class marks in each of the two subjects.
  2. Form the frequency table with sets for the marks of math and according to the table's data , calculate the mean and the standard deviation of your class marks in maths.
  3. Form the frequency table with sets for the marks of social studies and according to the table's data , calculate the mean and the standard deviation of your class marks in social studies.
  4. Indicate the subject in which the level of your class is more homogeneous.

# SKILLS

**TIMSS Problems**

## Accumulative basic skills

Choose the correct answer from the given ones :

1  $\{3\} \subset \dots\dots\dots$

(Alex. 16)

(a)  $(3, 7)$

(b)  $]3, 7]$

(c)  $]3, 7[$

(d)  $\{3, 7\}$

2  $[2, 7] - \{2, 7\} = \dots\dots\dots$

(Matrouh 17)

(a)  $[1, 6]$

(b)  $\emptyset$

(c)  $]2, 7[$

(d)  $\{0\}$

3 The next number in the pattern :  $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}$  is  $\dots\dots\dots$

(New Valley 20)

(a)  $\sqrt{50}$

(b)  $\sqrt{75}$

(c)  $\sqrt{60}$

(d)  $\sqrt{90}$

4  $2^{2017} = 2^{2016} + \dots\dots\dots$

(Luxor 17)

(a) 1

(b) 2

(c) 2016

(d)  $2^{2016}$

5 If  $[-1, x] \cap [y, 5] = [2, 3]$ , then  $x^y = \dots\dots\dots$

(a) 8

(b)  $\frac{1}{5}$

(c) 9

(d) -1

6 When the side length of a square increases by the ratio 10% , then its area increases by the ratio  $\dots\dots\dots\%$

(a) 10

(b) 15

(c) 20

(d) 21

7 The ratio between the area of a square shaped region of side length  $x$  cm. to the area of another square shaped region of side length  $2x$  cm. is  $\dots\dots\dots$

(Beni Suef 17)

(a) 1 : 2

(b)  $x : 4$

(c) 1 : 4

(d) 4 : 1

8 If  $F$  is an odd number , then the next odd number directly is  $\dots\dots\dots$  (Giza 17 - South Sinai 19)

(a)  $F^2$

(b)  $F^2 + F$

(c)  $F + 1$

(d)  $F + 2$

- 9 If  $M$  represents a negative number, which of the following represents a positive number? (Kafr El-Sheikh 17)
- (a)  $M^3$  (b)  $M^2$  (c)  $2M$  (d)  $\frac{M}{2}$
- 10 Half of the number  $2^{20}$  is ..... (Damietta 17)
- (a)  $2^{10}$  (b)  $1^{20}$  (c)  $2^{19}$  (d)  $1^{10}$
- 11 If  $(X - 3)^{\text{zero}} = 1$ , then  $X \in$  ..... (El-Monofia 18)
- (a)  $\mathbb{R}$  (b)  $\mathbb{R} - \{3\}$  (c)  $\mathbb{R} - \{4\}$  (d)  $\mathbb{R} - \{1\}$
- 12  $\left(\frac{\sqrt{5}+1}{2}\right)^{1000} \left(\frac{\sqrt{5}-1}{2}\right)^{1000} =$  ..... (El-Monofia 18)
- (a) zero (b) 1 (c)  $\frac{5^{1000}-1}{4}$  (d)  $4^{1000}$
- 13  $3^X + 3^X + 3^X =$  ..... (Suez 16)
- (a)  $9^X$  (b)  $3^{3X}$  (c)  $3^{X+1}$  (d)  $3^{X+3}$
- 14  $2^5 + 2^5 + 2^5 + 2^5 =$  ..... (Luxor 16)
- (a)  $2^7$  (b)  $2^6$  (c)  $2^4$  (d)  $2^{20}$
- 15 If  $X - y = 5$ ,  $X + y = \frac{1}{5}$ , then  $X^2 - y^2 =$  ..... (Kafr El-Sheikh 17 - Aswan 20)
- (a)  $\frac{1}{25}$  (b) 1 (c) 25 (d) 5
- 16 If  $X + y = yX = 5$ , then  $X^2y + y^2X =$  ..... (Aswan 16 - Ismailia 20)
- (a) 10 (b) 15 (c) 20 (d) 25
- 17 If  $(X - y)^2 = 20$ ,  $X^2 + y^2 = 10$ , then  $XY =$  ..... (Alex. 16)
- (a) 10 (b) 5 (c) -5 (d) 20
- 18 If  $1 < X < 3$ , then  $(3X - 1) \in$  ..... (Suez 16 - Giza 20)
- (a)  $[2, 8[$  (b)  $[2, 8]$  (c)  $]2, 8[$  (d)  $\{2, 8\}$
- 19 The S.S. of the inequality:  $5 - 3X > 11$  in  $\mathbb{R}$  is ..... (Kafr El-Sheikh 17)
- (a)  $] -\infty, -2[$  (b)  $] -2, \infty[$  (c)  $] -\infty, -2]$  (d)  $[-2, 2]$
- 20 The sum of the two square roots of  $2\frac{1}{4}$  is ..... (El-Monofia 17 - North Sinai 19)
- (a) zero (b)  $\frac{3}{2}$  (c) 3 (d)  $\frac{9}{4}$
- 21 Four times the number  $2^8 =$  ..... (Alex. 17 - Sohag 19)
- (a)  $2^{32}$  (b)  $8^8$  (c)  $2^{10}$  (d)  $4^8$
- 22 If  $X = \sqrt{3} + \sqrt{2}$ ,  $y = \frac{1}{\sqrt{3} + \sqrt{2}}$ , then  $(X + y)^2 =$  ..... (El-Gharbia 17)
- (a) 8 (b) zero (c) 9 (d) 12

- 23 If  $2^x = \frac{1}{8}$ , then  $x = \dots\dots\dots$  (El-Monofia 17)  
 (a)  $\frac{1}{2}$  (b)  $\frac{1}{3}$  (c) 3 (d) -3
- 24 A book consists of 56 pages. How many pages the number 5 appears in the pages serial of this book ?  
 (a) 6 (b) 7 (c) 12 (d) 13
- 25 If we put on one side of a road of length 12 km. some light poles from the begining to the end of the road, where the distance between each two consecutive poles is  $\frac{1}{2}$  km. , then the number of poles is .....  
 (a) 12 (b) 24 (c) 25 (d) 23
- 26 The decimal that lies between 0.07 and 0.08 is ..... (Alex. 17)  
 (a) 0.00075 (b) 0.0075 (c) 0.075 (d) -0.75
- 27 The square of double the number  $\frac{1}{2}$  is .....  
 (a)  $\frac{1}{4}$  (b)  $\frac{1}{8}$  (c) 1 (d) 2
- 28 If three times a number = 45 , then  $\frac{1}{5}$  of this number = ..... (El-Menia 19)  
 (a) 15 (b) 5 (c) 3 (d) 9
- 29 If  $\frac{5}{4} + \frac{5}{x} = \frac{5}{2}$ , then  $x = \dots\dots\dots$  (El-Monofia 20)  
 (a) 2 (b) 4 (c) 5 (d)  $\frac{5}{2}$
- 30  $]-1, 3] \cap \{-3, -1\} = \dots\dots\dots$  (Assiut 18)  
 (a)  $\emptyset$  (b)  $\{-3\}$  (c)  $\{-1\}$  (d)  $\{3\}$
- 31  $[2, 7] - ]2, 7[ = \dots\dots\dots$  (Beni Suef 18)  
 (a)  $\emptyset$  (b)  $\{2\}$  (c)  $\{7\}$  (d)  $\{2, 7\}$
- 32  $\mathbb{Z}^- \cup \mathbb{N} = \dots\dots\dots$  (Luxor 17 - Alex. 18)  
 (a)  $\emptyset$  (b)  $\mathbb{N}$  (c)  $\mathbb{Z}$  (d)  $\mathbb{R}$
- 33 The expression :  $(x - 2)^2 - x^2$  is of ..... degree. (Kafr El-Sheikh 20)  
 (a) first. (b) second. (c) third. (d) fourth.
- 34 The solution set of the equation :  $x - 1 = |-1|$  in  $\mathbb{N}$  is ..... (Suez 18)  
 (a) (1, 2) (b) 2 (c)  $\{2\}$  (d)  $\{-2\}$
- 35 If  $17x + 8 = 11$ , then  $17x + 11 = \dots\dots\dots$  (Ismailia 19)  
 (a) 8 (b) 11 (c) 14 (d) 17

Second

# Trigonometry and Geometry



UNIT **4** Trigonometry..... 81

UNIT **5** Analytical geometry..... 102

► Accumulative basic skills “TIMSS Problems” ..... 134

# UNIT 4

## Trigonometry



### ► Exercises of the unit :

1. The main trigonometrical ratios of the acute angle.
  2. The main trigonometrical ratios of some angles.
- 🔄 Unit Exams.

🔍 A research project on unit four



Scan the  
**QR code**  
to solve an  
interactive  
test on each  
lesson

## EXERCISE

# 1

## The main trigonometrical ratios of the acute angle



From the school book

Interactive test

1 Complete the following :

1  $85^{\circ} 38' 8'' = \dots\dots\dots$  (in degrees)

2  $56.18^{\circ} = \dots\dots\dots$  (in degrees , minutes and seconds)

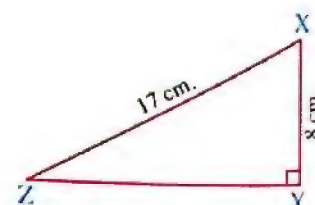
3 In the opposite figure :

XYZ is a right-angled triangle at Y  
in which  $XY = 8 \text{ cm.}$  ,  $XZ = 17 \text{ cm.}$

(a)  $\sin X = \dots\dots\dots$  ,  $\sin Z = \dots\dots\dots$

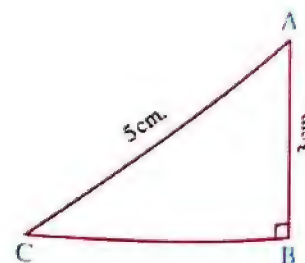
(b)  $\cos X = \dots\dots\dots$  ,  $\cos Z = \dots\dots\dots$

(c)  $\tan X = \dots\dots\dots$  ,  $\tan Z = \dots\dots\dots$



4 In the opposite figure :

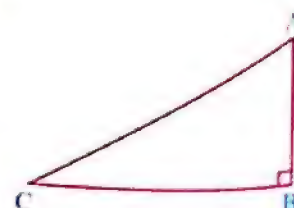
$\triangle ABC$  is a right-angled triangle at B  
 ,  $AB = 3 \text{ cm.}$  ,  $AC = 5 \text{ cm.}$   
 , then  $\sin C \times \cos C = \dots\dots\dots$



(Beni Suef 11)

5 In the opposite figure :

$\frac{BC}{AC} = \cos \dots\dots\dots$

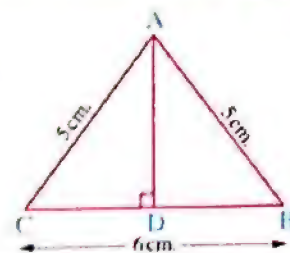


(Damietta 11)

6 In the opposite figure :

$\cos B = \dots\dots\dots$

(El Ghurba 12)



2 Choose the correct answer from those given :

1 For any acute angle A ,  $\tan A = \dots\dots\dots$

(Ismailia 12)

- (a)  $\frac{\cos A}{\sin A}$                       (b)  $\sin A \cos A$                       (c)  $\frac{\sin A}{\cos A}$                       (d)  $\sin A + \cos A$

2 If X , y are measures of two complementary angles and  $\sin X = \frac{3}{5}$

, then  $\cos y = \dots\dots\dots$

(Giza 17 - El-Beheira 18 - Giza 20)

- (a)  $\frac{4}{5}$                       (b)  $\frac{3}{5}$                       (c)  $\frac{3}{4}$                       (d)  $\frac{5}{3}$

3 For any two acute angles A and B if  $\sin A = \cos B$  , then  $m(\angle A) + m(\angle B) = \dots\dots\dots$

- (a)  $30^\circ$                       (b)  $60^\circ$                       (c)  $90^\circ$                       (d)  $180^\circ$

4 If  $\sin 70^\circ = \cos X$  where X is an acute angle , then  $X = \dots\dots\dots$

(El-Kalyoubia 18)

- (a)  $60^\circ$                       (b)  $45^\circ$                       (c)  $10^\circ$                       (d)  $20^\circ$

5 In  $\triangle ABC$  , if  $m(\angle A) = 85^\circ$  and  $\sin B = \cos B$  , then  $m(\angle C) = \dots\dots\dots$

(El-Beheira 17 - El-Dakahlia 19)

- (a)  $30^\circ$                       (b)  $45^\circ$                       (c)  $50^\circ$                       (d)  $60^\circ$

6 In  $\triangle ABC$  : if  $m(\angle B) = 90^\circ$  , then  $\sin A + \cos C = \dots\dots\dots$

(El-Monofia 17)

- (a)  $2 \sin A$                       (b)  $2 \sin C$                       (c)  $2 \sin B$                       (d)  $2 \cos A$

7  $\triangle ABC$  is a right-angled triangle at A , then cosine angle B : sine angle C equals  $\dots\dots\dots$

(El-Sharkia 18)

- (a)  $\frac{3}{5}$                       (b)  $\frac{4}{3}$                       (c)  $\frac{3}{4}$                       (d) 1

8 DEF is a right-angled triangle at E , which of the following relations is false ?

- (a)  $\tan D \times \tan F = 1$                       (b)  $\sin D = \cos F$                       (c)  $\cos D = \sin F$                       (d)  $\cos D = \sin E$

(El-Dakahlia 16)

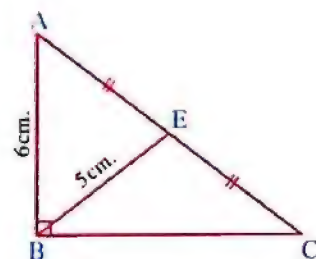
9 In the opposite figure :

$\triangle ABC$  is a right-angled triangle at B

,  $\overline{BE}$  is a median ,  $BE = 5$  cm.

,  $AB = 6$  cm. , then  $\sin C = \dots\dots\dots$

- (a)  $\frac{5}{6}$                       (b)  $\frac{3}{5}$   
(c)  $\frac{6}{5}$                       (d)  $\frac{5}{3}$



(Aswan 16)

- 3 If the ratio between the measures of two supplementary angles is 3 : 5 , find the measure of each one by degree measure. (El-Beheira 14 - Aswan 15 - El-Gharbia 19) «  $67^{\circ} 30'$  ,  $112^{\circ} 30'$  »

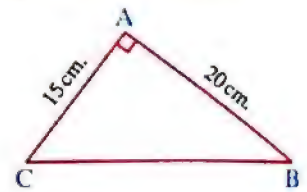
- 4 If the ratio between the measures of two complementary angles is 3 : 4 , find the degree measure of the greater angle in measure. «  $51^{\circ} 25' 43''$  »

- 5 If the ratio between the measures of the interior angles of the triangle is 3 : 4 : 7 , find the degree measure of each angle. (El-Beheira 13) «  $38^{\circ} 34' 17''$  ,  $51^{\circ} 25' 43''$  ,  $90^{\circ}$  »

- 6 In the opposite figure :

ABC is a triangle in which :  $m(\angle A) = 90^{\circ}$

, AC = 15 cm. and AB = 20 cm.



Prove that :  $\cos C \cos B - \sin C \sin B = \text{zero}$

(El-Beheira 17 - El-Kalyoubia 18 - El-Menia 19 - Giza 20)

- 7 XYZ is a right-angled triangle at Z where : XZ = 7 cm. and XY = 25 cm.

Find the value of each of the following :

1  $\tan X \times \tan Y$

2  $\sin^2 X + \sin^2 Y$

(Port Said 18) « 1 , 1 »

- 8 ABC is a right-angled triangle at B in which : BC = 4 cm. and AC = 5 cm.

Deduce that :  $\sin^2 A - \cos^2 A = 2 \sin^2 A - 1$

- 9 ABC is a right-angled triangle at B , if AB : AC = 3 : 5

Find the main trigonometrical of  $\angle A$

(Aswan 13) «  $\frac{4}{5}$  ,  $\frac{3}{5}$  ,  $\frac{4}{3}$  »

- 10 XYZ is a right-angled triangle at Y , if YZ = 2 XY

Find : The value of each of  $\tan Z$  ,  $\tan X$  ,  $\cos Z$  ,  $\cos X$

«  $\frac{1}{2}$  , 2 ,  $\frac{2}{\sqrt{5}}$  ,  $\frac{1}{\sqrt{5}}$  »

- 11 ABC is a right-angled triangle at B , if  $2 AB = \sqrt{3} AC$

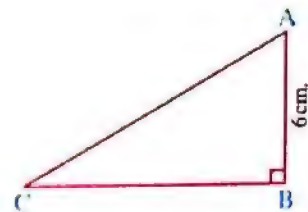
Find the main trigonometrical of the angle C

(Alexandria 15 - El-Dakahlia 18 - Aswan 19) «  $\frac{\sqrt{3}}{2}$  ,  $\frac{1}{2}$  ,  $\sqrt{3}$  »

12 In the opposite figure :

ABC is a right-angled triangle at B  
 $AB = 6$  cm. ,  $\tan C = \frac{3}{4}$  , find :

- 1 The length of each of  $\overline{BC}$  and  $\overline{AC}$
- 2  $\sin A + \cos A$

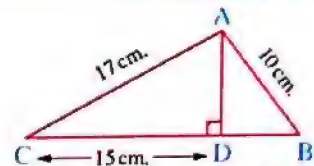


(Ismailia 12 - El-Monofia 15) « 8 cm. , 10 cm. ,  $\frac{7}{5}$  »

13 In the opposite figure :

$\overline{AD} \perp \overline{BC}$  ,  $AC = 17$  cm. ,  
 $DC = 15$  cm. ,  $AB = 10$  cm.

Find : The value of  $3 \tan C + \sin B$



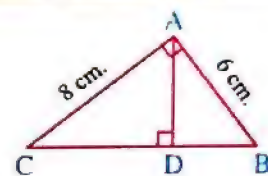
(Ismailia 14) «  $\frac{12}{5}$  »

14 In the opposite figure :

$m(\angle BAC) = 90^\circ$  ,  $\overline{AD} \perp \overline{BC}$   
 $AB = 6$  cm. ,  $AC = 8$  cm.

Find : 1  $\tan(\angle BAD)$

- 2 The value of :  $\cos(\angle DAC) + \cos(\angle DAB)$

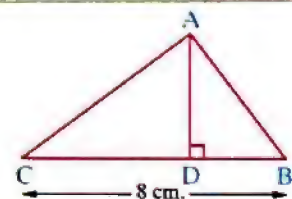


(El-Gharbia 16) «  $\frac{3}{4}$  ,  $\frac{7}{5}$  »

15 In the opposite figure :

$\Delta ABC$  is an acute-angled triangle  
 $BC = 8$  cm. ,  $\overline{AD} \perp \overline{BC}$

Find the value of :  $AB \cos B + AC \cos C$

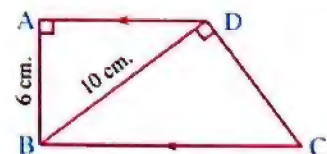


(El-Sharkia 17) « 8 cm. »

16 In the opposite figure :

ABCD is a trapezium in which  $\angle A$  is right  
 $\overline{AD} \parallel \overline{BC}$  ,  $m(\angle BDC) = 90^\circ$   
 $AB = 6$  cm. ,  $BD = 10$  cm.

Find :  $\tan(\angle ADB)$  and length of  $\overline{DC}$



(El-Dakahlia 17) «  $\frac{3}{4}$  , 7.5 cm. »

17 ABCD is an isosceles trapezoid in which :  $\overline{AD} \parallel \overline{BC}$  ,  $AD = 4$  cm. ,  $AB = 5$  cm. and  $BC = 12$  cm.

Prove that :  $\frac{5 \tan B \cos C}{\sin^2 C + \cos^2 B} = 3$

(New Valley 17)

18 ABCD is a trapezoid in which :  $\overline{AD} \parallel \overline{BC}$  ,  $m(\angle B) = 90^\circ$  if  $AB = 3$  cm.

$AD = 6$  cm. and  $BC = 10$  cm.

Prove that :  $\cos(\angle DCB) - \tan(\angle ACB) = \frac{1}{2}$

(El-Monofia 17 - Matrouh 18 - Giza 20)

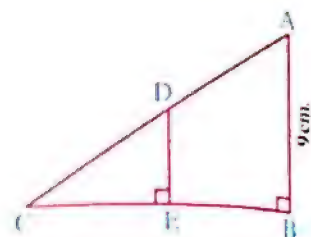
19 In the opposite figure :

ABC is a right-angled triangle at B in which :

$AB = 9 \text{ cm.}$  ,  $D \in \overline{AC}$  ,  $E \in \overline{BC}$

where  $\overline{DE} \perp \overline{BC}$  and  $4 DE = 3 EC$

**Calculate :** The area of  $\Delta ABC$



« 54 cm<sup>2</sup> »

20 ABC is an isosceles triangle in which :  $AB = AC$  and  $\sin \frac{A}{2} = \frac{4}{5}$

Find  $\cos B$  without using the calculator.

(Red Sea 13) «  $\frac{4}{5}$  »

21 If  $\Delta ABC$  is a right-angled at C , **prove that :**  $\sin B + \cos B > 1$

22 ABC is a right-angled triangle at B and  $\sin A = 0.6$

**Find :** The value of  $\sin A \cos C + \cos A \sin C$

(Kafr El-Sheikh 13) « 1 »

23 ABC is a right-angled triangle at B and  $7 \tan A - 24 = 0$

**Find :** The value of  $1 - \tan A \sin C$

«  $\frac{1}{25}$  »

24 If the following figures are formed from congruent squares , then find the required under each figure :

1



**Find :**  $\tan X$

2



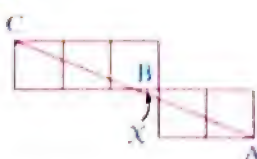
**Find :**  $\tan X$

3



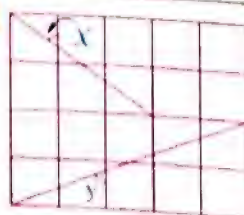
**Find :**  $\cos X$

4



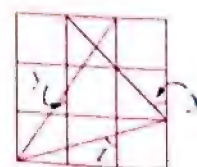
If A , B and C are collinear.  
**Find :**  $\tan X$

5



**Find :**  $\tan X + \frac{1}{\tan y}$

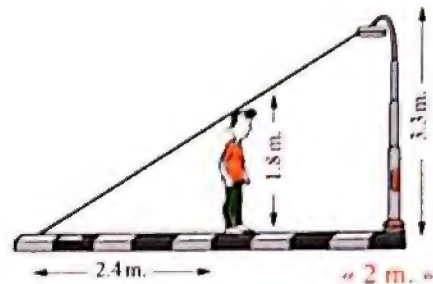
6



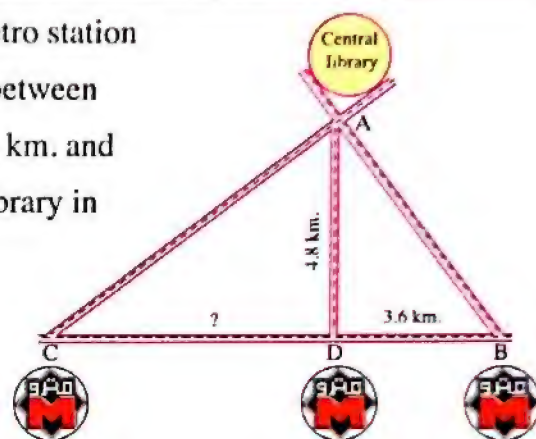
**Find :**  $\tan X + \tan y - \tan z$

### Life Applications

- 25 A man with a height of 1.8 m. stands in front of a lamppost with height 3.3 m. If the length of the man's shade (when the lamppost is turned on) is 2.4 m. Find the distance between the man and the base of the lamppost.



- 26 In one of the governorates, we want to build a metro station between two other stations such that the distance between that station and one of the other two stations is 3.6 km. and the shortest distance between it and the Central Library in the governorate is 4.8 km.



If you know that the two ways between the Central Library and the two metro stations B and C are orthogonal.

**Find in two different methods :**

The distance between the metro station we want to build and the metro station  $\approx 6.4 \text{ km.}$



### For excellent pupils

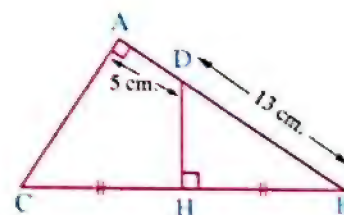
- 27 In the opposite figure :

$$m(\angle A) = 90^\circ, \overline{DH} \perp \overline{BC}$$

where H is the midpoint of  $\overline{BC}$

,  $AD = 5 \text{ cm.}$  and  $BD = 13 \text{ cm.}$

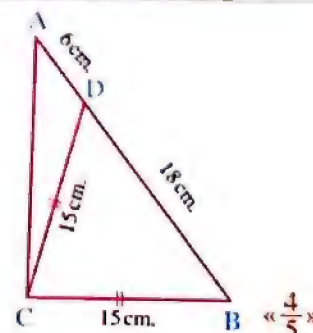
Find with proof  $\tan B$



(Damieta 17)  $\approx \frac{2}{3}$

- 28 From the opposite figure :

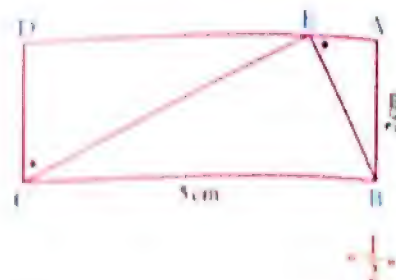
Find :  $\tan(\angle BAC)$



29 In the opposite figure :

ABCD is a rectangle in which :  
 $AE < ED$  ,  $AB = 2 \text{ cm}$  ,  $BC = 5 \text{ cm}$  ,  
 $m(\angle AEB) = m(\angle ECD)$

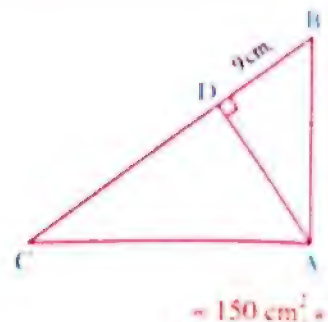
Find :  $\tan(\angle CED)$



30 In the opposite figure :

ABC is a triangle ,  $D \in \overline{BC}$  where :  
 $\overline{AD} \perp \overline{BC}$  ,  $BD = 9 \text{ cm}$  ,  
 $\sin(\angle BAD) = \cos(\angle CAD) = \frac{3}{5}$

Find : The area of  $\triangle ABC$

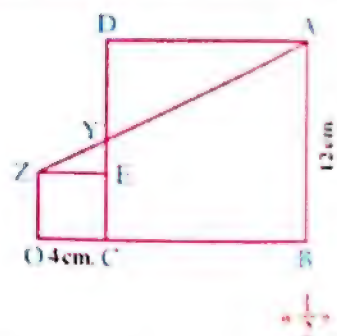


31 In any right-angled triangle ABC at B ,  
 prove that :  $\sin^2 A + \sin^2 C = 1$

32 In the opposite figure :

ABCD is a square ,  
 ECOZ is a square ,  
 $AB = 12 \text{ cm}$  ,  $CO = 4 \text{ cm}$  .

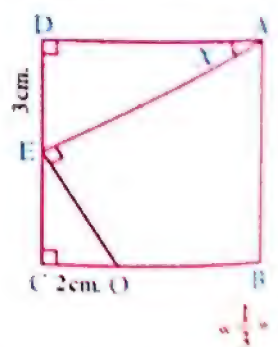
Find :  $\tan(\angle AZE)$



33 In the opposite figure :

ABCD is a square ,  $E \in \overline{DC}$  ,  
 $O \in \overline{BC}$  ,  $\overline{AE} \perp \overline{EO}$   
 $DE = 3 \text{ cm}$  ,  $CO = 2 \text{ cm}$  .

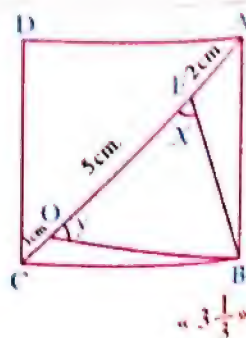
Find :  $\tan X$



34 In the opposite figure :

ABCD is a square ,  
 $E \in \overline{AC}$  ,  $O \in \overline{AC}$

Find : The value of  $\tan X + \tan y$





## EXERCISE

# 2

## The main trigonometrical ratios of some angles



From the school book

Interactive test

1 Without using the calculator , find each of the following :

1  $\sin 45^\circ - \cos 45^\circ$

2  $\cos 60^\circ + \sin 30^\circ$

3  $\sin 30^\circ + \cos 60^\circ - \tan 45^\circ$

4  $\sin 60^\circ + \cos 30^\circ + \tan 60^\circ$

5  $\sin^2 45^\circ + \cos^2 45^\circ$

6  $4 \cos 30^\circ \tan 60^\circ$

7  $\tan^2 60^\circ - 2 \sin 45^\circ \cos 45^\circ$

(North Sinai 17)

8  $\sin^2 60^\circ - \tan 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

(Assiut 17)

9  $2 \sin 30^\circ \cos 60^\circ + \sqrt{2} \sin 45^\circ$

10  $(\cos 30^\circ - \cos 60^\circ) (\sin 30^\circ + \sin 60^\circ)$

11  $\frac{\sin 30^\circ}{\cos 60^\circ} - \cos 30^\circ \sin 60^\circ$

(Ismailia 17)

12  $\frac{\cos^2 60^\circ + \cos^2 30^\circ + \tan^2 45^\circ}{\sin 60^\circ \tan 60^\circ - \sin 30^\circ}$

(El-Gharbia 17)

2 Without using the calculator , prove each of the following :

1  $\sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ$

(Souhag 18 - Giza 19 - North Sinai 20)

2  $\cos 60^\circ = 2 \cos^2 30^\circ - 1$

(Port Said 18 - North Sinai 19 - South Sinai 20)

3  $2 \cos^2 30^\circ - 1 = 1 - 2 \sin^2 30^\circ$

(El-Sharkia 15)

4  $\cos 60^\circ = \cos^2 30^\circ - \sin^2 30^\circ$

(Alex. 17 - El-Fayoum 18 - South Sinai 19)

5  $\tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$

(Matrouh 17 - Damietta 19 - Alex. 20)

6  $\cos^2 60^\circ = 5 \sin^2 30^\circ - \tan^2 45^\circ$

(Kaf El-Sheikh 11 - El-Menia 14 - Suez 17)

7  $\sin^3 30^\circ = 9 \cos^3 60^\circ - \tan 45^\circ$

(Luxor 17)

8  $\frac{\sin 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ}{\sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ} = \tan^2 45^\circ$

9  $\sin 30^\circ = \sqrt{\frac{1 - \cos 60^\circ}{2}}$

3 Choose the correct answer from those given :

1 If  $\cos X = \frac{1}{2}$  where  $X$  is an acute angle , then  $m(\angle X) = \dots\dots\dots$

(Cairo 13)

- (a)  $90^\circ$  (b)  $60^\circ$  (c)  $45^\circ$  (d)  $30^\circ$

2 If  $\sin X = \frac{1}{2}$  where  $X$  is an acute angle , then  $m(\angle X) = \dots\dots\dots$

(Cairo 20)

- (a)  $90^\circ$  (b)  $60^\circ$  (c)  $45^\circ$  (d)  $30^\circ$

3 If  $\tan X = \frac{1}{\sqrt{3}}$  where  $X$  is an acute angle , then  $\tan 2X = \dots\dots\dots$

- (a)  $\frac{2}{\sqrt{3}}$  (b)  $2\sqrt{3}$  (c)  $\sqrt{3}$  (d) 3

4 If  $\cos X = \frac{\sqrt{3}}{2}$  where  $X$  is a measure of an acute angle , then  $\sin 2X = \dots\dots\dots$

(El-Gharbia 18 - Red Sea 19)

- (a) 1 (b)  $\frac{\sqrt{3}}{2}$  (c)  $\frac{1}{2}$  (d)  $\frac{1}{\sqrt{2}}$

5 If  $2 \sin X = \tan 60^\circ$  where  $X$  is an acute angle , then  $m(\angle X) = \dots\dots\dots$

(Souhag 11)

- (a)  $30^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $40^\circ$

6 If  $\tan 3X = \sqrt{3}$  where  $3X$  is an acute angle , then  $m(\angle X) = \dots\dots\dots$

(Ismailia 15 - North Sinai 20)

- (a)  $20^\circ$  (b)  $30^\circ$  (c)  $45^\circ$  (d)  $60^\circ$

7 If  $\sin 2X = \frac{\sqrt{3}}{2}$  , then  $X = \dots\dots\dots$  (where  $2X$  is an acute angle)

(Giza 11)

- (a)  $20^\circ$  (b)  $30^\circ$  (c)  $45^\circ$  (d)  $60^\circ$

- 8 If  $\cos \frac{X}{2} = \frac{1}{2}$  where  $\frac{X}{2}$  is an acute angle, then  $m(\angle X) = \dots\dots\dots$   
 (a)  $30^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $120^\circ$
- 9 If  $\cos (X + 10^\circ) = \frac{1}{2}$  where  $(X + 10^\circ)$  is an acute angle, then  $X = \dots\dots\dots$  (El-Fayoum 11)  
 (a)  $30^\circ$  (b)  $40^\circ$  (c)  $50^\circ$  (d)  $70^\circ$
- 10 If  $\tan (2X - 5^\circ) = 1$  where  $X$  is an acute angle, then  $X = \dots\dots\dots^\circ$  (El-Gharbia 16 – Luxor 20)  
 (a) 45 (b) 35 (c) 25 (d) 15
- 11 If  $\sin (X + 5^\circ) = \frac{1}{2}$  where  $(X + 5^\circ)$  is the measure of an acute angle, then  $\tan (X + 20^\circ) = \dots\dots\dots$  (El-Dakahlia 11)  
 (a)  $\frac{\sqrt{2}}{2}$  (b)  $\frac{1}{2}$  (c)  $\frac{\sqrt{3}}{2}$  (d) 1
- 12 If  $X$  and  $y$  are complementary angles where  $X : y = 1 : 2$ , then  $\sin X + \cos y = \dots\dots\dots$   
 (a)  $\frac{1}{2}$  (b)  $\frac{1}{4}$  (c)  $\frac{\sqrt{3}}{2}$  (d) 1  
 (El-Beheira 15)
- 13 In the triangle ABC if :  $m(\angle A) : m(\angle B) : m(\angle C) = 3 : 4 : 5$ , then  $\cos B = \dots\dots\dots$   
 (a) 0 (b)  $\frac{1}{2}$  (c) 1 (d)  $\frac{\sqrt{3}}{2}$   
 (El-Gharbia 16)
- 14 The tangent of an acute angle of the right isosceles triangle is equal to  $\dots\dots\dots$   
 (a)  $\sqrt{3}$  (b)  $\frac{1}{\sqrt{3}}$  (c) 1 (d)  $\frac{\sqrt{2}}{2}$   
 (El-Dakahlia 16)
- 15  $\triangle ABC$  is right-angled at A, if  $\tan B = 1$ , then  $\tan C - \sin C \cos C = \dots\dots\dots$   
 (a) zero (b) 1 (c) 2 (d)  $\frac{1}{2}$   
 (Red Sea 16)
- 16 If the straight line :  $y = X \sin 30^\circ + c$  passes through the point (4, 6), then  $c = \dots\dots\dots$   
 (a) 4 (b) 6 (c) 8 (d) 2  
 (El-Monofia 16)

**4 Find the value of  $X$  :**

1  $X \sin^2 45^\circ = \tan^2 60^\circ$

(Souhag 17) « 6 »

2  $X \sin 30^\circ \cos^2 45^\circ = \sin^2 60^\circ$

(South Sinai 16 - Alex. 19 - Assiut 20) « 3 »

3  $X \sin 45^\circ \cos 45^\circ \tan 60^\circ = \tan^2 45^\circ - \cos^2 60^\circ$

«  $\frac{\sqrt{3}}{2}$  »

4  $4X = \cos^2 30^\circ \tan^2 30^\circ \tan^2 45^\circ$

(Alex. 17 - El-Fayoum 19 - Suez 20) «  $\frac{1}{16}$  »**5 Find the value of  $X$  in each of the following :**

1  $\tan X = 4 \sin 30^\circ \cos 60^\circ$  where  $X$  is an acute angle.

(Damietta 18 - El-Gharbia 19 - Giza 20) «  $45^\circ$  »

2  $\sin X = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$  where  $0^\circ < X < 90^\circ$

(Cairo 17) «  $30^\circ$  »

3  $2 \sin X = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$  where  $X$  is an acute angle. (Giza 18) «  $30^\circ$  »

4  $6 \sin X \cos 45^\circ \sin 45^\circ = 1 - \cos^2 60^\circ$  where  $0^\circ < X < 90^\circ$

(Aswan 13) «  $14^\circ 28' 39''$  »

5  $\cos X = \frac{\sin 60^\circ \sin 30^\circ}{\tan 45^\circ \sin^2 45^\circ}$  where  $X$  is an acute angle.

(El-Dakahlia 18) «  $30^\circ$  »

6  $\cos (3X + 6^\circ) = \sin 30^\circ$  where  $(3X + 6^\circ)$  is an acute angle.

«  $18^\circ$  »

7  $\sqrt{3} \sin X \tan 30^\circ = \tan 45^\circ \cos 2X$  where  $X$  is an acute angle.

(El-Monofia 20) «  $30^\circ$  »**6 Find  $E$  in each of the following where  $E$  is the measure of an acute angle :**

1  $\sin^2 45^\circ = \cos E \tan 30^\circ$

(Damietta 16 - El-Monofia 17 - Beni Suef 19) «  $30^\circ$  »

2  $\sin E \sin^2 60^\circ = 3 \sin^2 45^\circ \cos^2 45^\circ \cos 60^\circ$

(Beni Suef 18) «  $30^\circ$  »

3  $3 \tan E - 4 \sin^2 30^\circ = 8 \cos^2 60^\circ$

«  $45^\circ$  »

7 If  $\tan X = \frac{1}{\sqrt{3}}$ ,  $X$  is an acute angle, find :  $\sin X \tan \left(\frac{3X}{2}\right) + \cos 2X$

(Damietta 13) « 1 »

8 If  $\sin X = \tan 30^\circ \sin 60^\circ$  where  $X$  is a measure of an acute angle, then find without using the calculator the value of :  $4 \cos X \sin X$

(El-kalyoubia 20) «  $\sqrt{3}$  »

9 Complete the following tables where the using angles are acute angles :

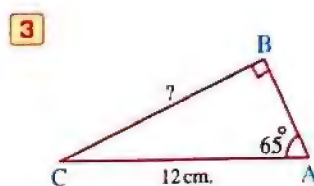
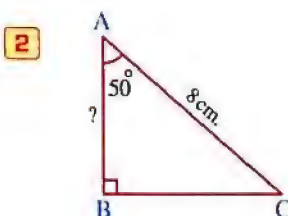
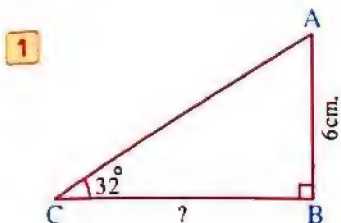
Ratio \ Measure of angle	30°	.....	.....
sin	.....	.....	.....
cos	.....	$\frac{1}{2}$	.....
tan	.....	.....	1

Without using the calculator

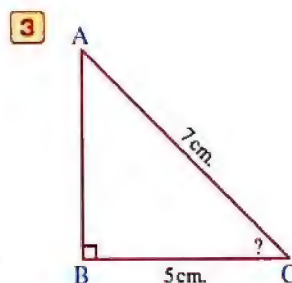
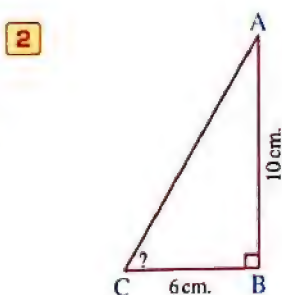
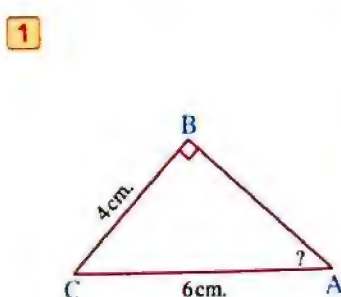
Ratio \ Measure of angle	34° 12'	.....	.....	.....
sin	.....	.....	.....	0.6
cos	.....	0.6217	.....	.....
tan	.....	.....	2.2203	.....

By using the calculator

10 Find the length of the side marked by the sign (?) in each of the following figures to the nearest 2 decimal digits :



11 Find in each of the following figures the measure of the angle marked by the sign (?) in degrees , minutes and seconds :



12 In the opposite figure :

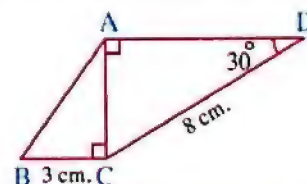
$m(\angle D) = 30^\circ$

$m(\angle CAD) = m(\angle ACB) = 90^\circ$

$BC = 3 \text{ cm.}, CD = 8 \text{ cm.}$

Find : 1  $\tan B$

2  $m(\angle BAD)$



(El-Sharkia 18)

$\frac{4}{3}, 126^\circ 52' 12''$

13 ABC is an isosceles triangle in which  $AB = AC = 7 \text{ cm.}$  and  $BC = 10 \text{ cm.}$

Find : 1  $m(\angle B)$

2 The area of  $\triangle ABC$

$\ll 44^\circ 24' 55'', 10\sqrt{6} \text{ cm}^2 \gg$

- 14 ABC is an isosceles triangle in which  $AB = AC = 12.6$  cm, and  $m(\angle C) = 84^\circ 24'$

Find the length of  $\overline{BC}$  to the nearest one decimal number.

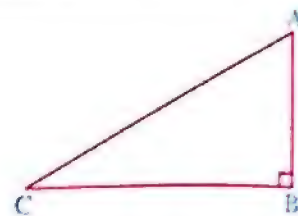
« 2.5 cm. »

- 15 In the opposite figure :

ABC is a right-angled triangle at B ,

$m(\angle A) = 2 m(\angle C)$

Find : The value of  $\cos^2 A + \tan^2 C$



(El-Sharkia 13) «  $\frac{7}{12}$  »

- 16 In the opposite figure :

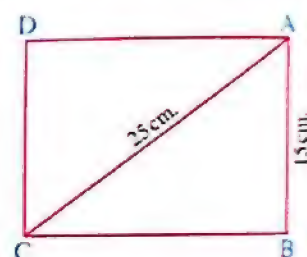
ABCD is a rectangle in which :

$AB = 15$  cm, and  $AC = 25$  cm.

Find :

1  $m(\angle ACB)$

2 The area of the rectangle ABCD (Alex. 16 – Qena 17 – El-Fayoum 20) «  $36^\circ 52' 12''$  ,  $300 \text{ cm}^2$  »



- 17 ABCD is a rectangle whose diagonal length  $AC = 24$  cm, ,  $m(\angle ACB) = 25^\circ$

Find : The length of  $\overline{BC}$

« 21.8 cm. »

- 18 In the opposite figure :

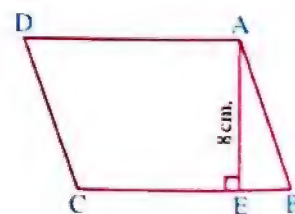
ABCD is a parallelogram of surface area =  $96 \text{ cm}^2$

$BE : EC = 1 : 3$  ,  $\overline{AE} \perp \overline{BC}$  and  $AE = 8$  cm.

Find : 1 The length of  $\overline{AD}$

2  $m(\angle B)$

3 The length of  $\overline{AB}$  to the nearest one decimal (use more than way)



« 12 cm. ,  $69^\circ 26' 38''$  , 8.5 cm. »

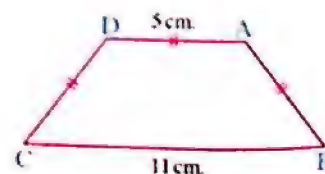
- 19 In the opposite figure :

ABCD is an isosceles trapezium in which :

$AB = AD = DC = 5$  cm.,  $BC = 11$  cm. Find :

1  $m(\angle B)$  ,  $m(\angle A)$

2 The area of the trapezium ABCD



(Matrouh 13) «  $53^\circ 7' 48''$  ,  $126^\circ 52' 12''$  ,  $32 \text{ cm}^2$  »

- 20 ABCD is a trapezium in which :  $\overline{AD} \parallel \overline{BC}$  and  $m(\angle ABC) = 90^\circ$

If  $AB = 12$  cm,  $AD = 16$  cm, and  $BC = 25$  cm. Find :

1 The length of  $\overline{DC}$

2  $m(\angle C)$

3  $\sin(\angle DCB) - \tan(\angle ACB)$

« 15 cm,  $53^\circ 7' 48''$ ,  $\frac{8}{25}$  »

### Life Applications

- 21 A ladder  $\overline{AB}$  is of length 6 metres, its upper edge A lies on a vertical wall and its other edge B on a horizontal floor. If C is the projection of point A on the surface of the floor and its angle of slope on the surface of the floor was of measure  $60^\circ$ , then find the length of  $\overline{AC}$

(Kafr El-Sheikh 17) «  $3\sqrt{3}$  m. »

- 22 A person walks up an inclined plane which makes with the horizontal plane an angle of measure  $22^\circ$ . If this person walks 500 m. up the plane, calculate the height of this plane above the ground surface to the nearest metre.

« 187 m. »

- 23 The wind broke the upper point of a tree to make  $60^\circ$  angle with the ground level, if the top of the tree meets the ground 4 metres away from the bottom of the tree, Find the height of the tree to the nearest metre.

(El-Fayoum 14) « 15 m. »



### For excellent pupils

- 24 Find the value of A where A is the measure of an acute angle if :

$$\cos A \times \tan A = \frac{1}{2}$$

«  $30^\circ$  »

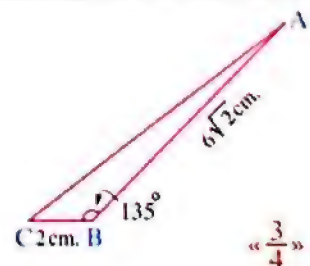
- 25 In the opposite figure :

$$\text{If } m(\angle B) = 135^\circ$$

$$\text{, } AB = 6\sqrt{2} \text{ cm.}$$

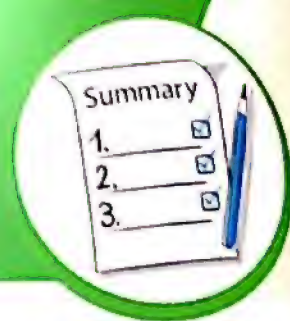
$$\text{, } BC = 2 \text{ cm.}$$

Find :  $\tan C$



«  $\frac{3}{4}$  »

# Summary of Unit 4



## The main trigonometrical ratios of the acute angle :

★ If  $\triangle ABC$  is a right-angled triangle at B , then :

$$\bullet \sin A = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\bullet \cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\bullet \tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{BC}{AB}$$

$$\bullet \sin C = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$\bullet \cos C = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$\bullet \tan C = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{AB}{BC}$$



★  $\tan A = \frac{\sin A}{\cos A}$

★ If  $\angle A$  ,  $\angle B$  are complementary angles , then :  $\sin A = \cos B$  and  $\cos A = \sin B$  and vice versa

*i.e.* If  $\angle A$  and  $\angle B$  are acute angles and  $\sin A = \cos B$  or  $\cos A = \sin B$

, then  $m(\angle A) + m(\angle B) = 90^\circ$

## The trigonometrical ratios of some angles :

$$\bullet \sin 30^\circ = \frac{1}{2}$$

$$\bullet \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\bullet \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\bullet \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\bullet \cos 60^\circ = \frac{1}{2}$$

$$\bullet \tan 60^\circ = \sqrt{3}$$

$$\bullet \sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\bullet \cos 45^\circ = \frac{1}{\sqrt{2}}$$

$$\bullet \tan 45^\circ = 1$$



## Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1 For any acute angle  $A$  ,  $\tan A = \dots\dots\dots$

- (a)  $\frac{\cos A}{\sin A}$  (b)  $\sin A \cos A$  (c)  $\frac{\sin A}{\cos A}$  (d)  $\sin A + \cos A$

2 If  $\cos X = \frac{\sqrt{2}}{2}$  where  $X$  is an acute angle , then  $\sin 2X = \dots\dots\dots$

- (a)  $\frac{1}{\sqrt{2}}$  (b)  $-\frac{\sqrt{2}}{2}$  (c) 1 (d)  $\frac{2}{\sqrt{2}}$

3 ABC is a right-angled triangle at B where  $3AC = 5BC$  , then  $\tan A = \dots\dots\dots$

- (a)  $\frac{3}{5}$  (b)  $\frac{5}{3}$  (c)  $\frac{3}{4}$  (d)  $\frac{4}{3}$

4 If  $\cos (X + 15^\circ) = \frac{1}{2}$  , then  $\sin (75^\circ - X) = \dots\dots\dots$

- (a)  $\frac{1}{2}$  (b)  $\frac{\sqrt{3}}{2}$  (c)  $\frac{1}{\sqrt{2}}$  (d) 1

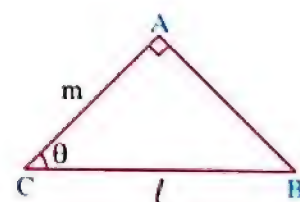
5 If XYZ is an isosceles triangle and right at Z , then  $\tan X = \dots\dots\dots$

- (a)  $\frac{1}{\sqrt{3}}$  (b)  $\sqrt{3}$  (c) 1 (d)  $\frac{1}{3}$

6 In the opposite figure :

If the length of  $\overline{BC}$  is  $l$  and the length of  $\overline{AC}$  is  $m$  , then which of the following equations can be used to find  $l$  ?

- (a)  $l = \frac{m}{\cos \theta}$  (b)  $l = \frac{m}{\sin \theta}$   
(c)  $l = m \cos \theta$  (d)  $l = m \sin \theta$

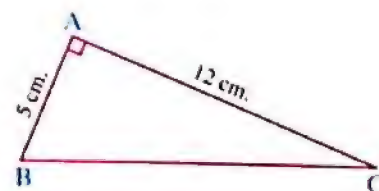


2 [a] In the opposite figure :

ABC is a triangle in which :  $m(\angle A) = 90^\circ$

,  $AC = 12$  cm. and  $AB = 5$  cm.

Prove that :  $\cos C \cos B - \sin C \sin B = \text{zero}$



[b] Find the value of  $X$  in each of the following :

1  $X \sin 30^\circ = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$

2  $2 \sin X = \tan^2 60^\circ - 2 \tan 45^\circ$  where  $X$  is an acute angle.

3 [a] ABCD is an isosceles trapezoid in which :

$\overline{AD} \parallel \overline{BC}$ ,  $AD = 4$  cm.,  $AB = 5$  cm. and  $BC = 12$  cm.

Find the value of :  $\frac{\tan B \cos C}{\cos^2 C + \sin^2 C}$

[b] Find the value of :  $\frac{\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ}{\sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ}$

4 [a] In the opposite figure :

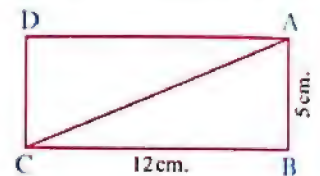
ABCD is a rectangle in which :

$AB = 5$  cm. and  $BC = 12$  cm.

Find :

1 The length of  $\overline{AC}$

2 The value of :  $5 \tan (\angle ACD) - 13 \sin (\angle DAC)$



[b] The wind broke the upper point of a tree to make  $30^\circ$  angle with the ground level , if the top of the tree meets the ground 3 metres away from the bottom of the tree , Find the height of the tree to the nearest metre.

5 [a] If ABC is a right-angled triangle at B ,  $\sin A + \cos C = 1$  Find :  $m(\angle A)$

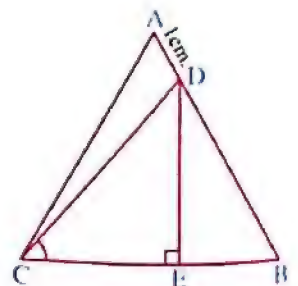
[b] In the opposite figure :

ABC is an equilateral triangle of side length 5 cm.

,  $D \in \overline{AB}$  where  $AD = 1$  cm.

,  $\overline{DE} \perp \overline{BC}$

Find :  $\tan (\angle DCE)$



## ? Model 2

Answer the following questions :

1 Choose the correct answer from those given :

- 1 If  $\tan (X + 15) = 1$  where  $X$  is an acute angle , then  $X = \dots\dots\dots$   
 (a)  $60^\circ$  (b)  $45^\circ$  (c)  $30^\circ$  (d)  $15^\circ$
- 2 If  $\sin 30^\circ = \cos E$  where  $E$  is an acute angle , then  $m(\angle E) = \dots\dots\dots$   
 (a)  $15^\circ$  (b)  $30^\circ$  (c)  $60^\circ$  (d)  $90^\circ$
- 3 If  $X$  is an acute angle ,  $2 \sin X - 1 = 0$  , then  $m(\angle X) = \dots\dots\dots$   
 (a)  $60^\circ$  (b)  $90^\circ$  (c)  $45^\circ$  (d)  $30^\circ$
- 4 If  $A$  and  $B$  are two complementary angles where  $A : B = 1 : 2$  , then  $\sin A + \cos B = \dots\dots\dots$   
 (a)  $\frac{1}{2}$  (b)  $\frac{1}{4}$  (c)  $\frac{\sqrt{3}}{2}$  (d) 1
- 5 If  $m(\angle A) = 70^\circ$  ,  $\sin B = \cos B$  in  $\triangle ABC$  , then  $m(\angle C) = \dots\dots\dots$   
 (a)  $50^\circ$  (b)  $45^\circ$  (c)  $70^\circ$  (d)  $65^\circ$

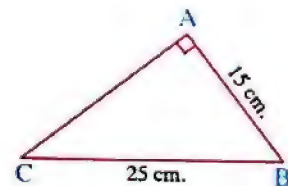
6 In the opposite figure :

$ABC$  is a triangle in which :

$m(\angle A) = 90^\circ$  ,  $AB = 15$  cm.

,  $BC = 25$  cm. , then  $\tan B = \dots\dots\dots$

- (a)  $\frac{3}{5}$  (b)  $\frac{4}{5}$  (c)  $\frac{4}{3}$  (d)  $\frac{3}{4}$



2 [a] Find the value of  $X$  in degrees if :  $\tan 2X = 4 \sin 30^\circ \cos 30^\circ$  where  $0^\circ < X^\circ < 90^\circ$

[b] If the ratio between the measures of two complementary angles is  $3 : 5$  ,  
find the measure of each one by degree measure.

3 [a] In the opposite figure :

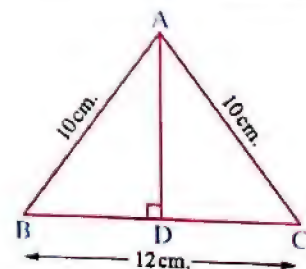
$ABC$  is a triangle where  $AB = AC = 10$  cm.

,  $BC = 12$  cm. ,  $\overline{AD} \perp \overline{BC}$

Find : 1  $\cos B$

2  $m(\angle B)$

3  $\sin (90^\circ - B)$



[b] Without using calculator , prove that :  $\tan^2 60^\circ - \tan^2 45^\circ = \sin^2 60^\circ + \cos^2 60^\circ + 2 \sin 30^\circ$

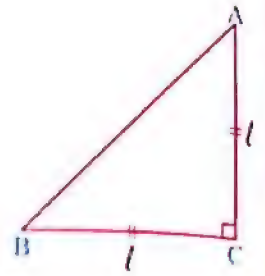
**4 [a] In the opposite figure :**

ABC is an isosceles triangle and right at C

, the length of each leg is  $l$  length unit :

**Find :** **1** The ratio between the lengths of sides of the triangle AC : BC : AB

**2**  $\tan B$  ,  $\sin A$

**[b] In the opposite figure :**

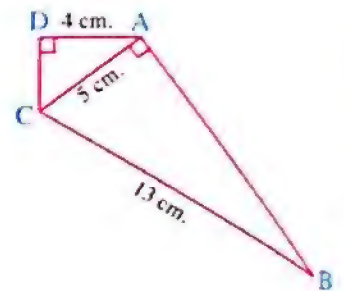
$m(\angle ADC) = m(\angle BAC) = 90^\circ$

,  $AD = 4$  cm. ,  $AC = 5$  cm. and  $BC = 13$  cm.

**Calculate the value of each of :**

**1**  $\tan(\angle ACB) + \tan(\angle ACD)$

**2**  $\sin(\angle B) \cos(\angle CAD) + \cos(\angle B) \sin(\angle CAD)$

**5 [a] ABC is a right-angled triangle at B**

**1 Prove that :**  $\sin^2 A + \cos^2 A = 1$

**2** If  $AB = 5$  cm. ,  $AC = 13$  cm. , **find :**  $m(\angle C)$

**[b] Find without using the calculator :**  $\sin 45^\circ \cos 45^\circ + 3 \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$

# A Research Project

## On Unit Four



### Project aims :

- Finding the measure of an angle knowing one of its trigonometric ratios.
- Applying Pythagoras' theorem.
- Associating mathematics with sports.
- Associating mathematics with history.
- Associating mathematics with science.

### Do a research project on the following topic :

*"Football game is considered one of the most popular team sports around the world. It is the most practiced in most countries".*

### Discuss the following points using available resources :

- Write down about the start of football and how it has developed over the years.
- Mention the dimensions of the football pitch, the dimensions of the goal and the dimensions of the penalty area.
- How far is the penalty point to the goal line ?
- If a player kicks a ball at the penalty point towards the goal and the ball then hits the horizontal crossbar exactly at the middle, calculate the following providing the ball moves in a straight line :
  1. The distance that the ball covered to hit the horizontal crossbar.
  2. The measure of the angle that the path of the ball made with the ground.
  3. To score a goal, in this case, write the interval which the measure of the angle that the ball path made with the ground belongs to.
  4. The average velocity that the ball moved in if it hits the beam in 0.4 seconds after it was kicked.

# UNIT 5

## Analytical geometry



### ► Exercises of the unit :

3. Distance between two points.
  4. The two coordinates of the midpoint of a line segment.
  5. The slope of the straight line.
  6. The equation of the straight line given its slope and the intercepted part of y-axis.
- 🟢 Unit Exams.

🔍 A research project on unit five



Scan the  
**QR code**  
to solve an  
interactive  
test on each  
lesson



## EXERCISE

# 3

## Distance between two points



From the school book

Interactive test.

**1** Find the length of  $\overline{AB}$  in each of the following cases :

**1** A (1 , 2) , B (4 , 6)

**3** A (-2 , 7) , B (3 , -5)

**5** A (15 , 0) , B (6 , 0)

**2** A (2 , -1) , B (5 , -5)

**4** A (-2 , 5) , B (3 , 0)

**6** A (6 , 0) , B (0 , -8)

**2** Choose the correct answer from those given :

**1** The distance between the two points (3 , a) and (-1 , a) is ..... length unit. (Ismailia 17)

- (a) 16                      (b) 9                      (c) 5                      (d) 4

**2** The distance between the point ( $\sqrt{3}$  , 1) and the point of origin is ..... (Souhag 18)

- (a) 4                      (b) 3                      (c) 2                      (d) 1

**3** If the distance between the two points (a , 0) , (0 , 1) is unit length

, then a = .....

(El-Gharbia 20)


- (a) 1                      (b) -1                      (c) 0                      (d) 2

**4** The radius length of the circle whose centre is (7 , 4) and passes through (3 , 1) equals ..... length units.

- (a) 5                      (b) -5                      (c) 2.5                      (d) 25

**5** If ABCD is a square and A (3 , 5) and B (4 , 2) , then the area of the square ABCD equals ..... area unit.

- (a)  $\sqrt{10}$                       (b) 10                      (c)  $4\sqrt{10}$                       (d) 40

- 6 If ABCD is a rhombus and  $A(-1, 7)$ ,  $B(-3, 1)$ , then the perimeter of the rhombus ABCD = ..... length unit.  
 (a) 40 (b)  $4\sqrt{52}$  (c)  $8\sqrt{10}$  (d)  $2\sqrt{10}$
- 7 In the Cartesian coordinates plane, a point that is at a distance 2 length unit from the origin may be ..... (Cairo 16)  
 (a)  $(1, 2)$  (b)  $(2, 1)$  (c)  $(0, 2)$  (d)  $(-3, 5)$
- 8 The distance between the point  $(-5, -2)$  and y-axis is ..... length unit. (El-Gharbia 16)  
 (a) -5 (b) -2 (c) 2 (d) 5
- 9 The distance between the point  $(5, \tan^2 60^\circ)$  and the X-axis is ..... length unit. (Suez 17)  
 (a) 5 (b)  $\sqrt{5}$  (c) 3 (d)  $\sqrt{3}$
- 10 The distance between the point  $(l, -4)$  and y-axis is ..... length unit, where  $l \in \mathbb{R}$ . (Damietta 18)  
 (a) 4 (b)  $l$  (c) -4 (d)  $|l|$
- 11 The perpendicular distance between the two straight lines,  $y - 3 = 0$ ,  $y + 2 = 0$  equals ..... length units. (Alex. 17 - El-Fayoum 17)  
 (a) 5 (b) 1 (c) 2 (d) 3
- 12  A circle its centre is the origin and its radius length is 2 length unit, which of the following points belongs to the circle? (El-Gharbia 14 - Beni Suef 16 - El-Beheira 17)  
 (a)  $(1, 2)$  (b)  $(-2, 1)$  (c)  $(\sqrt{3}, 1)$  (d)  $(\sqrt{2}, 1)$
- 3 If  $A(3, 1)$ ,  $B(1, 2)$  and  $C(5, 4)$ , prove that :  $BC = 2 AB$  (El-Monofia 13 - Luxor 16)
- 4 Prove that : The points  $A(4, 3)$ ,  $B(1, 1)$  and  $C(-5, -3)$  are collinear. (Assiut 14 - Kafr El-Sheikh 15 - El-Fayoum 17)
- 5 If  $A(-2, 2)$ ,  $B(1, -1)$ , then prove that the point  $C(3, 4)$  lies on axis of symmetry of  $\overline{AB}$
- 6 Show which of the following sets of points are collinear :  
 1  $A(1, 4)$ ,  $B(3, -2)$  and  $C(-3, 16)$   
 2  $A(7, 0)$ ,  $B(-3, 6)$  and  $C(22, 9)$  (Cairo 18)  
 3  $A(-1, 4)$ ,  $B(3, -14)$  and  $C(-5, -6)$

- 7 Show the type of  $\triangle ABC$  such that  $A(-2, 4)$  ,  $B(3, -1)$  ,  $C(4, 5)$  according to its side lengths.

(New Valley 16 - Giza 17 - Damietta 19 - El-Behara 20)

- 8 Show the type of each of the following triangles according to its angles if its vertices are :

1  $A(2, 1)$  ,  $B(4, -2)$  and  $C(7, 5)$

2  $A(3, 5)$  ,  $B(-1, 1)$  and  $C(5, -5)$

3  $A(4, 4)$  ,  $B(3, -1)$  and  $C(-2, 4)$

4  $A(0, 0)$  ,  $B(6, 0)$  and  $C(0, 8)$

5  $A(1, -1)$  ,  $B(2, 1)$  and  $C(-3, -2)$

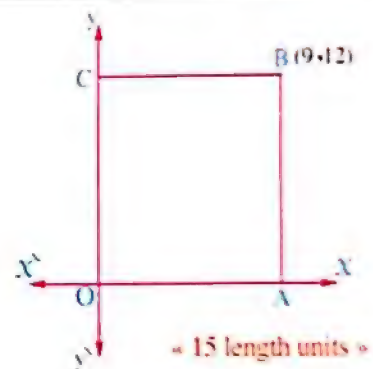
- 9 Prove that the triangle with vertices of points :  $A(5, -5)$  ,  $B(-1, 7)$  and  $C(15, 15)$  is a right-angled triangle at  $B$  , then calculate its area.

(Beni Suef 13 - El-Monofia 14 - Qena 16) « 120 square units »

- 10 If the points  $A(5, 0)$  ,  $B(7, 2\sqrt{3})$  and  $C(3, 2\sqrt{3})$  are three points in a Cartesian coordinates plane. **Prove that :**  $\triangle ABC$  is equilateral and find its area. «  $4\sqrt{3}$  square units »

- 11 In the opposite figure :

If  $ABCO$  is a rectangle  
 , then find the length of :  $\overline{AC}$



- 12 In each of the following , prove that the points  $A$  ,  $B$  ,  $C$  and  $D$  are vertices of a parallelogram where :

1  $A(-1, 1)$  ,  $B(0, 5)$  ,  $C(5, 6)$  and  $D(4, 2)$

(Suez 11)

2  $A(-2, 4)$  ,  $B(5, -3)$  ,  $C(7, 1)$  and  $D(0, 8)$

(Souhag 08)

- 13 **Prove that :** The points  $A(0, 1)$  ,  $B(4, 5)$  ,  $C(1, 8)$  and  $D(-3, 4)$  are vertices of a rectangle and find its diagonal length.

(Souhag 09) «  $5\sqrt{2}$  length units »




- 14 **Prove that :** The points  $A(3, 3)$  ,  $B(0, 3)$  ,  $C(0, 0)$  and  $D(3, 0)$  in the Cartesian coordinates plane are vertices of a square and calculate the length of its diagonal and its area.

(Luxor 09) «  $3\sqrt{2}$  length units , 9 square units »

- 15  $ABCD$  is a quadrilateral where  $A(5, 3)$  ,  $B(6, -2)$  ,  $C(1, -1)$  and  $D(0, 4)$

**Prove that :**  $ABCD$  is a rhombus , then find its area.

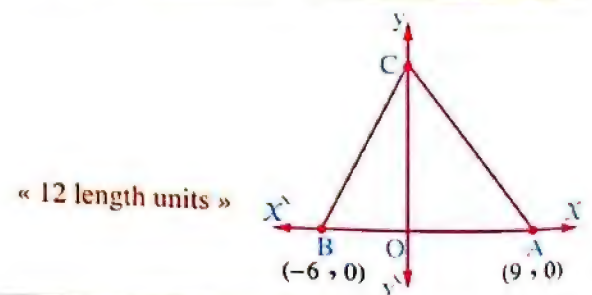
(Qena 19) « 24 square units »

- 16**  **Prove that :** The points A (− 2 , 5) , B (3 , 3) and C (− 4 , 2) are non-collinear and if D (− 9 , 4)  
**Prove that :** The figure ABCD is a parallelogram. (Port Said 17)
- 17** ABCD is a quadrilateral where A (2 , 4) , B (− 3 , 0) , C (− 7 , 5) and D (− 2 , 9)  
**Prove that :** The figure ABCD is a square. (El-Beheira 17 – Cairo 19 – El-Monofia 20)
- 18** **Prove that :** The points A (3 , − 1) , B (− 4 , 6) and C (2 , − 2) lie on the same circle whose centre is M (− 1 , 2) , then find the circumference of the circle where  $\pi = 3.14$   
(Cairo 15 – North Sinai 16 – El-Kalyoubia 18 – Alex. 19 – Aswan 20) « 31.4 length units »
- 19** If the distance between the point (X , 5) and the point (6 , 1) equals  $2\sqrt{5}$  length units.  
**Find :** the value of X (El-Monofia 18 – Matrouh 18 – El-Kalyoubia 19 – Damietta 20) « 4 or 8 »
- 20**  **Find the value of a in each of the following cases :**
- 1** If the distance between the two points (a , 7) , (− 2 , 3) equals 5 length unit.  
(Matrouh 17 – Alex. 18 – El-Menia 19 – El-Fayoum 20) « 1 or − 5 »
  - 2** If the distance between the two points (a , 7) , (3 a − 1 , − 5) equals 13 length unit.  
« − 2 or 3 »
- 21**  If A (X , 3) , B (3 , 2) and C (5 , 1) and  $AB = BC$  , then find the value of X  
(Port Said 14 – El-Beheira 15 – El-Beheira 17 – El-Beheira 19) « 5 or 1 »

**22** **In the opposite figure :**

If  $AB = AC$

**Find :** The length of  $\overline{CO}$



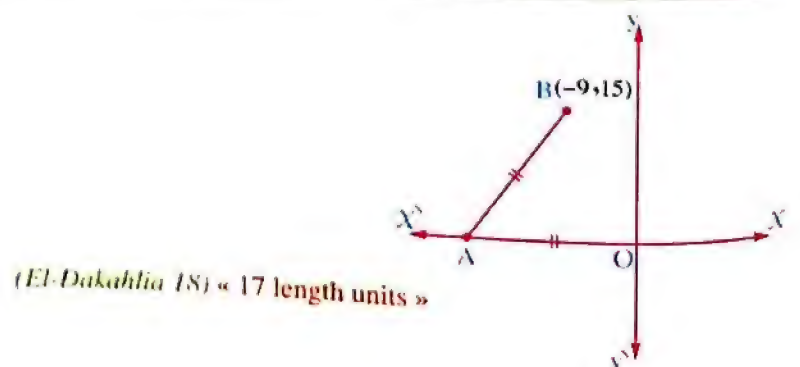
- 23** If the axis of symmetry of  $\overline{CD}$  is passing through the point A (6 , m) where C (3 , 1) , D (− 3 , 7) , then find the value of m  
(El-Dakahlia 16) « 10 »

**24** **In the opposite figure :**

If  $A \in$  the X-axis

and  $AO = AB$

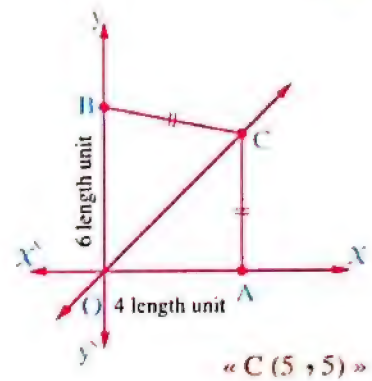
**Find :** The length of  $\overline{AB}$



25 In the opposite figure :

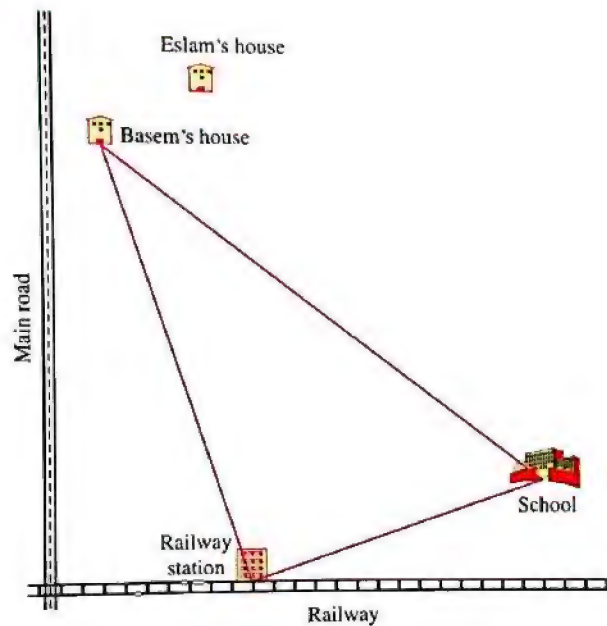
If  $A \in \overline{OX}$ ,  $B \in \overline{OY}$  where  $OA = 4$  length unit  
 ,  $OB = 6$  length unit , the straight line  $\overline{OC}$  represents  
 the function  $f : f(x) = x$  and  $AC = BC$

Find the coordinates of the point C



### Life Application

26



If the distance between Basem's house and the main road is 1 km. and the distance between Basem's house and the railway lines is 9 km.

Eslam's house is 3 km. away from the main road and 10 km. away from the railway lines.

The school is 10 km. away from the main road and 2 km. away from the railway lines.

The railway is 4 km. away from the main road.

- 1 Which is nearer to school, Basem's house or Eslam's house ?
- 2 Is the way (school – railway station) perpendicular to the way (Basem's house – railway station) ? Mention the reason.



### For excellent pupils

27

If the points  $A(4, -2)$ ,  $B(x, 2)$  and  $C(3, 5)$  are three points in the Cartesian coordinates plane. Find the value of  $x$  which makes  $\triangle ABC$  a right-angled triangle at B and find its area.

« 0 or 7, 12 square units or 12.5 square units »

## EXERCISE

# 4

## The two coordinates of the midpoint of a line segment



From the school book

Interactive test

1 Find the coordinates of the midpoint of  $\overline{AB}$  in each of the following cases :

1  $A(3, 5)$  ,  $B(7, 1)$

2  $A(5, -3)$  ,  $B(-1, 3)$

3  $A(-5, 4)$  ,  $B(5, -4)$

4  $A(0, 4)$  ,  $B(8, 0)$

5  $A(2, 4)$  ,  $B(6, 0)$

6  $A(7, -6)$  ,  $B(-1, 0)$

2 If the point  $(x, 0)$  is the midpoint of  $\overline{AB}$  where  $A(1, -5)$  and  $B(2, 5)$  ,  
find the value of  $x$

$x = \frac{3}{2}$

3 If the point  $(5, 3)$  is the midpoint of  $\overline{AB}$  where  $A(15, y)$  and  
 $B(-5, -2)$  , find the value of  $y$

$y = 8$

4 If  $C(6, -4)$  is the midpoint of  $\overline{AB}$  where  $A(5, -3)$  ,  
Find the coordinates of the point  $B$  (Aswan 17 - El-Dakahlia 18 - Beni Suef 19 - Cairo 19)  $\approx (7, -5)$

5 If  $C$  is the midpoint of  $\overline{AB}$  , then find  $x, y$  in each of the following cases :

1  $A(1, 5)$  ,  $B(3, 7)$  ,  $C(x, y)$

$\approx (2, 6)$

2  $A(-3, y)$  ,  $B(9, 11)$  ,  $C(x, -3)$

(Aswan 18)  $\approx (3, -17)$

3  $A(x, -6)$  ,  $B(9, -11)$  ,  $C(-3, y)$

$\approx -15, -8.5$




4  $A(x, 3)$  ,  $B(6, y)$  ,  $C(4, 6)$


(Cairo 15 - Matrouh 17)  $\approx (2, 9)$

## 6 Choose the correct answer from those given :

- 1 If  $(4, -3)$  is the midpoint of  $\overline{XY}$  where  $X(5, -2)$ , then  $Y$  is ..... (El-Menia 16)  
 (a)  $(4.5, 4.5)$  (b)  $(4, 3)$  (c)  $(3, 4)$  (d)  $(3, -4)$
- 2 If  $C(-3, y)$  is the midpoint of  $\overline{AB}$  where  $A(X, -6)$  and  $B(1, -8)$  ..... (Qena 18)  
 , then  $X + y =$  .....  
 (a)  $-11$  (b)  $11$  (c)  $-18$  (d)  $-14$
- 3 If  $\overline{AB}$  is a diameter in a circle where  $A(3, -5)$  and  $B(5, 1)$  ..... (El-Fayoum 18 - Matrouh 19)  
 , then the centre of the circle is .....  
 (a)  $(4, -2)$  (b)  $(4, 2)$  (c)  $(2, -2)$  (d)  $(8, -2)$
- 4 If  $ABCD$  is a square where  $A(3, 4)$  ,  $C(5, 6)$  , then the midpoint of its diagonal is ..... (El-Menia 18)  
 (a)  $(8, 10)$  (b)  $(10, 8)$  (c)  $(4, 5)$  (d)  $(15, 24)$
- 5 If  $M(1, 2)$  is the intersection point of the two diagonals of the parallelogram  $ABCD$  where  $A(2, 5)$  , then  $C$  is .....  
 (a)  $(0, 2)$  (b)  $(0, -1)$  (c)  $(-4, 1)$  (d)  $(-1, 0)$
- 6 If  $(\frac{1}{2}, \frac{5}{2})$  is the midpoint of  $\overline{AB}$  where  $A(1, -1)$  and  $B(X, 6)$  , then  $X =$  .....  
 (a)  $0$  (b)  $1$  (c)  $2$  (d)  $\frac{1}{2}$
- 7 If the  $X$ -axis bisects  $\overline{AB}$  such that  $A(3, 2)$  and  $B(-2, y)$  , then  $y =$  ..... (El-Dakahlia 17)  
 (a)  $3$  (b)  $2$  (c)  $-2$  (d)  $4$

## 7 Complete the following :

- 1  If the point of the origin is the midpoint of a line segment  $\overline{AB}$  where  $A(5, -2)$  , then the point  $B$  is ..... (Assiut 11)
- 2 If  $A(3, y)$  ,  $B(5, -2)$  and  $C$  is the midpoint of  $\overline{AB}$  where  $C(X, 2)$  , then  $X =$  ..... and  $y =$  .....
- 3 If  $B \in \overline{AC}$  where  $AB = BC$  ,  $A(0, 5)$  and  $C(-4, -1)$  , then  $B$  is .....
- 4  If  $A, B, C$  and  $D$  are four collinear points and  $AB = BC = CD$  ,  $A(1, 3)$  and  $C(5, 1)$  , then the point  $B$  is (..... , ..... ) and the point  $D$  is (..... , ..... )
- 5   $\overline{AD}$  is a median in  $\triangle ABC$  ,  $M$  is the midpoint of  $\overline{AD}$  where  $A(0, 8)$  ,  $B(3, 2)$  and  $C(-3, 6)$  , then the point  $D$  is (..... , ..... ) and the point  $M$  is (..... , ..... )

- 8  If  $A(1, -6)$  and  $B(9, 2)$  , find the coordinates of the points which divide  $\overline{AB}$  into four equal parts in length. (Souhag 18) =  $(5, -2)$  ,  $(3, -4)$  ,  $(7, 0)$  »

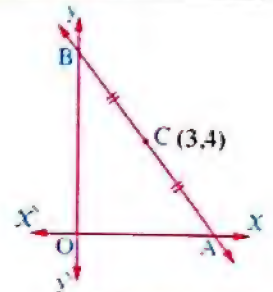
- 9 If the origin point O is the midpoint of  $\overline{AB}$  where A ( $X-2$ ,  $y$ ) and B ( $-2$ ,  $2$ ), find ( $X$ ,  $y$ ) « (4, -2) »
- 10 Find the value of each of  $a$  and  $b$  that satisfies that  $(2a-3, a-b)$  is the midpoint of the line segment whose terminals ( $7$ ,  $-1$ ) and ( $3$ ,  $7$ ) (El-Fayoum 12) « 4, 1 »
- 11  $\overline{AB}$  is a diameter in a circle M, if B ( $8$ ,  $11$ ) and M ( $5$ ,  $7$ )  
Find : 1 The coordinates of A      2 The perimeter of the circle, where ( $\pi = 3.14$ )  
(El-Kalyoubia 16 - North Sinai 17 - Kafr El-Sheikh 18) « A (2, 3), 31.4 length unit »

- 12 ABC is a triangle where A ( $1$ ,  $3$ ), B ( $5$ ,  $1$ ) and C ( $3$ ,  $7$ ), if D is the midpoint of  $\overline{AB}$  and E is the midpoint of  $\overline{AC}$ , by using the coordinates, prove that :  $DE = \frac{1}{2}BC$

- 13 In the opposite figure :

C ( $3$ ,  $4$ ) is the midpoint of  $\overline{AB}$

Find : The perimeter of triangle OAB



(Alex. 17 - El-Kalyoubia 20)

« 24 length unit »

- 14 In the opposite figure :

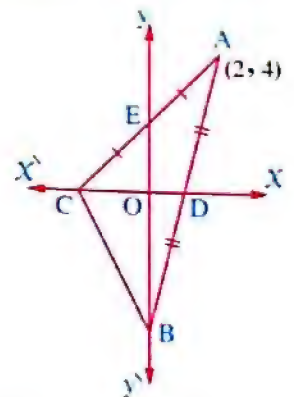
D is the midpoint of  $\overline{AB}$

, E is the midpoint of  $\overline{AC}$

If A ( $2$ ,  $4$ )

, then find :

The length of  $\overline{BC}$  and from it deduce the length of  $\overline{DE}$



«  $2\sqrt{5}$  length unit,  $\sqrt{5}$  length unit »

- 15  $\overline{AD}$  is the median in  $\triangle ABC$ , M is the midpoint of  $\overline{AD}$  where M ( $0$ ,  $6$ ), B ( $3$ ,  $2$ ), C ( $-3$ ,  $6$ ) find the coordinates of the point A (Kafr El-Sheikh 17) « A (0, 8) »
- 16 If A ( $-1$ ,  $-1$ ), B ( $2$ ,  $3$ ), C ( $6$ ,  $0$ ) and D ( $3$ ,  $-4$ ) are four points in the Cartesian coordinates plane, prove that :  $\overline{AC}$  and  $\overline{BD}$  bisect each other. (Suez 19)

- 17 Prove that : The points A ( $3$ ,  $-2$ ), B ( $-5$ ,  $0$ ), C ( $0$ ,  $-7$ ) and D ( $8$ ,  $-9$ ) are the vertices of a parallelogram. (El-Fayoum 19)

- 18 If the points  $A(3, 2)$ ,  $B(4, -3)$ ,  $C(-1, -2)$  and  $D(-2, 3)$  are vertices of the rhombus. Find :
- The coordinates of the point of intersection of the two diagonals.
  - The area of the rhombus  $ABCD$   $(El\text{-}Fayoum\ 17 - Port\text{-}Said\ 18 - Alex\ 20) = (1, 0) ; 24 \text{ square unit} =$
- 19  $ABCD$  is a parallelogram where  $A(3, 2)$ ,  $B(4, -5)$  and  $C(0, -3)$ . Find the coordinates of the intersection point of its diagonals, then find the coordinates of the point  $D$   $(Assiut\ 17 - El\text{-}Behera\ 18 - Alex\ 19 - El\text{-}Dakahlia\ 20) = (1\frac{1}{2}, -\frac{1}{2}) ; (-1, 4) =$
- 20 **Prove that :** The points  $A(6, 0)$ ,  $B(2, -4)$  and  $C(-4, 2)$  are the vertices of a right-angled triangle at  $B$ , then find the coordinates of  $D$  that make the figure  $ABCD$  a rectangle.  $(Assiut\ 11 - Kafr\ El\text{-}Sheikh\ 14 - El\text{-}Behera\ 19) = D(0, 6) =$
- 21 **Prove that :** The points  $A(5, 3)$ ,  $B(3, -2)$ ,  $C(-2, -4)$  are the vertices of an obtuse-angled triangle at  $B$ , then find the coordinates of the point  $D$  that makes the figure  $ABCD$  a rhombus and find its area.  $= (0, 1) ; 21 \text{ square units} =$
- 22 **Prove that :** The points  $A(-3, 0)$ ,  $B(3, 4)$  and  $C(1, -6)$  are the vertices of an isosceles triangle of vertex  $A$ , then find the length of the drawn line segment from  $A$  perpendicular on  $\overline{BC}$   $(El\text{-}Kalyoubia\ 12 - El\text{-}Monofia\ 16 - Qena\ 19) = \sqrt{26} \text{ length unit} =$
- 23  $\triangle ABC$  whose vertices are  $A(1, 1)$ ,  $B(3, 1)$  and  $C(1, 3)$ . **Prove that :**  $\triangle ABC$  is an isosceles triangle then find its area.  $(El\text{-}Sharkia\ 18) = 2 \text{ square unit} =$
- 24  $ABCD$  is a parallelogram where  $A(3, 4)$ ,  $B(2, -1)$  and  $C(-4, -3)$ , find the coordinates of  $D$ , take  $E \in \overline{AD}$  where  $AE = 2 AD$ . What are the coordinates of the point  $E$ ?  $= (-3, 2) ; (-9, 0) =$
- 25  $ABCD$  is a quadrilateral,  $X(2, 3)$ ,  $Y(m, 3)$ ,  $Z(1, -1)$  and  $L(-4, n)$  are midpoints of  $\overline{AB}$ ,  $\overline{AD}$ ,  $\overline{BC}$  and  $\overline{DC}$  respectively. **Find :** The value of :  $m + n$   $= -4 =$



## For excellent pupils

- 26  $ABCD$  is a trapezium in which  $BC = 2 AD$  and  $A(6, 4)$ ,  $B(4, -2)$ ,  $C(-2, -4)$ . Find the coordinates of  $D$  where  $\overline{BC} \parallel \overline{AD}$ . **(Hint :** Complete the parallelogram  $ABCE$  and use it to find  $D$ )  $= (3, 3) =$

## EXERCISE

# 5

## The slope of the straight line



From the school book

Interactive test

### 1 Complete the following :

- 1 The slope of the straight line parallel to X-axis is ..... *(El-Kalyoubia 11)*
- 2 The slope of the straight line parallel to y-axis is .....
- 3 If  $\overline{AB} \parallel \overline{CD}$  and the slope of  $\overline{AB} = \frac{2}{3}$ , then the slope of  $\overline{CD}$  equals ..... *(New Valley 12)*
- 4 If  $\overline{AB} \perp \overline{CD}$  and the slope of  $\overline{AB} = \frac{1}{2}$ , then the slope of  $\overline{CD}$  equals ..... *(El-Gharbia 11)*
- 5 ABC is a right-angled triangle at B where A = (1 , 5) and B = (0 , 1), then the slope of  $\overline{BC}$  equals ..... *(Suez 11)*
- 6 ABCD is a parallelogram where A (-1 , 4) and B (0 , 1), then the slope of  $\overline{DC}$  = ..... *(Ismailia 11)*
- 7 If ABCD is a square whose diagonals  $\overline{AC}$  and  $\overline{BD}$  where A (3 , 5) and C (5 , -1), then the slope of  $\overline{BD}$  = .....

### 2 Choose the correct answer from those given :

#### 1 In the opposite figure :

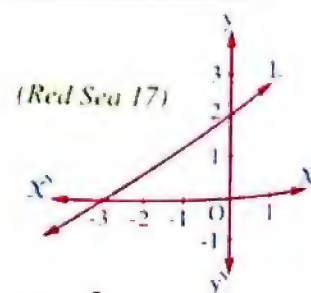
The slope of the straight line L equals .....

(a)  $\frac{2}{3}$

(b)  $\frac{-2}{3}$

(c)  $\frac{3}{2}$

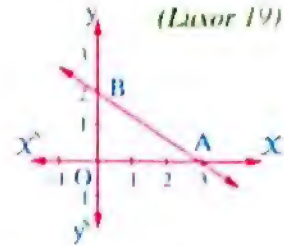
(d)  $\frac{-3}{2}$



## 2 In the opposite figure :

The slope of  $\overline{AB}$  = .....

- (a)  $\frac{2}{3}$  (b)  $-\frac{2}{3}$   
 (c)  $\frac{3}{2}$  (d)  $-\frac{3}{2}$



(Luxor 19)

3 The slope of the straight line that makes with the positive direction of the  $X$ -axis a positive angle of measure  $\theta$  equals .....

(Giza 17)

- (a)  $\sin \theta$  (b)  $\cos \theta$  (c)  $\frac{\sin \theta}{\cos \theta}$  (d)  $\sin \theta + \cos \theta$

4 If the slope of a straight line is more than zero, then the type of the positive angle which it makes with the positive direction of  $X$ -axis is .....

(Damietta 11)

- (a) zero. (b) acute. (c) right. (d) obtuse.

5 If  $m_1$  and  $m_2$  are the slopes of two perpendicular straight lines, then .....

(Qena 12)

- (a)  $m_1 = m_2$  (b)  $m_1 = -m_2$  (c)  $m_1 m_2 = -1$  (d)  $m_1 m_2 = 1$

6 If  $m_1$  and  $m_2$  are the slopes of two parallel straight lines, then .....

- (a)  $m_1 - m_2 = 0$  (b)  $m_1 + m_2 = 0$  (c)  $m_1 m_2 = 0$  (d)  $m_1 - m_2 \neq 0$

7 The straight line that passes through the two points  $(0, 0)$  and  $(2, 3)$  is parallel to the straight line whose slope is .....

- (a)  $\frac{3}{2}$  (b)  $\frac{2}{3}$  (c)  $-\frac{3}{2}$  (d)  $-\frac{2}{3}$

8 If the straight line  $L$  is perpendicular to the straight line which passes through the two points  $(-1, 2)$  and  $(0, 5)$ , then the slope of the straight line  $L$  = .....

- (a) 3 (b) -3 (c)  $\frac{1}{3}$  (d)  $-\frac{1}{3}$

9 If  $m_1$  and  $m_2$  are the slopes of two perpendicular straight lines and  $m_1 = 0.75$ , then  $m_2$  = .....

(El-Sharkia 13)

- (a)  $-\frac{3}{4}$  (b)  $\frac{4}{3}$  (c)  $-\frac{4}{3}$  (d)  $\frac{3}{4}$

10 If the two straight lines whose slopes are  $-\frac{2}{3}$  and  $\frac{k}{2}$  are parallel, then  $k$  = .....

(Alex. 17 - Matrouh 19)

- (a)  $-\frac{3}{4}$  (b)  $\frac{1}{3}$  (c) 3 (d)  $-\frac{4}{3}$

11 If the straight line which passes through the two points  $(X, 5)$  and  $(2, 3)$  is parallel to the straight line which passes through the two points  $(3, 4)$  and  $(5, 2)$ , then  $X$  = .....

- (a) 2 (b) -2 (c) zero (d) 1

12 The straight line which passes through the two points  $(-1, -1)$  and  $(4, 4)$  makes with the positive direction of  $X$ -axis a positive angle of measure .....

(El-Monofia 15 - North Sinai 17)

- (a)  $30^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $135^\circ$

- 13 The straight line which passes through two points  $(k, 0)$  and  $(0, 4)$  is perpendicular to a straight line which makes an angle of measure  $45^\circ$  with the positive direction of  $X$ -axis, then  $k = \dots\dots\dots$  (Aswan 13)

(a) 4

(b) -4

(c) 1

(d) -1

- 14 In the opposite figure :

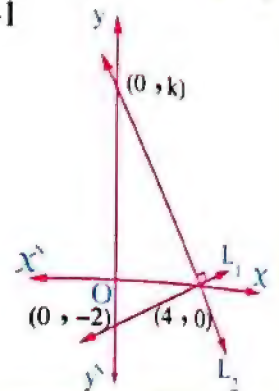
If  $L_1 \perp L_2$ , then  $k = \dots\dots\dots$ 

(a) 2

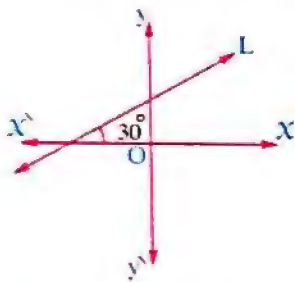
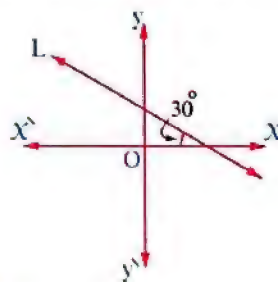
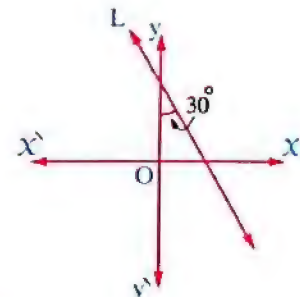
(b) 4

(c) 6

(d) 8



- 3 Write under each figure the slope of the straight line  $L$  :

1 The slope of  $L$  is .....2 The slope of  $L$  is .....3 The slope of  $L$  is .....

- 4 Find the slope of the straight line which makes with the positive direction of  $X$ -axis a positive angle of measure :

1 zero

2  $30^\circ$ 3  $45^\circ$ 4  $57^\circ$ 5  $60^\circ$ 6  $90^\circ$ 7  $86^\circ 42'$ 8  $135^\circ$ 

- 5 Using the calculator, find the measure of the positive angle which the straight line (whose slope is  $m$ ) makes with the positive direction of  $X$ -axis in the following cases.

1  $m = 0.3$ 2  $m = 0.3673$ 3  $m = 1.0246$ 4  $m = \frac{4}{5}$ 

- 6 Prove that : The straight line which passes through the two points  $(4, 2)$  and  $(5, 6)$  is parallel to the straight line which passes through the two points  $(0, 5)$  and  $(-1, 1)$

- 7 Prove that : The straight line passing through the two points  $A(-3, 4)$  and  $C(-3, -2)$  is perpendicular to the straight line passing through the two points  $B(1, 2)$  and  $D(-3, 2)$

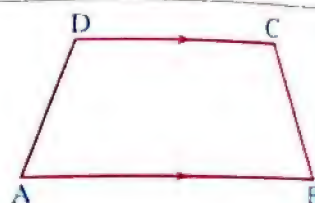
- 8 Prove that : The straight line passing through the two points  $(2, -1)$  and  $(6, 3)$  is parallel to the straight line that makes a positive angle of measure  $45^\circ$  with the positive direction of the  $X$ -axis.

(Alex. 17 - El-Menia 18 - Suc 20)

- 9 Prove that : The straight line which passes through the two points  $(4, 3\sqrt{3})$  and  $(5, 2\sqrt{3})$  is perpendicular to the straight line which makes a positive angle of measure  $30^\circ$  with the positive direction of X-axis. (El-Beheira 13 - Cairo 16 - Beni Suef 16)
- 10 In the Cartesian coordinates plane if A (1, 5) , B (X - 1, 3) , C (4, 7) and D (2, 1) are four points satisfying  $\overline{AD} \parallel \overline{BC}$ , find the value of X « 6 »
- 11 If the triangle whose vertices Y = (4, 2) , X = (3, 5) , Z = (-5, a) is right - angled at Y find the value of : a (El-Monofia 17 - Damietta 17 - Assiut 20) « 1 »
- 12 If the straight line  $\overline{AB} \parallel$  the y-axis , where A (X, 7) and B (3, 5) , then find the value of X (Luxor 19) « 3 »
- 13 If the straight line  $\overline{CD} \parallel$  the X-axis , where C (4, 2) and D (-5, y) , find the value of y « 2 »
- 14 If the straight line  $L_1$  passes through the two points (3, 1) and (2, k) and the straight line  $L_2$  makes with the positive direction of the X-axis a positive angle whose measure is  $45^\circ$ , then find k if the two straight lines  $L_1$  and  $L_2$  are :  
 1 parallel                      2 perpendicular (Assiut 17 - Alex. 18 - Aswan 20) « 0, 2 »
- 15 Find the measure of the positive angle which the straight line L makes with the positive direction of X-axis if the straight line L passes through the two points (4, 3) and (2, -5) «  $75^\circ$   $57^\circ$   $50^\circ$  »
- 16 Find the measure of the positive angle which the straight line L makes with the positive direction of X-axis if the straight line L passes through the two points (0, 0) and (2, -2) «  $135^\circ$  »
- 17 Find the measure of the positive angle which the straight line L makes with the positive direction of X-axis if the straight line L is perpendicular to the straight lines which passes through the two points (-2, 5) and (4, -1) «  $45^\circ$  »
- 18 Prove that : The points A (1, 1) , B (2, 3) and C (0, -1) are collinear. (Cairo 13)
- 19 If the points (0, 1) , (A, 3) and (2, 5) are located on one straight line. Then find the value of A (El-Gharbia 14 - Qena 16 - Sohag 18 - Damietta 19 - Cairo 20) « 1 »
- 20 If A (1, 7) , B (-1, 5) and C (4, 2) , prove that :  $C \notin \overline{AB}$
- 21 If A (-1, -1) , B (2, 3) and C (6, 0) , prove that : the triangle ABC is a right-angled triangle at B (Suez 14 - Kafr El-Sheikh 17 - Aswan 19)

- 22 **Prove that :** The points A  $(-1, 1)$  , B  $(0, 5)$  , C  $(4, 2)$  and D  $(5, 6)$  are the vertices of the parallelogram ABDC (Luxor 12 – Beni Suef 18)
- 23 **Prove by using the slope that the points** A  $(-1, 3)$  , B  $(5, 1)$  , C  $(6, 4)$  and D  $(0, 6)$  are the vertices of the rectangle ABCD (Beni Suef 13 – Souhag 17 – North Sinai 18)
- 24 **Prove that :** The points A  $(1, 3)$  , B  $(6, 4)$  , C  $(7, 9)$  and D  $(2, 8)$  are vertices of the rhombus ABCD
- 25 **Prove that :** The points A  $(-1, -1)$  , B  $(2, 3)$  , C  $(6, 0)$  and D  $(3, -4)$  are vertices of a square.

- 26 **In the drawn figure :**  
ABCD is a trapezoid where  $\overline{AB} \parallel \overline{CD}$  , A  $(9, -2)$  , B  $(3, 2)$  , C  $(x, -x)$  and D  $(4, -3)$   
Find the coordinates of the point C



(Alex. 14 – Suez 19) «  $(1, -1)$  »

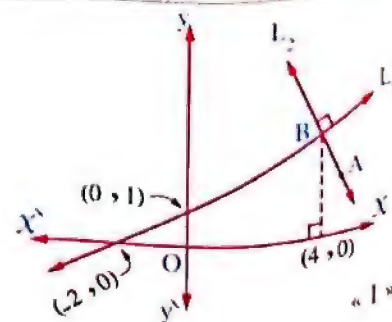
- 27 **Prove that :** The points A  $(4, 3)$  , B  $(7, 0)$  and C  $(1, -2)$  are vertices of a triangle and if the point D  $(1, 2)$  , then prove that the figure ABCD is a trapezoid and find the ratio between AD and BC «  $1:2$  »



### For excellent pupils

- 28 Find the slope of the straight line which makes with the positive direction of X-axis a positive angle whose sine =  $\frac{3}{5}$  «  $\frac{3}{4}$  »
- 29 If the points A  $(1, 1)$  , B  $(3, 3)$  , C  $(0, -3x)$  and D  $(x, y)$  are vertices of the rectangle ABCD , find the value of each of x and y «  $-2, 4$  »
- 30 ABCD is a rhombus in which : A  $(3, 2)$  , B  $(4, k)$  and C  $(-1, -2)$  (Ismailia 12)  
**Find :** 1 The value of k «  $-3$  »  
2 The length of  $\overline{BD}$  «  $6\sqrt{2}$  length unit »

- 31 **In the opposite figure :**  
If  $\overline{L_1} \perp \overline{L_2}$   
, A  $\in L_2$  where A  $(5m, m)$   
**Find :** The value of m





## EXERCISE

# 6

The equation of the straight line given its slope and the intercepted part of y-axis



From the school book

Interactive test

**1** Find the slope and the intercepted part of y-axis by each of the following straight lines :

**1**  $y = 5x - 3$

**2**  $2y = 4 - x$

**3**  $2x - 3y - 6 = 0$

**4**  $y + x - 1 = 0$

**5**  $\frac{x}{2} + 3y = 6$

**6**  $\frac{x}{2} + \frac{y}{3} = 1$  (Matrouh 19 – El-Kalyoubia 20)

**2** Find the equation of the straight line if :

**1** Its slope = 2 and intercepts from the positive part of y-axis 7 units. (Damietta 19 – Suez 20)

**2** Its slope = -1 and intercepts from the positive part of y-axis 3 units.

**3** Its slope =  $2\frac{1}{2}$  and intercepts from the negative part of y-axis one unit.

**4** Its slope =  $-\frac{3}{4}$  and intercepts from the negative part of y-axis  $2\frac{1}{2}$  units.

**5** Its slope = zero and intercepts from the negative part of y-axis 2 units.

**6** Its slope =  $-\frac{1}{3}$  and passes through the point (0, 2)

(El-Menia 17)

**7** Its slope = -2 and passes through the origin point.

**3** Find the equation of the straight line :

**1** Which passes through the origin point and makes with the positive direction of X-axis a positive angle of measure  $135^\circ$






**2** Passing through the point (3, 2) and makes with the positive direction of X-axis a positive angle of measure  $45^\circ$

(El-Sharkia 17)

**3** Which cuts a part of length 3 units from the negative part of y-axis and is parallel to the line whose equation :  $2x - 3y = 6$

(El-Beheira 11)

**4** Which is perpendicular to the straight line :  $3x - 4y + 7 = 0$  and intercepts from the positive part of y-axis a part of length 6 units.

- 5 Which intercepts a positive part from y-axis of length 5 units and perpendicular to the straight line which passes through the two points  $(-2, 1)$  and  $(2, 7)$
- 6 Find the equation of the straight line which intercepts from the positive parts of the coordinate axes «X-axis and y-axis» two parts of lengths 4 and 9 length unit respectively. *(Luxor 17 – Kafr El-Sheikh 18 – El-Kalyoubia 19)*
- 7 Which passes through the point  $(2, -1)$  and its slope equals 2 *(El-Kalyoubia 11)*
- 8 Passing through the point  $(-2, 3)$  and perpendicular to the straight line whose equation :  $y = \frac{1}{2}x - 5$  *(El-Dakahlia 13)*
- 9  Passing through the point  $(3, -5)$  and it is parallel to the straight line :  $x + 2y - 7 = 0$  *(Aswan 20)*
- 10 Which passes through the point  $(3, 2)$  and parallel to the straight line passing through the two points  $(5, 6)$  and  $(-1, 2)$  *(Helwan 09)*
- 11 Passing through the point  $(1, 2)$  and perpendicular to the straight line passing through the two points A  $(2, -3)$  and B  $(5, -4)$  *(El-Gharbia 14 – Luxor 18 – Suez 19 – Port Said 20)*
- 12 Passing through the point  $(2, -2)$  and perpendicular to the straight line which makes an angle of measure  $45^\circ$  with the positive direction of X-axis *(Luxor 11)*
- 13  Which passes through the two points  $(2, -1)$  and  $(1, 1)$  *(El-Gharbia 13 – El-Kalyoubia 16)*
- 14  Which passes through the two points  $(4, 2)$  and  $(-2, -1)$  then prove that it passes through the origin point. *(El-Beheira 17 – Cairo 19)*
- 15  Whose slope equals the slope of the straight line :  $\frac{y-1}{x} = \frac{1}{3}$  and intercepts a negative part of y-axis of 3 length units. *(Damietta 18 – Suez 19)*
- 16 Which is perpendicular to  $\overline{AB}$  from the point A where A  $(-3, 6)$  and B  $(2, 1)$
- 17  The perpendicular to  $\overline{AB}$  from its midpoint where A  $(1, 3)$  and B  $(3, 5)$  *(Qena 18)*
- 18 Passing through the midpoint of the line segment  $\overline{AB}$  where A  $(4, 8)$  and B  $(-2, 4)$  and parallel to the straight line whose equation is  $2y = 4x - 5$
- 19 Which passes through the midpoint of the line segment  $\overline{AB}$  where A  $(3, 6)$  and B  $(-1, 4)$  and perpendicular to the straight line whose equation is  $2y - 4x + 11 = 0$  *(Cairo 09)*
- 20 Passing through the point  $(2, 3)$  and intercepts from the positive part of X-axis a part of length 4 units. *(El-Sharkia 18)*

#### 4 Complete the following :

- 1 The slope of the straight line whose equation is :  $3x - 4y - 15 = 0$  is ..... and the slope of its perpendicular straight line is .....

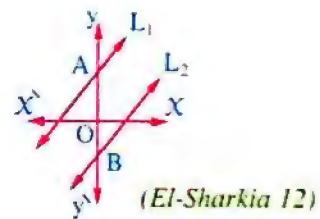
- 2 The straight line whose equation is :  $2x + 3y - 6 = 0$  cuts the y-axis at the point .....  
(Souhag 11)
- 3 The equation of the straight line whose slope = 2 and cuts the y-axis at the point (0 , 3) is .....  
(El-Dakahlia 11)
- 4 The equation of the X-axis is ..... while the equation of the y-axis is .....
- 5 The equation of the straight line which passes through the point (3 , 5) and is parallel to X-axis is .....  
(Aswiut 12)
- 6 The equation of the straight line which passes through the point (- 3 , 2) and is parallel to y-axis is .....
- 7 The equation of the straight line passing through the point (2 , -5) and its slope = zero is .....
- 8 The equation of the straight line which is parallel to the straight line :  $y = 2x - 3$  and passes through the origin point is .....

9 In the opposite figure :

$L_1 \parallel L_2$  ,  $AB = 7$  units of length.

If the equation of  $L_1$  is :  $y = 2x + 4$

, then the equation of  $L_2$  is .....



10 In the opposite figure :

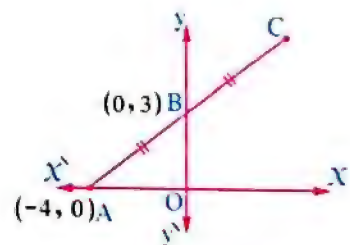
$B \in \overline{AC}$  ,  $A(-4, 0)$  ,  $B(0, 3)$  and  $AB = BC$

, then :

(1) The coordinates of point C (..... , .....)

(2) In  $\triangle OAB$  ,  $\tan A = \dots\dots\dots$

(3) The equation of  $\overline{AC}$  is  $y = \dots\dots\dots x + \dots\dots\dots$



5 Choose the correct answer from those given :

- 1 The slope of the straight line whose equation is :  $3y = 2x - 5$  is .....  
(a) 3 (b) 2 (c) -5 (d)  $\frac{2}{3}$
- 2 The straight line whose equation is :  $3x - 3y + 5 = 0$  makes a positive angle with the positive direction of X-axis , its measure = .....  
(El-Monofia 11)  
(a)  $30^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $90^\circ$
- 3 The straight line whose equation is :  $2x - 3y - 6 = 0$  intercepts from y-axis a part of length ..... units.  
(El-Fayoum 13 - Cairo 14 - Qena 17 - El-Kalyoubia 18)  
(a) -6 (b) -2 (c)  $\frac{2}{3}$  (d) 2

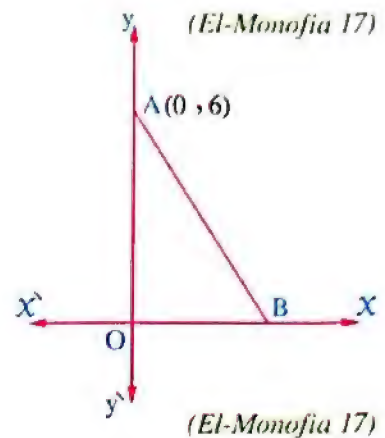
- 4 The straight line whose equation is :  $2x + 5y - 10 = 0$  cuts from  $x$ -axis a part of length ..... units. (El-Dakahlia 11)  
 (a)  $\frac{2}{5}$  (b) 2 (c)  $\frac{5}{2}$  (d) 5
- 5 The equation of the straight line which intercepts a part of length 4 units from the positive part of  $y$ -axis and parallel to the straight line :  $y = 3x + 5$  is .....  
 (a)  $y = 3x + 4$  (b)  $y = 4x + 3$  (c)  $y = 3x - 4$  (d)  $y = -3x + 4$
- 6 The two straight lines :  $y = 3x - 5$  and  $2y = 6x + 5$  are .....  
 (a) parallel (b) coincident  
 (c) intersecting and not perpendicular (d) perpendicular
- 7 If the two straight lines :  $3x - 4y - 3 = 0$  and  $ky + 4x - 8 = 0$  are perpendicular , then  $k =$  ..... (El-Beheira 15 - Giza 16 - Red Sea 19)  
 (a) -4 (b) -3 (c) 3 (d) 4
- 8 If the two straight lines :  $x + y = 5$  and  $kx + 2y = 0$  are parallel , then  $k =$  .....  
(El-Dakahlia 15 - Souhag 16 - Qena 17 - El-Menia 19)  
 (a) -2 (b) -1 (c) 1 (d) 2
- 9 If the straight line whose equation is :  $y = kx + 5$  is parallel to  $x$ -axis , then  $k =$  ..... (El-Gharbia 18)  
 (a) 0 (b) 1 (c) 2 (d) 3
- 10 The two straight lines :  $y = ax + b$  and  $y = cx + d$  are perpendicular , then ..... = -1 (El-Gharbia 08 - Souhag 16)  
 (a)  $a \times d$  (b)  $b \times c$  (c)  $a \times c$  (d)  $b \times d$
- 11 The straight line passing through the two points (5 , 4) and (1 , 5) is perpendicular to the straight line .....  
 (a)  $4x = 3 - 4y$  (b)  $5y + x = 4$  (c)  $y = 4x$  (d)  $x + 2y = 4$
- 12 The slope of the straight line whose equation is :  $3y = ax - 5$  and passes through the point (20 , 5) is .....  
 (a) -1 (b) 1 (c) -2 (d)  $\frac{1}{3}$
- 13 If the straight line whose equation is :  $ax + (2 - a)y = 5$  is parallel to the straight line which passes through (1 , 4) , (3 , 5) , then  $a =$  ..... (Kafir El-Sheikh 20)  
 (a) 3 (b) -2 (c) 6 (d) 4

- 14 The area of the triangle in square units which is bounded by the straight lines  $3x - 4y = 12$ ,  $x = 0$ ,  $y = 0$  equals .....  
 (a) 6 (b) 7 (c) 12 (d) -6  
 (El-Kalyoubia 15 - El-Fayoum 20)

15 In the opposite figure :

If the area of  $\triangle AOB = 9$  square unit,  
 , then the equation of the straight line  $\overline{AB} = \dots\dots\dots$

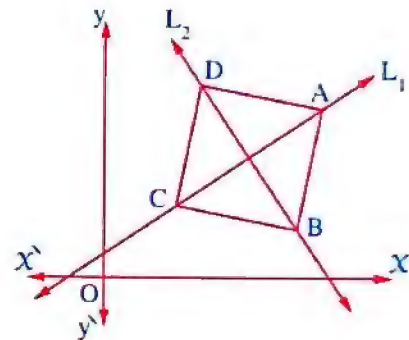
- (a)  $y = 2x + 6$   
 (b)  $y = 6 - 2x$   
 (c)  $y = 2x - 6$   
 (d)  $y = \frac{1}{2}x - 6$



16 In the opposite figure :

If ABCD is a square  
 , the equation of the straight line  $L_1$  is :  $y = \frac{2}{3}x + 1$   
 and the equation of the straight line  $L_2$  is :  $y = kx + 14$  , then  $k = \dots\dots\dots$

- (a)  $\frac{2}{3}$  (b)  $\frac{3}{2}$   
 (c)  $-\frac{2}{3}$  (d)  $-\frac{3}{2}$



- 6 Prove that : The straight line which passes through the two points A (3 , 1) and B (1 , 2) is parallel to the straight line :  $2x + 4y - 3 = 0$   
 (El-Sharkia 17)

- 7 Prove that : The straight line whose equation :  $2x + y + 8 = 0$  is perpendicular to the straight line passing through A (2 , 3) and B (-2 , 1)  
 (Aswan 12)

- 8 Find the equations of the two straight lines which pass through the point (-3 , 2) and parallel to the two axes.

- 9 Find the measure of the positive angle which is made by the straight line :  $3x - 2y + 6 = 0$  with the positive direction of the X-axis , then find the coordinates of its intersection point with the y-axis.

- 10 If the straight line whose equation :  $2x - 3y - 6 = 0$  cuts the X-axis at point A and the y-axis at point B , find :  
 (El-Sharkia 13)

- 1 The coordinates of two points A and B  
 2 The equation of the straight line passing through the midpoint of  $\overline{AB}$  and parallel to the y-axis.

- 11 If the straight line which passes through the two points  $(2, -1)$  and  $(5, 1)$  is parallel to the straight line whose equation is :  $aX + 3Y + 5 = 0$  Find the value of :  $a$  (El-Gharbia 18) \* 2 \*
- 12 If the straight line which passes through the two points  $(5, 2)$  and  $(6, -3)$  is perpendicular to the straight line whose equation is :  $5Y - aX + 3 = 0$  , find the value of  $a$  \* 1 \*
- 13 If  $A(2, -3)$  and  $B(5, y)$  , find the value of  $y$  if the straight line  $\overline{AB}$  is parallel to the straight line  $L : 3Y - 4X + 1 = 0$  \* 1 \*
- 14 If the straight line :  $Y - (2k - 1)X = 7$  and the straight line which makes with the positive direction of the  $X$ -axis a positive angle of measure  $45^\circ$  are parallel, then find the value of  $k$  (El-Sharkia 16) \* 1 \*
- 15 Find the equation of the axis of symmetry of  $\overline{XY}$  , where  $X(3, -2)$  and  $Y(-5, 6)$  (El-Dakahlia 12 - Port Said 14)
- 16 A  $(5, -6)$  ,  $B(3, 7)$  and  $C(1, -3)$  , find the equation of the straight line which passes through the point  $A$  and the midpoint of  $\overline{BC}$  (Port Said 19 - El-Fayoum 20)
- 17  $ABC$  is a triangle whose vertices  $A(0, 6)$  ,  $B(5, -1)$  and  $C(-2, 1)$   
Find the equation of the straight line passing through the vertex  $A$  and perpendicular to  $\overline{BC}$
- 18  $ABC$  is a triangle in which  $A(1, 2)$  ,  $B(5, -2)$  and  $C(3, 4)$  ,  $D$  is the midpoint of  $\overline{AB}$  and  $\overline{DE} \parallel \overline{BC}$  and intersects  $\overline{AC}$  at  $E$   
Find : 1 The length of  $\overline{DE}$   
2 The equation of the straight line  $\overline{DE}$  (Alexandria 15 - Matrouh 18)
- 19  $ABCD$  is a square in which :  $A(5, 4)$  and  $C(-1, 6)$   
Find the equations of  $\overline{BD}$  (El-Monofia 15)
- 20  $ABCD$  is a rhombus ,  $M$  is the point of intersection of its two diagonals where  $A(1, 3)$  and  $C(6, 0)$  , find the equation of the straight line which passes through the two points  $B$  and  $D$  (Aswan 09)
- 21 Find the equation of the straight line passing through two points  $A(2, 3)$  ,  $B(-1, -3)$   
Show that for any point  $C(2k + 1, 4k + 1)$  , then  $C \in \overline{AB}$  (El-Dakahlia 14)

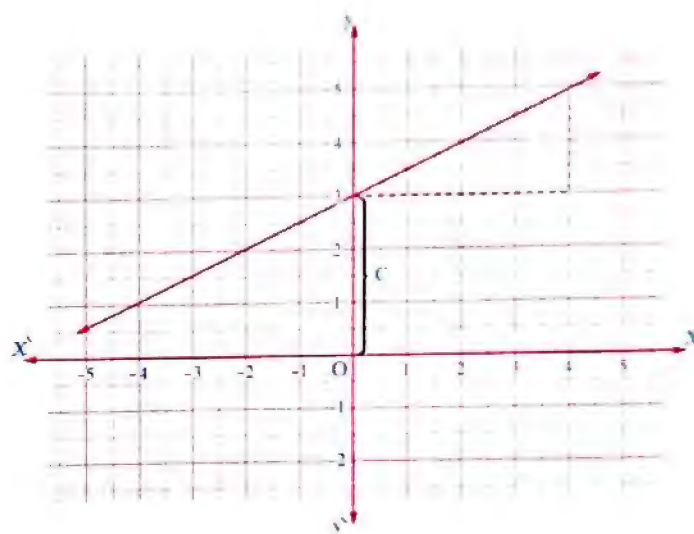
22 Draw the straight line in each of the following cases :

- 1 The slope =  $-\frac{1}{2}$  and intercepts from the positive part of y-axis a part of one unit.
- 2 The slope = 2 and intercepts from the negative part of y-axis equal to 3 length units.
- 3 Intercepts from the positive parts of the two axes (X-axis , y-axis) two parts of lengths 2 and 3 length units respectively.

23 Find the slope of the straight line :  $y - 2x - 3 = 0$  , then find the length of the intercepted part from y-axis , also draw this line. (Helwan 11)

24 From the opposite graph , find :

- 1 The slope of the straight line (m)
- 2 The intercepted part of y-axis (c)
- 3 The equation of the straight line given (m) and (c)
- 4 The length of the intercepted part of X-axis.
- 5 The area of the triangle bounded by the straight line and the two axes.



25 The opposite table represents a linear relation :

- 1 Find the equation of the straight line.
- 2 Find the length of the intercepted part from y-axis.
- 3 Find the value of a

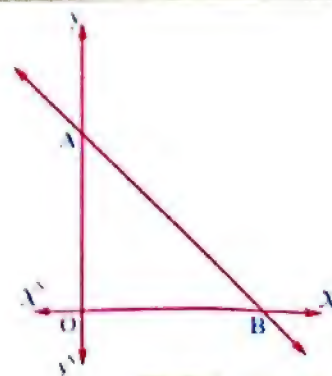
$x$	1	2	3
$y = f(x)$	1	3	a

(El-Kalyoubia 13 - Alexandria 15)

26 The opposite figure represents the straight line  $\overleftrightarrow{AB}$  whose equation is  $y = kx + c$  and cuts from the two axes two equal parts and passes through the point (2 , 3)

Find : 1 The values of  $k$  ,  $c$

- 2 The area of the triangle ABO



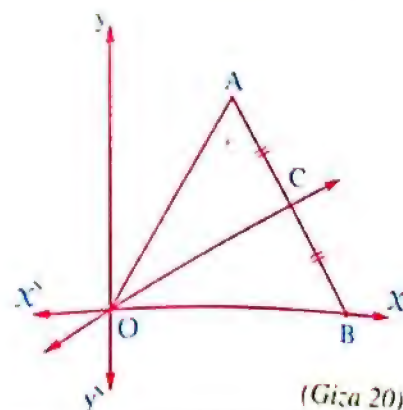
(El-Dakahlia 19)

27 In the opposite figure :

ABO is an equilateral triangle ,

C is the midpoint of  $\overline{AB}$

Find the equation of the straight line  $\overline{OC}$



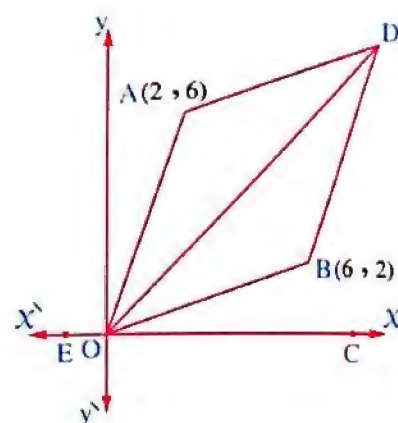
(Giza 20)

28 In the opposite figure :

The points A (2 , 6) , O (0 , 0) , B (6 , 2) and D are vertices of the rhombus.

Find :

- 1 The coordinates of the point D
- 2 The equation of the straight line  $\overline{OD}$
- 3  $m(\angle DOE)$



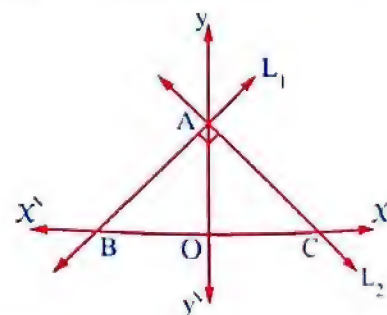
(El-Sharkia 14)

29 In the opposite figure :

If  $L_1 \perp L_2$

and the equation of  $L_1$  is :  $2x - y + 2 = 0$

Find the equation of the straight line  $L_2$



30 In the opposite figure :

$\overline{AB}$  cuts y-axis at point A (0 , 8) and cuts X-axis at point B

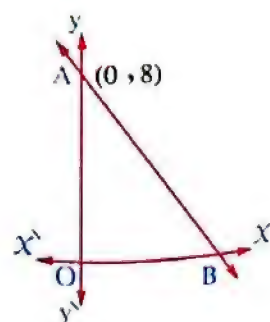
, If  $\tan(\angle ABO) = \frac{4}{3}$  , find :

- 1 First :  $m(\angle BAO)$

Second : The coordinate of B

- 2 First : The slope of  $\overline{AB}$

Second : The equation of the straight line passes through the point O and perpendicular to  $\overline{AB}$



(El-Sharkia 13)

31 In the opposite figure :

The point C is the midpoint of  $\overline{AB}$  where C (4 , 3) :

1 Find the coordinates of each of :

O , A and B

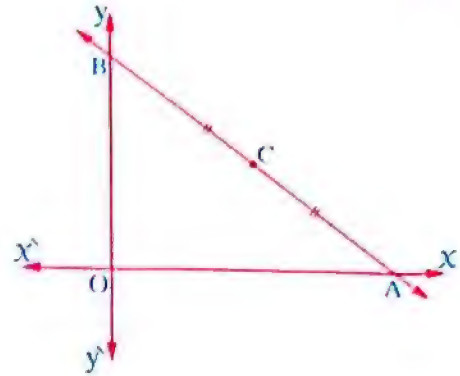
2 Find the length of each of :

$\overline{OA}$  ,  $\overline{OB}$  ,  $\overline{CA}$  ,  $\overline{CB}$  and  $\overline{CO}$

3 Find the slope of each of :

$\overline{AB}$  ,  $\overline{OC}$  ,  $\overline{OA}$  and  $\overline{OB}$

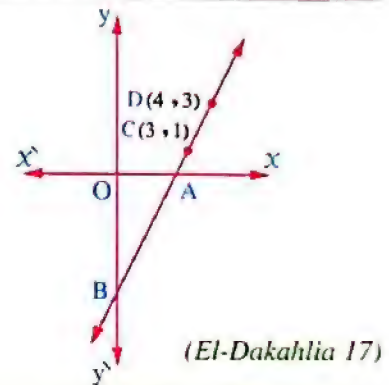
4 Find the equation of each of :  $\overline{AB}$  and  $\overline{CO}$



32 In the opposite figure :

The straight line  $\overline{AB}$  passes through two points C (3 , 1) , D (4 , 3) and intersects the two axis in A , B respectively.

Find the lengths of  $\overline{AO}$  ,  $\overline{OB}$  , where O is the origin point.



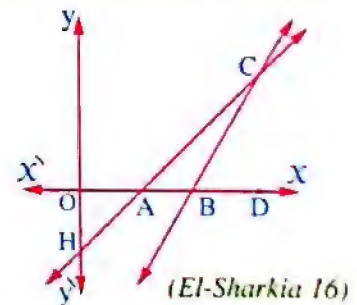
33 In the opposite figure :

O is the original point A , B , D  $\in$  X-axis , the slope of  $\overline{BC} = \sqrt{3}$  , the equation of  $\overline{AC}$  is :  $x - y = 3$

Find : 1 The slope of  $\overline{AC}$  and the length of  $\overline{OH}$

2  $m(\angle CBD)$  and  $m(\angle CAD)$

3  $m(\angle ACB)$



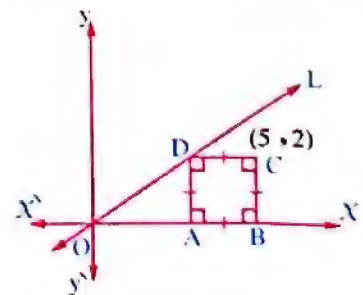
34 In the opposite figure :

ABCD is a square

, D  $\in$  the straight line L

and C (5 , 2)

Find the equation of the straight line L

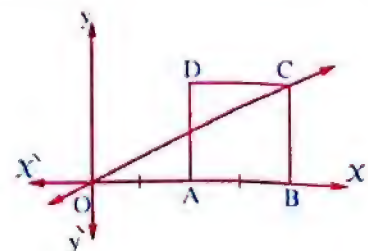


35 In the opposite figure :

ABCD is a square

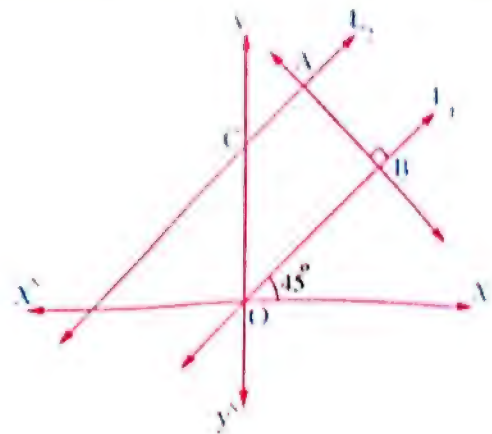
, OA = AB

Find the equation of the straight line  $\overline{OC}$



**36** In the opposite figure :

$L_1$  and  $L_2$  are parallel lines,  $L_1$  makes with the positive direction of  $X$ -axis an angle of measure  $45^\circ$  and passes through the origin point  $O$ ,  $A \in L_2$  where  $A(1, 5)$ ,  $\overline{AB} \perp L_1$ ,  $L_2$  intersects the  $y$ -axis at the point  $C$



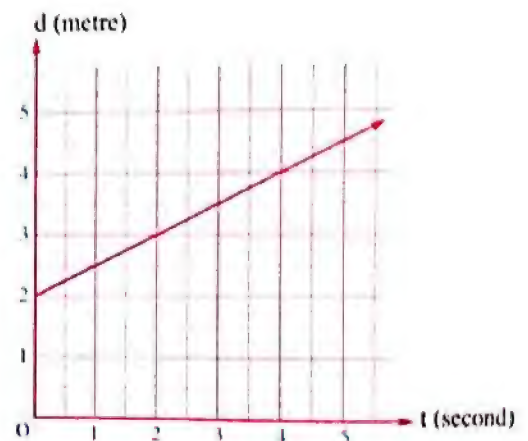
(El-Sharkia 15)

### Life Applications

**37** The opposite graph represents the motion of a particle moving with uniform velocity ( $v$ ) where the distance ( $d$ ) is measured in metre and the time ( $t$ ) in seconds.

**Find :**

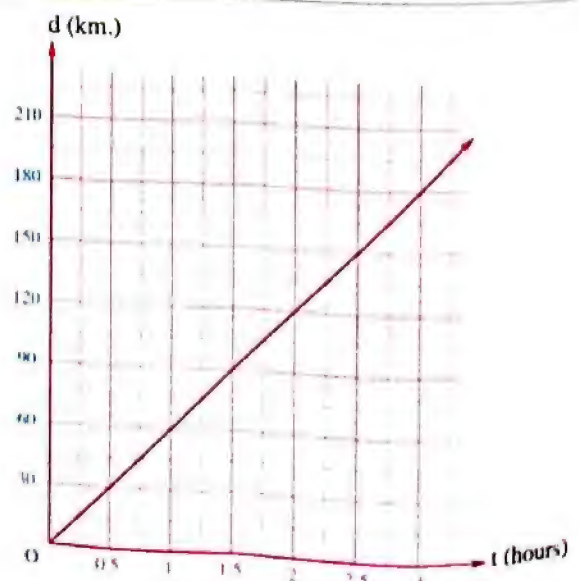
- 1 The distance at the beginning of the motion.
- 2 The velocity of the particle.
- 3 The equation of the straight line representing the motion of the particle.
- 4 The covered distance after 4 seconds from the beginning of the motion.
- 5 The time in which the particle covers a distance of 3.5 metres from the beginning of the motion.



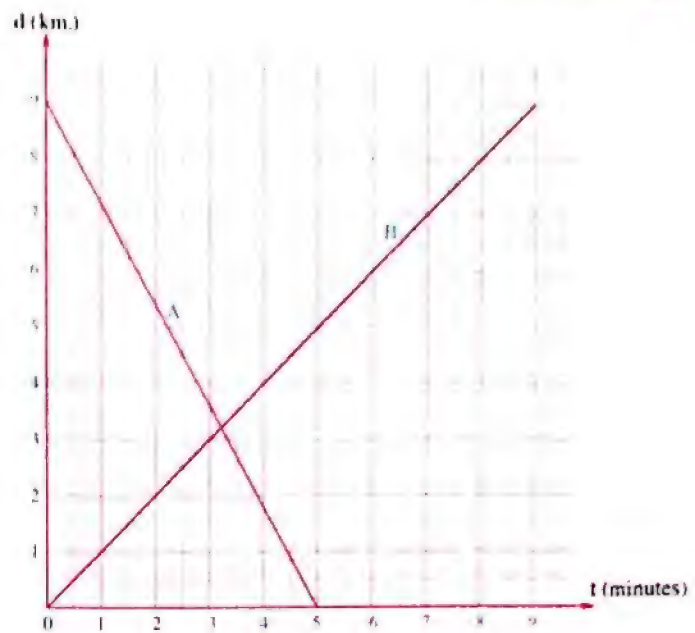
**38** The opposite graph represents the relation between distance the car covers ( $d$  in km.) and the time the car covers in ( $t$  in hour).

**Find :**

- 1 The covered distance after 90 minutes.
- 2 The time which the car took to cover a distance of 150 km.
- 3 The velocity of the car.
- 4 The equation of the straight line which represents the relation between the distance ( $d$ ) and the time ( $t$ ).



39 The opposite graph represents the relation between the covered distance ( $d$ ) in km. and the time ( $t$ ) in minutes for the two objects A and B in the two ends of a straight road moving in two opposite directions :



- 1 Did A and B move at the same time ?
- 2 After how many minutes did A and B meet together ?
- 3 What is the velocity of A ?
- 4 Write the equation of the straight line which represents the relation between the distance ( $d$ ) and the time ( $t$ ) of the motion of the object B

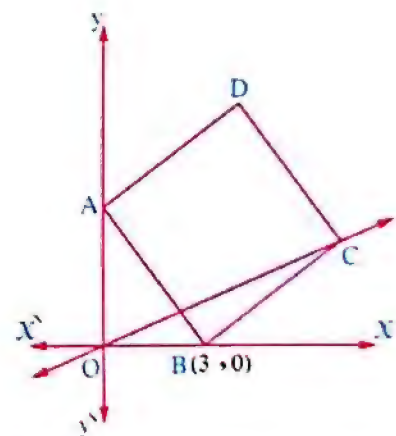


For excellent pupils

40 In the opposite figure :

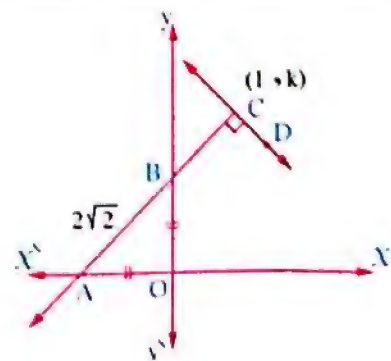
If the area of a square ABCD = 25 square units.

Find : the equation of  $\overrightarrow{CO}$



41 From the opposite figure :

Find : the equation of  $\overline{CD}$



# Summary of Unit 5



★ If A ( $x_1, y_1$ ) and B ( $x_2, y_2$ ), then :

- The length of  $\overline{AB}$

$$= \sqrt{(\text{difference between } x\text{-coordinates})^2 + (\text{difference between } y\text{-coordinates})^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- The midpoint of  $\overline{AB} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

- The slope of  $\overline{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \tan \theta$

(where  $\theta$  is the measure of the positive angle which the straight line makes with the positive direction of the  $x$ -axis)

★ If  $L_1$  and  $L_2$  are two straight lines of slopes  $m_1$  and  $m_2$  respectively, then

- $L_1 \parallel L_2$  if  $m_1 = m_2$  and vice versa
- $L_1 \perp L_2$  if  $m_1 \times m_2 = -1$  and vice versa

★ If the equation of a straight line in the form :  $y = m x + c$ , then :

- The slope of the straight line =  $m$
- The length of the intercepted part from  $y$ -axis =  $|c|$   
and it passes through the point  $(0, c)$

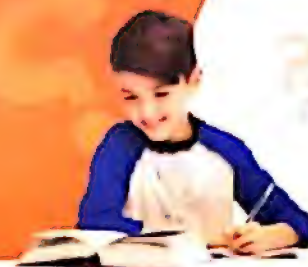
★ If the equation of a straight line in the form :  $a x + b y + c = 0$ , then :

- The slope of the straight line =  $\frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b}$
- The length of the intercepted part from  $y$ -axis =  $\left| \frac{-c}{b} \right|$   
and the straight line cuts  $y$ -axis at the point  $\left( 0, \frac{-c}{b} \right)$

★ The equation of the straight line which passes through the origin point O  $(0, 0)$  is  $y = m x$ , where  $m$  is the slope of the straight line.

★ The equation of  $x$ -axis is  $y = 0$

★ The equation of  $y$ -axis is  $x = 0$



## Model 1

Answer the following questions :

1 Choose the correct answer from those given :

1 The distance between the point  $(-7, -3)$  and y-axis is ..... length unit.

- (a)  $-7$  (b)  $-3$  (c)  $7$  (d)  $3$

2 The point  $(0, 4)$  bisects the distance between the two points  $(-1, -1)$ ,  $(X, y)$ , then the point  $(X, y)$  is .....

- (a)  $(1, 9)$  (b)  $(-1, 9)$  (c)  $(-\frac{1}{2}, \frac{3}{2})$  (d)  $(-1, 3)$

3 If the two straight lines whose slopes are  $(-\frac{1}{4})$  and  $(4k)$  are perpendicular, then  $k = \dots\dots\dots$

- (a)  $1$  (b)  $4$  (c)  $-4$  (d)  $\frac{1}{4}$

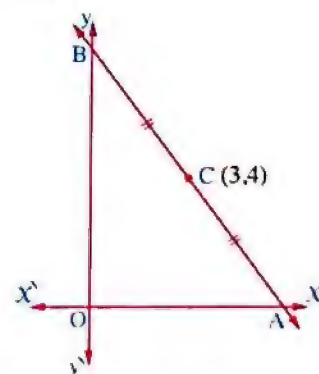
4 The slope of the straight line whose equation is :  $X - 5 = 0$  is .....

- (a)  $5$  (b)  $\frac{1}{5}$  (c) undefined (d)  $0$

5 In the opposite figure :

$C(3, 4)$  is the midpoint of  $\overline{AB}$ , then  $OA = \dots\dots\dots$  length unit

- (a)  $3$  (b)  $4$   
(c)  $6$  (d)  $8$



6 If the straight line which passes through the two points  $(X, -1)$  and  $(4, 2)$  is parallel to the straight line which passes through the two points  $(3, 4)$  and  $(-3, -2)$ , then  $X = \dots\dots\dots$

- (a)  $-3$  (b)  $2$  (c)  $7$  (d)  $1$

2 [a] ABCD is a quadrilateral where  $A(-1, 3)$ ,  $B(5, 1)$ ,  $C(7, 4)$  and  $D(1, 6)$

Prove that : The figure ABCD is a parallelogram.

[b] Find the equation of the straight line which passes through the point  $(3, 4)$  and it is perpendicular to the straight line whose equation is :  $5X - 2y + 7 = 0$

**3** [a] If the point A (5, 2) lies on the circle whose centre is M (1, -1), find :

- 1 The area of the circle in terms of  $\pi$
- 2 The equation of the straight line passing through the two points A, M

[b] If the points : A (3, 2) , B (4, -3) , C (-1, -2) and D (-2, 3) are the vertices of a rhombus, find :

- 1 The coordinates of the point of intersection of its diagonals.
- 2 The area of the rhombus ABCD

**4** [a] If the distance between the point (X, 5) and the point (6, 1) equals  $2\sqrt{5}$  length unit , then find the value of X

[b] **Prove that :** The points A (-3, 0) , B (3, 4) and C (1, -6) are the vertices of an isosceles triangle at vertex A , then find the length of the drawn line segment from A perpendicular on BC

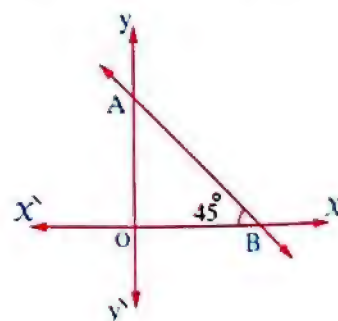
**5** [a] Find the equation of the straight line which passes through the two points (3, 7) , (6, 13)

[b] **In the opposite figure :**

$\overline{AB}$  intercepts from the positive part of the X-axis 3 units

,  $m(\angle ABO) = 45^\circ$

Find the equation of  $\overline{AB}$



### Model 2

**Answer the following questions :**

**1** Choose the correct answer from those given :

1 If ABCD is a rectangle where : A (-4, -1)

, C (4, 5) , then the length of  $\overline{BD}$  = ..... length unit.

- (a) 10                      (b) 6                      (c) 5                      (d) 4

2 If (4, -3) is the midpoint of  $\overline{AB}$  where A (3, -4) , then B is .....

- (a) (5, -2)              (b) (2, 5)              (c) (5, 2)              (d) (3.5, -3.5)

3 If the two straight lines :  $3x - 4y - 3 = 0$  and  $kx + 3y - 8 = 0$  are perpendicular , then  $k = \dots\dots\dots$

- (a)  $-4$  (b)  $-3$  (c)  $3$  (d)  $4$

4 The straight line whose equation is :  $2x - 3y - 6 = 0$  intercepts from y-axis a part of length  $\dots\dots\dots$  units.

- (a)  $-6$  (b)  $-2$  (c)  $\frac{2}{3}$  (d)  $2$

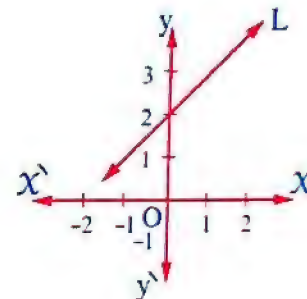
5 The equation of the straight line which passes through the point  $(2, -3)$  and is parallel to X-axis is  $\dots\dots\dots$

- (a)  $x = 2$  (b)  $y = -3$   
(c)  $x = -2$  (d)  $y = 3$

6 In the opposite figure :

Which of the following represents  
the equation of the straight line L ?

- (a)  $y = x$  (b)  $y = 2$   
(c)  $y + x = 2$  (d)  $y - x = 2$



2 [a] If ABCD is a square where :

$A(2, 4)$  ,  $B(-3, 0)$  ,  $C(-7, 5)$  , find :

- 1 The coordinates of D  
2 The area of the square ABCD

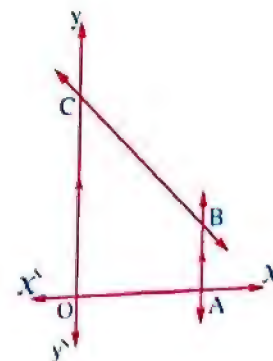
[b] In the opposite figure :

The straight line  $\overleftrightarrow{AB}$  is parallel to y-axis  
, the straight line  $\overleftrightarrow{BC}$  has equation  $y = -x + 3$   
, the point B  $(2, 1)$

Find : 1 The length of  $\overline{BC}$

2 The area of the figure OABC

3  $m(\angle OCB)$

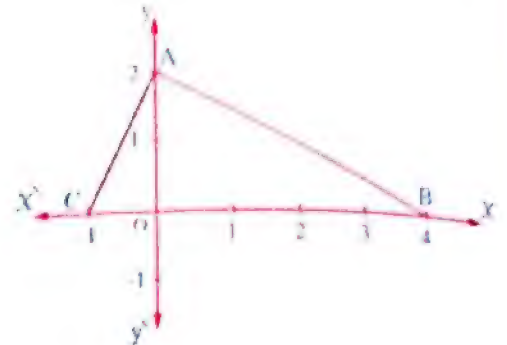


- 3 [a] Prove that : The points  $A(-2, 5)$ ,  $B(3, 3)$  and  $C(-4, 2)$  are not collinear.  
 [b] If  $A(X, 3)$ ,  $B(3, 2)$  and  $C(5, 1)$  and  $AB = BC$ , then find the value of  $X$

- 4 [a] In the opposite figure :

A triangle  $ABC$  is drawn in the orthogonal Cartesian coordinates plane

Prove that :  $\triangle ABC$  is a right-angled triangle  
 • then find its area



- [b]  $ABC$  is a triangle in which :  $A(1, 2)$ ,  $B(5, -2)$  and  $C(3, 4)$

Find the equation of the straight line passing through the vertex  $A$  and perpendicular to  $\overline{BC}$

- 5 [a]  $ABCD$  is a quadrilateral in which :  $A(0, 6)$ ,  $B(-1, 3)$ ,  $C(5, 1)$  and  $D(6, 4)$

Prove by using the slope the figure  $ABCD$  is a rectangle.

- [b] If the axis of symmetry of  $\overline{CD}$  is passing through the point  $A(6, m)$  where  $C(3, 1)$  and  $D(-3, 7)$ , then find the value of  $m$

# A Research Project

## On Unit Five



### Project aims :

- Finding the distance between two points in the coordinates plane.
- Finding the coordinates of the midpoint of a line segment.
- Calculate the perimeter and area of a triangle.
- Finding the slope of a straight line.
- Finding the equation of a straight line.
- Association of geometry with history.

### Do a research project on the following topic :

*"Analytic geometry is a branch of mathematics that uses a coordinates system to study geometry".*

Discuss the following points using available resources :

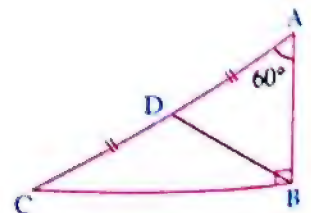
- Write a short note on the scientist, René Descartes who discovered the Cartesian coordinates and about his achievements in the field of mathematics.
- Draw the two axes of coordinates on a sheet of graph paper.
- On the coordinates plane locate three points that represent the vertices of an isosceles triangle. Then find :
  1. The perimeter of this triangle.
  2. The area of this triangle.
  3. The slope of each side of this triangle.
  4. The equation of the straight line that contains each side of this triangle.



## Accumulative basic skills

Choose the correct answer from those given :

- 1 The number of diagonals of the hexagon is ..... (Qena 20)  
 (a) 6 (b) 3 (c) 12 (d) 9
- 2 The two angles of base of an isosceles triangle are ..... (Alexandria 16 – North Sinai 17)  
 (a) congruent. (b) supplementary.  
 (c) vertically opposite angles. (d) corresponding.
- 3 The measure of an exterior angle of an equilateral triangle is .....  
(Alex. 17 – Beni Suef 18 – Kafr El-Sheikh 19 – Cairo 20)  
 (a)  $60^\circ$  (b)  $150^\circ$  (c)  $120^\circ$  (d)  $30^\circ$
- 4 The number of axes of symmetry of the isosceles triangle equals ..... (Matrouh 17 – Alex. 18)  
 (a) 0 (b) 1 (c) 2 (d) 3
- 5 In the opposite figure :  
 If  $m(\angle ABC) = 90^\circ$  ,  $m(\angle A) = 60^\circ$   
 and  $\overline{BD}$  is a median in  $\triangle ABC$  , then  $m(\angle DBC) = \dots\dots\dots$   
 (a)  $20^\circ$  (b)  $30^\circ$   
 (c)  $60^\circ$  (d)  $45^\circ$
- 6 The triangle whose side lengths are 5 cm. , 5 cm. , ..... is an isosceles triangle. (El-Menia 17)  
 (a) 9 cm. (b) 10 cm. (c) 11 cm. (d) 12 cm.
- 7 The triangle whose side lengths are 5 cm. , 12 cm. and 13 cm. , its area = .....  $\text{cm}^2$   
(Matrouh 18)  
 (a) 30 (b) 32.5 (c) 78 (d) 144



- 8 In any triangle, the sum of the lengths of any two sides is ..... the length of the third side. (El-Fayoum 18 – El-Menia 19)  
 (a) greater than (b) smaller than (c) equal to (d) half
- 9 The point of concurrence of the medians of the triangle divides the median in the ratio of ..... from the base. (El-Fayoum 18)  
 (a) 1 : 3 (b) 2 : 1 (c) 3 : 1 (d) 1 : 2
- 10 The sum of the measures of the accumulative angles at a point equals .....° (Assiut 18 – El-Fayoum 19)  
 (a) 90 (b) 180 (c) 270 (d) 360
- 11 If ABCD is a square, then  $m(\angle CAB) = \dots\dots\dots$  (Alex. 17 – El-Beheira 18)  
 (a)  $90^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $30^\circ$
- 12 If the lengths of the diagonals of the rhombus are 6 cm. , 10 cm. , then its area equals .....  $\text{cm}^2$  (Kafr El-Sheikh 17)  
 (a) 30 (b) 60 (c) 15 (d) 10
- 13 The image of the point  $(-4, 5)$  by the translation  $(2, -3)$  is ..... (Kafr El-Sheikh 17)  
 (a)  $(-2, -2)$  (b)  $(2, -2)$  (c)  $(2, 2)$  (d)  $(-2, 2)$
- 14 The image of the point  $(-2, 5)$  by reflection in X-axis is ..... (Ismailia 16)  
 (a)  $(-2, -5)$  (b)  $(2, 5)$  (c)  $(2, -5)$  (d)  $(5, -2)$
- 15 The quadrilateral which its diagonals are equal in length and perpendicular is the ..... (Beni Suef 20)  
 (a) square. (b) rhombus. (c) rectangle. (d) parallelogram.
- 16 The volume of the cuboid whose dimensions are  $\sqrt{2}, \sqrt{3}, \sqrt{6}$  centimetres equals .....  $\text{cm}^3$  (South Sinai 16)  
 (a)  $2\sqrt{6}$  (b)  $3\sqrt{6}$  (c)  $3\sqrt{2}$  (d) 6
- 17 If 3, 7, L are lengths of sides of a triangle, then L may be equal ..... (Souhag 20)  
 (a) 3 (b) 4 (c) 7 (d) 10
- 18  $\triangle ABC$  in which  $m(\angle B) = 3m(\angle A) = 90^\circ$ , then  $m(\angle C) = \dots\dots\dots$  (Aswan 16)  
 (a)  $30^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $90^\circ$
- 19 ABC is a triangle, if  $m(\angle B) > m(\angle C)$ , then ..... (Suez 16)  
 (a)  $AC - AB < 0$  (b)  $AC - AB \leq 0$  (c)  $BC \leq AB$  (d)  $AC - AB > 0$

20 The circumference of the circle with diameter length 14 cm. is ..... cm. (where  $\pi = \frac{22}{7}$ )  
(El-Fayoum 17)

- (a) 7 (b) 22 (c) 44 (d) 14

21 If  $m(\angle X) = m(\angle Y)$ ,  $\angle X$ ,  $\angle Y$  are complementary  
, then  $m(\angle X) =$  .....  
(North Sinai 17)

- (a)  $90^\circ$  (b)  $60^\circ$  (c)  $45^\circ$  (d)  $30^\circ$

22 If  $\overline{XY}$  is the axis of symmetry of  $\overline{AB}$ , then  $XA$  .....  $XB$   
(Suez 20)

- (a)  $>$  (b)  $<$  (c)  $=$  (d)  $\perp$

23 ABCD is a parallelogram in which  $m(\angle A) + m(\angle C) = 200^\circ$   
, then  $m(\angle B) =$  .....  
(El-Ismailia 17 – Suez 19)

- (a)  $50^\circ$  (b)  $80^\circ$  (c)  $100^\circ$  (d)  $160^\circ$

24 If ABCD is a parallelogram, then  $AB + CD =$  .....  
(Suez 18)

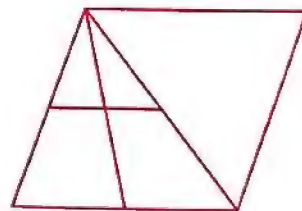
- (a)  $2 AC$  (b)  $2 BC$  (c)  $2 BD$  (d)  $2 CD$

25 If  $L_1 \parallel L_2$ ,  $L_3 \perp L_1$ ,  $L_4 \perp L_2$ , then .....  
(El-Beheira 17)

- (a)  $L_2 \parallel L_3$  (b)  $L_1 \parallel L_4$  (c)  $L_3 \parallel L_4$  (d)  $L_3 \perp L_4$

26 The number of triangles in the opposite figure = ..... triangles.  
(New Valley 16)

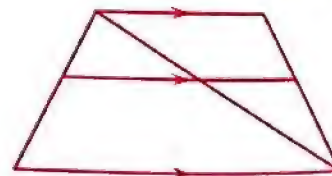
- (a) 5 (b) 6  
(c) 7 (d) 8



27 In the opposite figure :

The number of trapeziums = .....

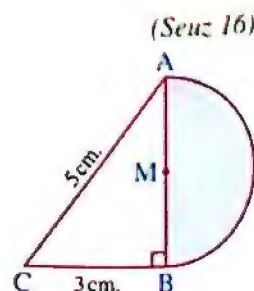
- (a) 2 (b) 3  
(c) 4 (d) 5



28 In the opposite figure :

$\overline{AB}$  is a diameter of a circle, then the surface  
area of the shaded shape = .....  $\text{cm}^2$

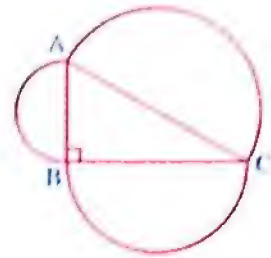
- (a)  $4\pi$  (b)  $16\pi$   
(c)  $2\pi$  (d)  $9\pi$



**29 In the opposite figure :**

ABC is a right-angled triangle at B what is the area of the semicircle drawn on the hypotenuse AC if the areas of the two semicircles drawn on AB and BC are 36 and 64 square units respectively ?

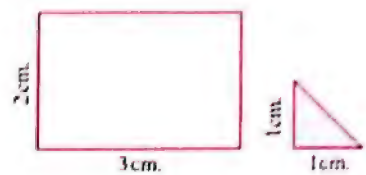
- (a) 80 square units    (b) 96 square units    (c) 100 square units    (d) 120 square units



**30 In the opposite figure :**

The number of the coloured right-angled triangle needed to cover the rectangle surface completely is .....

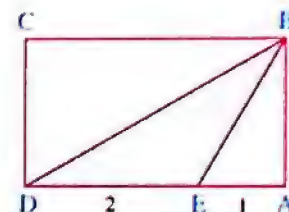
- (a) 4    (b) 6    (c) 8    (d) 12



**31 In the opposite figure :**

If  $AE : ED = 1 : 2$  , then the ratio between the area of  $\triangle BED$  and the rectangle ABCD is .....

- (a) 1 : 2    (b) 1 : 3    (c) 2 : 3    (d) 2 : 5

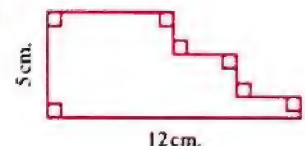


**32 In the opposite figure :**

The perimeter of the figure = ..... cm.

- (a) 17    (b) 22  
(c) 29    (d) 34

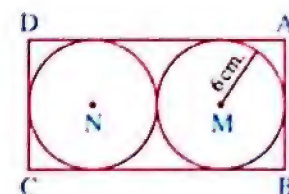
(Aswan 18)



**33 In the opposite figure :**

Two circles M and N inside a rectangle , the radius length of each one is 6 cm. , then the area of the rectangle = .....  $\text{cm}^2$

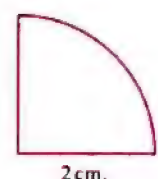
- (a) 288    (b) 252    (c) 216    (d) 144



**34 The opposite figure represents quarter a circle**

with radius 2 cm. long , then its perimeter = ..... cm. (Giza 19)

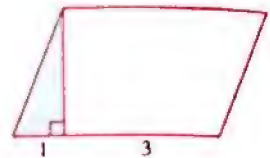
- (a)  $2\pi$     (b)  $5\pi$   
(c)  $\pi + 4$     (d)  $4\pi + 4$



35 In the opposite figure :

If the base of the parallelogram is divided by the ratio 1 : 3 , then the ratio between the area of the coloured triangle and the area of the parallelogram is .....

- (a) 1 : 3                      (b) 1 : 6                      (c) 1 : 8

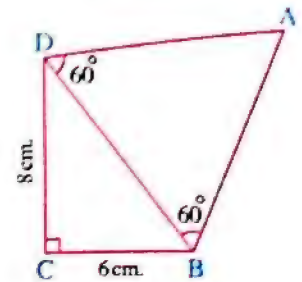


- (d) 1 : 9

36 In the opposite figure :

The perimeter of the figure = ..... cm.

- (a) 44    (b) 34  
(c) 24    (d) 14



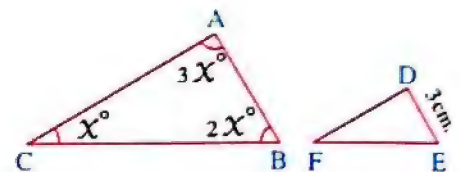
37 In the opposite figure :

If  $\triangle ABC \sim \triangle DEF$

,  $DE = 3$  cm.

, then  $EF =$  ..... cm.

- (a) 3    (b) 9    (c) 4



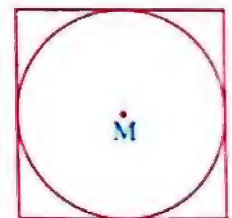
- (d) 6

38 In the opposite figure :

If the side length of the square = 10 cm.

, then the area of the circle = .....  $\text{cm}^2$

- (a)  $100\pi$     (b)  $25\pi$   
(c)  $50\pi$     (d)  $40\pi$



(Luxor 16)

39 In the opposite figure :

If  $A \in \overline{EF}$  ,  $B \in \overline{EF}$  ,  $m(\angle C) = 90^\circ$

, then  $x + y =$  .....  $^\circ$

- (a) 90    (b) 180  
(c) 270    (d) 360





**EL-MOASSER**

# In Mathematics Notebook

For **3<sup>rd</sup>** Prep.  
First Term

- Quizzes
- Final Revision
- Final Examinations



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By  
A group of supervisors

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(2 model examinations + model for the merge students)
  - 27 governorates' examinations.



## Second Trigonometry and Geometry

- 6 Quizzes.
- Final revision.
- Final examinations :
  - School book examinations  
(2 model examinations + model for the merge students)
  - 27 governorates' examinations.



**First**

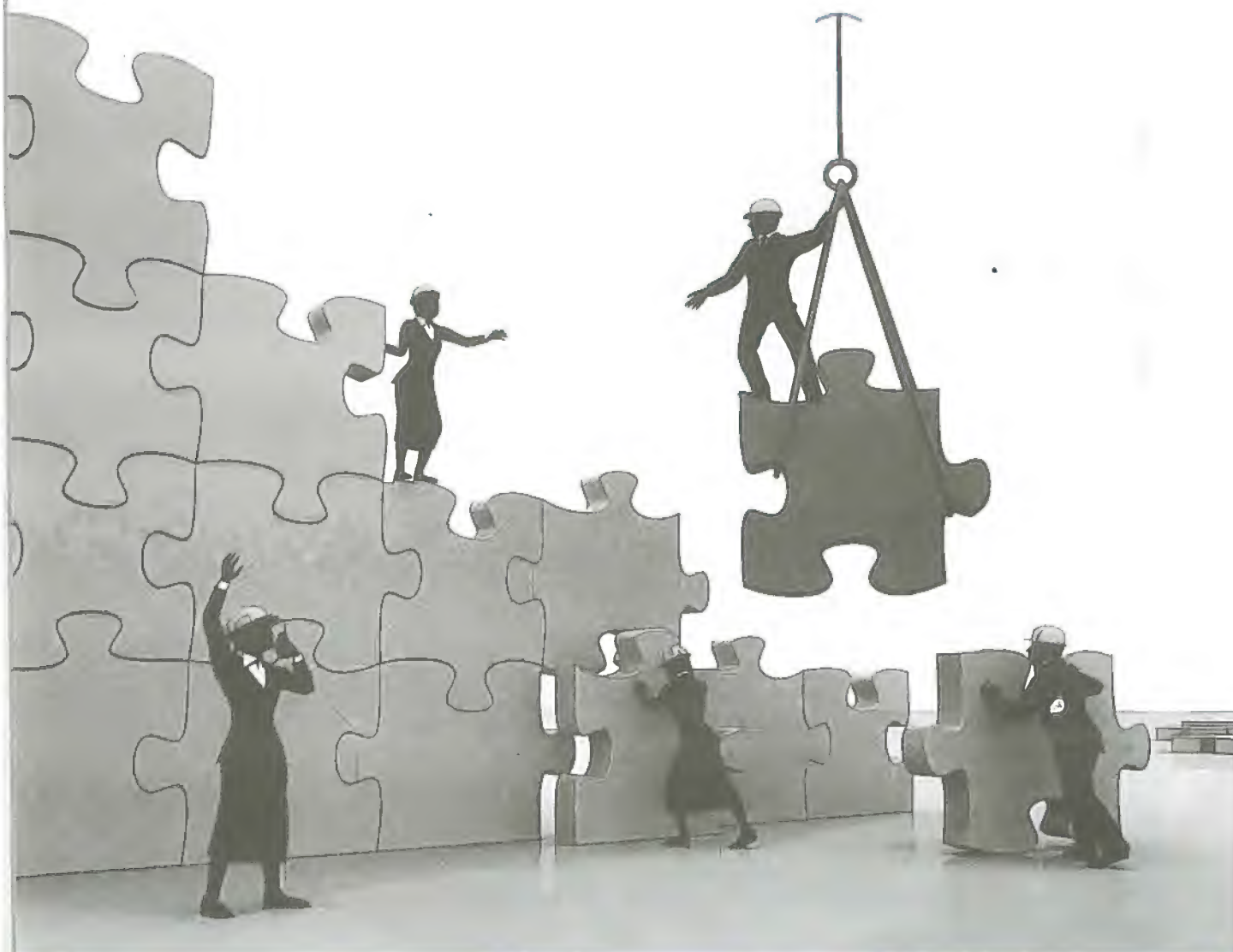
# Algebra and Statistics



- **10 quizzes.** ..... 5
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- **Final examinations :** ..... 27
  - School book examinations.  
(2 model examinations + model for the merge students)
  - 27 governorates' examinations.

# Quizzes

on Algebra and Statistics



## Quiz

1

on lesson 1 – unit 1

time  
15 min.

1 Choose the correct answer from those given :

- 1 If  $(2, 3) \in \{2, 5\} \times \{x, 6\}$ , then  $x = \dots\dots\dots$   
 (a) 6 (b) 5 (c) 3 (d) 2
- 2 If  $n(X^2) = 9$ ,  $n(X \times Y) = 6$ , then  $n(Y^2) = \dots\dots\dots$   
 (a) 3 (b) 2 (c) 9 (d) 4
- 3 If  $(x - 1, 13) = (8, y - 3)$ , then  $\sqrt{x + y} = \dots\dots\dots$   
 (a) 25 (b) 7 (c) 5 (d)  $\sqrt{5}$

2 [a] If  $X \times Y = \{(2, 2), (2, 5), (2, 7)\}$ , then find :

- 1 Y 2  $n(Y^2)$  3  $Y \times X$

[b] If  $X = \{1, 2, 6\}$ ,  $Y = \{2, 4, 5, 6\}$  and  $Z = \{4\}$ , then find :

- 1  $(X \cap Y) \times Z$  2  $(Y - Z) \times X$

## Quiz

2

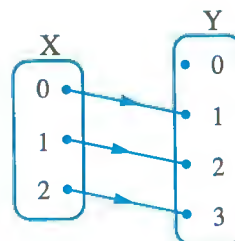
Till lesson 2 – unit 1

time  
15 min.

1 Choose the correct answer from those given :

- 1 The opposite figure represents a function from  $X$  to  $Y$ , its range =  $\dots\dots\dots$

- (a)  $\{0, 1, 2\}$  (b)  $\{1, 2, 3\}$   
 (c)  $\{0, 1, 2, 3\}$   
 (d)  $\{(0, 1), (1, 2), (2, 3)\}$



- 2 If  $(b + 3, 5)$  lies on y-axis, then  $b = \dots\dots\dots$

- (a) zero (b) 3 (c) -3 (d) 5

- 3 If  $X = \{1, 2\}$ , then  $n(X \times \emptyset) = \dots\dots\dots$

- (a) 2 (b)  $\emptyset$  (c) 0 (d)  $\{(1, 0), (2, 0)\}$

2 [a] If  $X = \{-2, -1, 0, 1, 2\}$  and  $R$  is a relation on  $X$ , where " $a R b$ " means " $a$  is the additive inverse of  $b$ " for each  $a \in X, b \in X$

- 1 Write  $R$  and represent it by an arrow diagram.

- 2 Is  $R$  a function? Why?

[b] If  $X = \{4, 6, 8, k\}$ ,  $Y = \{5, 4, 3, 2\}$  and  $R$  is a relation from  $X$  to  $Y$ , where " $a R b$ " means " $b = \frac{a}{2}$ " for each  $a \in X, b \in Y$

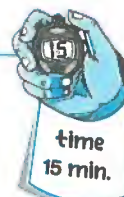
- 1 Find the value of  $k$  that makes  $R$  a function from  $X$  to  $Y$

- 2 Represent the function by a Cartesian diagram.

## Quiz

3

Till lesson 3 – unit 1



## 1 Choose the correct answer from those given :

1 If  $(4, a) \in$  the set of the function  $f$  where  $f(x) = x + 1$ , then  $a = \dots\dots\dots$

- (a) 3 (b) 4 (c) 5 (d) 6

2 If  $X \times Y = \{(2, 3), (2, 4), (2, 5)\}$ , then  $n(X^2) = \dots\dots\dots$

- (a)  $\{2\}$  (b) 4 (c) 1 (d)  $\{4\}$

3 The function  $f : f(x) = x^3 - (x^3 + 7)$  is a polynomial function of the  $\dots\dots\dots$  degree.

- (a) first (b) second (c) third (d) zero

2 [a] If the function  $f : \mathbb{Z} \longrightarrow \mathbb{Z}$  where  $f : x \longmapsto 2x - 1$ 

1 Find :  $f(1)$ ,  $f(-2)$ ,  $f(0)$

2 If  $f(x) = 9$ , then find the value of  $x$

[b] If the set of the function  $f = \{(0, 4), (1, 3), (2, 2), (3, 1)\}$

1 Write each of the domain and the range of the function  $f$

2 Write the rule of the function  $f$

## Quiz

4

Till lesson 4 – unit 1



## 1 Choose the correct answer from those given :

1 The linear function which is defined by the rule  $y = 1 - 2x$  is represented graphically by a straight line intersects  $x$ -axis at the point  $\dots\dots\dots$

- (a)  $(1, 0)$  (b)  $(0, 1)$  (c)  $(\frac{1}{2}, 0)$  (d)  $(0, \frac{1}{2})$

2 If  $f(x) = 5$ , then  $f(5) + f(-5) = \dots\dots\dots$

- (a) 0 (b) -10 (c) 10 (d)  $f(10)$

3 If  $f(x) = 2x + b$ ,  $f(3) = \text{zero}$ , then  $b = \dots\dots\dots$

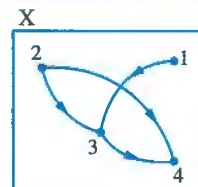
- (a) 0 (b) -6 (c) 6 (d) 3

## 2 [a] The opposite arrow diagram

shows the relation  $R$  on  $X$

, where  $X = \{1, 2, 3, 4\}$

Write  $R$ , then show if  $R$  is a function or not. Give reasons.



[b] Graph the function  $f : f(x) = 3 - 2x - x^2$  in  $\mathbb{R}$  taking  $x \in [-4, 2]$ , then find :

1 The maximum value or the minimum value of the function.

2 The equation of the axis of symmetry.

## Quiz

5

Till lesson 1 – unit 2



1 Choose the correct answer from those given :

1 If  $a, 3x, b$  and  $9x$  are proportional quantities, then  $\frac{a}{b} = \dots\dots\dots$ 

- (a) 9 (b) 3 (c) 6 (d)  $\frac{1}{3}$

2 If  $f(x) = 5$ , then  $f(-2x) = \dots\dots\dots$ 

- (a) -10 (b) 10 (c) -5 (d) 5

3 If  $4x^2 - 12xy + 9y^2 = 0$ ,  $x \in \mathbb{R}$ ,  $y \in \mathbb{R}$ , then  $\frac{x}{y} = \dots\dots\dots$ 

- (a)  $\frac{2}{3}$  (b)  $-\frac{2}{3}$  (c)  $\frac{3}{2}$  (d)  $-\frac{3}{2}$

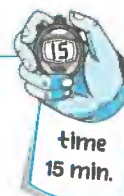
2 [a] If  $X = \{1, 2, 3\}$ ,  $Y = \{9, 1, 6, 3\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a = \frac{1}{3}b$ " for each  $a \in X$  and  $b \in Y$ , write  $R$  and represent it by an arrow diagram. Is  $R$  a function from  $X$  to  $Y$ ? and if " $3 R x$ ", then find:  $\sqrt{x}$

[b] If  $\frac{a}{b-a} = \frac{c}{d-c}$ , then prove that:  $a, b, c$  and  $d$  are proportional.

## Quiz

6

Till lesson 2 – unit 2



1 Choose the correct answer from those given :

1 If  $\frac{a}{b} = \frac{c}{d} = \frac{2}{3}$ , then  $\frac{a+2c}{b+2d} = \dots\dots\dots$ 

- (a)  $\frac{4}{9}$  (b)  $\frac{2}{3}$  (c)  $\frac{3}{2}$  (d)  $\frac{1}{3}$

2 If  $(a, -a)$  is an element of  $R$  of the function  $f$  where  $f(x) = 2x - 3$ , then  $a = \dots\dots\dots$ 

- (a) 1 (b) 2 (c) 3 (d) -3

3  $\frac{a}{2} = \frac{c}{3} = \frac{e}{4} = \frac{2a+3c-2e}{\dots\dots\dots}$ 

- (a) -5 (b) 9 (c) 5 (d) 3

2 [a] If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ , prove that:  $\frac{7a+5c-e}{7b+5d-f} = \frac{c}{d}$ 

[b] Represent graphically the function  $f: f(x) = (x-2)^2$  taking  $x \in [-1, 5]$ , from the graph deduce :

- 1 The maximum or minimum value of the function.  
2 The equation of the axis of symmetry.

## Quiz

7

Till lesson 3 – unit 2



1 Choose the correct answer from those given :

1 If  $x, 4, 6, 2$  are four proportional quantities , then  $x = \dots\dots\dots$ 

(a) 12

(b)  $\frac{4}{3}$ (c)  $\frac{3}{4}$ (d)  $\frac{1}{12}$ 2 The proportional mean between  $(x - 3)$  ,  $(x + 3)$  is  $\dots\dots\dots$ (a)  $\sqrt{x+3}$ (b)  $x^2 - 9$ (c)  $\pm\sqrt{x^2 - 9}$ (d)  $\pm(x^2 - 9)$ 3 If  $\frac{a}{b} = \frac{b}{c} = \frac{c}{5} = 2$  , then  $a = \dots\dots\dots$ (a)  $5 \times 2^2$ 

(b) 40

(c) -1

(d)  $2 \times 5^3$ 2 [a] If  $a, b, c, d$  are in continued proportion , prove that :  $\frac{a^3 + b^3}{b^3 + c^3} = \frac{a^2}{bc}$ [b] If  $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3x}$  , find the value of :  $x$ 

## Quiz

8

Till lesson 4 – unit 2



1 Choose the correct answer from those given :

1 If  $y \propto \frac{1}{x^2}$  , then  $y$  varies  $\dots\dots\dots$ (a) directly as  $x$ (b) directly as  $x^3$ (c) directly as  $x^2$ (d) inversely as  $x^2$ 2 If  $\frac{a}{3} = \frac{b}{4}$  , then  $4a - 3b + 5 = \dots\dots\dots$ 

(a) 4

(b) 3

(c) 5

(d) 7

3 The function  $f : f(x) = x(2x - 1)^2 + 3$  is a polynomial of the  $\dots\dots\dots$  degree.

(a) first

(b) second

(c) third

(d) fourth

2 [a] If  $\frac{a^2 + b^2}{b^2} = \frac{b^2 + c^2}{c^2}$  , then prove that :  $b$  is the middle proportion between  $a$  and  $c$ 

[b] If the value of speed  $v$  of water to pass through a hose nozzle inversely changes with the square of the hose nozzle radius length  $r$  and  $v = 5$  cm./sec.  
when  $r = 3$  cm. , find  $v$  when  $r = 3\frac{3}{4}$  cm.

## Quiz

9

Till lesson 1 – unit 3



## 1 Choose the correct answer from those given :

- 1 Choosing a sample from the society's layers in statistics is called a ..... sample.

(a) homogeneous (b) layer (c) intentional (d) cluster

- 2 The relation that represents a direct variation between  $x$  and  $y$  is .....

(a)  $xy = 5$  (b)  $y = x - 4$  (c)  $\frac{x}{3} = \frac{4}{y}$  (d)  $\frac{x}{7} = \frac{y}{5}$

- 3 If  $(x - 5, x + 3)$  lies in the second quadrant, then  $x =$  .....

(a) 3 (b) 5 (c) 7 (d) 9

- 2 [a] One of the car factories produced 3000 cars of model (A), 1000 cars of model (B), 2000 cars of model (C) and 4000 cars of model (D). If we want to draw a layer sample of size 500 cars to represent each layer due to its size, calculate the number of individuals of each layer in the sample.

[b] If  $4x^2 + 9y^2 = 12xy$ , then prove that :  $x$  varies directly as  $y$

## Quiz

10

Till lesson 2 – unit 3



## 1 Choose the correct answer from those given :

- 1 If  $X = \{3\}$ , then  $X^2 =$  .....

(a)  $\{3\}$  (b)  $\{9\}$  (c)  $\{(3, 9)\}$  (d)  $\{(3, 3)\}$

- 2 If  $\sum (x - \bar{x})^2 = 36$  of a set of values and the number of these values equals 9, then  $\sigma =$  .....

(a) 2 (b) 4 (c) 18 (d) 27

- 3 From the measures of dispersion is the .....

(a) mean. (b) median.  
(c) mode. (d) standard deviation.

- 2 The following is the frequency distribution of the number of damaged units which were found in 100 boxes of the produced units :

Number of damaged units	zero	1	2	3	4	5
Number of boxes	3	16	17	25	20	19

Find the standard deviation of the damaged units.

# Final Revision

of Algebra and Statistics



## Revision for the important rules of algebra and statistics

### First Algebra

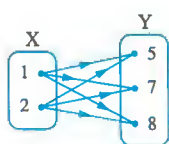
#### Remember The cartesian product of two finite sets and representing it

If  $X = \{1, 2\}$  ,  $Y = \{5, 7, 8\}$  , then :

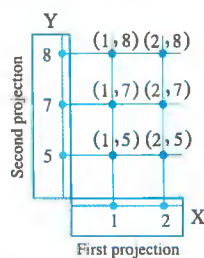
$X \times Y$

is the set of all ordered pairs whose first projection of each of them belongs to  $X$  and the second projection of each of them belongs to  $Y$

$$\text{i.e. } X \times Y = \{(1, 5), (1, 7), (1, 8), (2, 5), (2, 7), (2, 8)\}$$



The arrow diagram

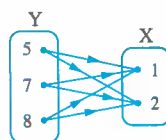


The graphical diagram  
(The Cartesian diagram)

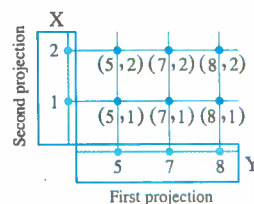
$Y \times X$

is the set of all ordered pairs whose first projection of each of them belongs to  $Y$  and the second projection of each of them belongs to  $X$

$$\text{i.e. } Y \times X = \{(5, 1), (5, 2), (7, 1), (7, 2), (8, 1), (8, 2)\}$$



The arrow diagram

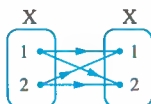


The graphical diagram  
(The Cartesian diagram)

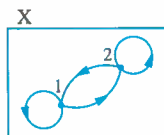
$X \times X$

is the set of all ordered pairs whose first projections and second projections belong to  $X$

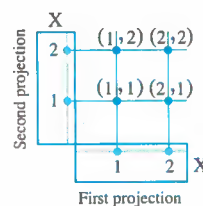
$$\text{i.e. } X \times X = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$$



The arrow diagram



The arrow diagram



The graphical diagram  
(The Cartesian diagram)

#### Remarks

- (1)  $X \times Y \neq Y \times X$  , where  $X \neq Y$
- (2)  $n(X \times Y) = n(Y \times X) = n(X) \times n(Y)$  where  $n$  is the number of elements
- (3)  $n(X \times X) = n(X^2) = [n(X)]^2$
- (4)  $X \times \emptyset = \emptyset \times X = \emptyset$

### Remember The relation and its representing

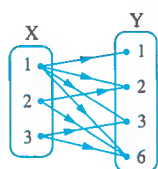
- The relation from the set  $X$  to the set  $Y$  is a connecting joining some or all the elements of  $X$  with some or all the elements of  $Y$
- If  $R$  is a relation from the set  $X$  to the set  $Y$ , then :
  - 1  $R$  is a set of ordered pairs where the first projection of each belongs to  $X$  and the second projection belongs to  $Y$
  - 2  $R \subset X \times Y$
  - 3 The relation can be represented by an arrow diagram or by a Cartesian diagram (graphically)
- If  $R$  is a relation from  $X$  to  $X$ , then  $R$  is a relation on  $X$  and  $R \subset X \times X$

### Example

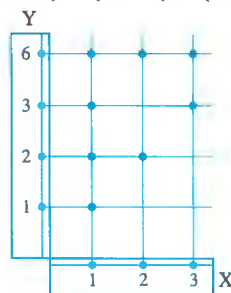
If  $X = \{1, 2, 3\}$ ,  $Y = \{1, 2, 3, 6\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a$  is a factor of  $b$ " for each  $a \in X, b \in Y$ , then write  $R$  and represent it by an arrow diagram and a Cartesian diagram.

### Solution

$$R = \{(1, 1), (1, 2), (1, 3), (1, 6), (2, 2), (2, 6), (3, 3), (3, 6)\}$$



The arrow diagram



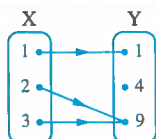
The Cartesian diagram

### Remember The function

- A relation from  $X$  to  $Y$  is said to be a function if :
  - 1 Each element of the set  $X$  appears only once as a first projection in one of the ordered pairs of the relation.
  - 2 Each element of the set  $X$  has one and only one arrow going out of it to one element of  $Y$  in the arrow diagram which represents the relation.
  - 3 Each vertical line has one and only one point lying on it of the points which represent the relation, in the Cartesian diagram which represents the relation.
- If  $f$  is a function from the set  $X$  to the set  $Y$  is written as  $f : X \longrightarrow Y$ , then :
  - 1  $X$  is called the domain of the function  $f$
  - 2  $Y$  is called the codomain of the function  $f$
  - 3 The set of images of the elements of the set  $X$  by the function  $f$  is called the range of the function  $f$  which is a subset of the codomain  $Y$

**For example :**

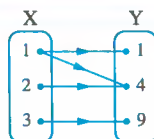
If  $X = \{1, 2, 3\}$ ,  $Y = \{1, 4, 9\}$ , then the following diagrams show some of the relations from  $X$  to  $Y$  and we note which of the following relations represent a function from  $X$  to  $Y$  and which does not represent :



**Note :** Going out only one arrow from each element of the elements of  $X$

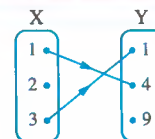
**Then :** The relation is a function from  $X$  to  $Y$

- The domain =  $\{1, 2, 3\}$
- The range =  $\{1, 9\}$



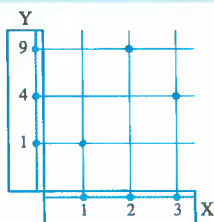
**Note :** Going out two arrows from the element 1 in  $X$

**Then :** The relation is not a function from  $X$  to  $Y$



**Note :** There are not arrows going out from the element 2 in  $X$

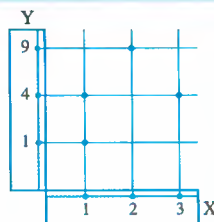
**Then :** The relation is not a function from  $X$  to  $Y$



**Note :** Each vertical line has only one point lying on it

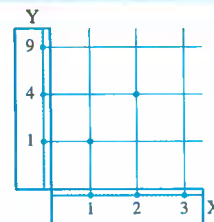
**Then :** The relation is a function from  $X$  to  $Y$

- The domain =  $\{1, 2, 3\}$
- The range =  $\{1, 4, 9\}$



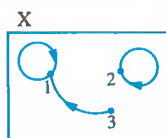
**Note :** There are two points lying on the vertical line at the element 1 in  $X$

**Then :** The relation is not a function from  $X$  to  $Y$



**Note :** There is not a point lying on the vertical line at the element 3 in  $X$

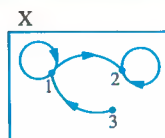
**Then :** The relation is not a function from  $X$  to  $Y$



**Note :** Going out only one arrow from each element of the elements of  $X$

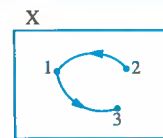
**Then :** The relation is a function on  $X$

- The domain =  $\{1, 2, 3\}$
- The range =  $\{1, 2\}$



**Note :** Going out two arrows from the element 1 in  $X$

**Then :** The relation is not a function on  $X$



**Note :** There are not arrows going out from the element 3 in  $X$

**Then :** The relation is not a function on  $X$

### Remember The polynomial functions

The polynomial function is a function whose rule is a term or an algebraic expression in condition that the following should be identified :

- 1 Each of the domain and the codomain of the function is the set of real numbers.
- 2 The power (The index) of the variable  $x$  in any of its terms is a natural number with noticing that : the degree of the function is the highest power of the variable  $x$

**For example :**

- The function  $f : f(x) = 3$  is a polynomial function of zero degree.
- The function  $f : f(x) = 2x + 1$  is a polynomial function of the first degree.
- The function  $f : f(x) = x^3 - 5x^2 + 1$  is a polynomial function of the third degree.

**While :**

The function  $f : f(x) = \frac{1}{x^2} + x^2$  is not a polynomial function because :  $\frac{1}{x^2} = x^{-2}$

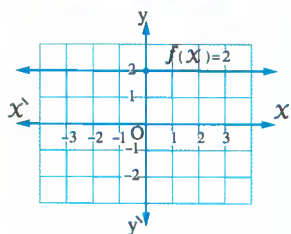
*i.e.* The index of the symbol  $x$  is not a natural number.

### Remember The graphical representation of the polynomial function

#### 1 The constant function

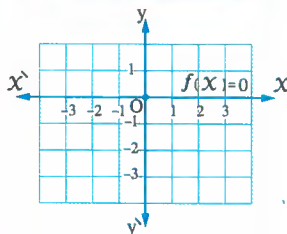
The function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = b$ ,  $b \in \mathbb{R}$  is represented by a straight line parallel to  $x$ -axis and intersects  $y$ -axis at the point  $(0, b)$

$$f : f(x) = 2$$



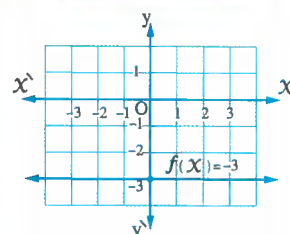
The straight line is above  $x$ -axis and passes through the point  $(0, 2)$   
(is of zero degree)

$$f : f(x) = 0$$



The straight line is coincident with  $x$ -axis and passes through the point  $(0, 0)$   
(has not degree)

$$f : f(x) = -3$$



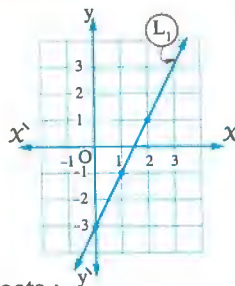
The straight line is below  $x$ -axis and passes through the point  $(0, -3)$   
(is of zero degree)

## 2 The linear function

The function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = ax + b$ ,  $a \in \mathbb{R} - \{0\}$ ,  $b \in \mathbb{R}$  is called a linear function (function of the first degree) and is represented by a straight line intersecting y-axis at  $(0, b)$  and x-axis at  $(-\frac{b}{a}, 0)$

$$f : f(x) = 2x - 3$$

$x$	0	1	2
$f(x)$	-3	-1	1

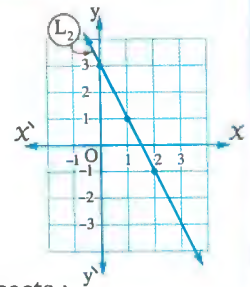


The straight line  $L_1$  intersects :

- x-axis at  $(1\frac{1}{2}, 0)$
- y-axis at  $(0, -3)$

$$f : f(x) = 3 - 2x$$

$x$	0	1	2
$f(x)$	3	1	-1



The straight line  $L_2$  intersects :

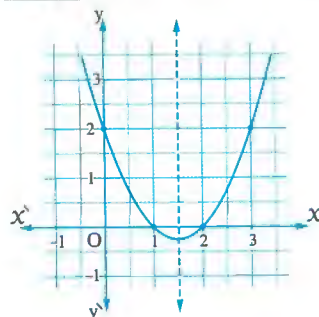
- x-axis at  $(1\frac{1}{2}, 0)$
- y-axis at  $(0, 3)$

## 3 The quadratic function

The function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = ax^2 + bx + c$ ,  $a, b$  and  $c \in \mathbb{R}$ ,  $a \neq 0$  is called a quadratic function and it is a polynomial function of the second degree and it is represented by a curve whose vertex is  $(-\frac{b}{2a}, f(-\frac{b}{2a}))$

$$f : f(x) = x^2 - 3x + 2, x \in [0, 3]$$

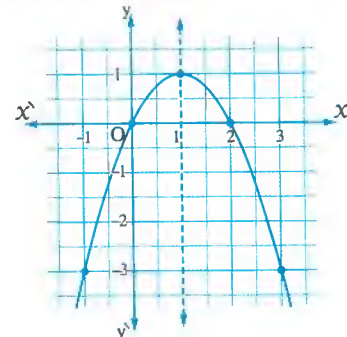
$x$	0	1	2	3
$f(x)$	2	0	0	2



- The vertex of the curve =  $(\frac{3}{2}, -\frac{1}{4})$
- The minimum value of the function =  $-\frac{1}{4}$
- The equation of line of symmetry :  $x = \frac{3}{2}$

$$f : f(x) = 2x - x^2, x \in [-1, 3]$$

$x$	-1	0	1	2	3
$f(x)$	-3	0	1	0	-3



- The vertex of the curve =  $(1, 1)$
- The maximum value of the function = 1
- The equation of line of symmetry :  $x = 1$

**Remember** The ratio and its properties

- The ratio between the two real numbers  $a$  and  $b$  is written as  $a : b$  or  $\frac{a}{b}$  and  $a$  is called the antecedent of the ratio,  $b$  is called the consequent and  $a, b$  are called the two terms of the ratio.
- The value of the ratio **does not change** if each of its terms is multiplied or divided by the same non-zero real number.
- The value of the ratio **changes** if we add or subtract (to or from) each of its two terms the same non-zero real number.
- If the ratio between two numbers is  $a : b$ , then :
 

The first number =  $am$

The second number =  $bm$ ,  $m \neq 0$

**Example**

Two numbers, their sum is 28 and the ratio between them is  $3 : 4$  what are the two numbers ?

**Solution**

Let the two numbers be  $3m, 4m$   $\therefore 3m + 4m = 28$   $\therefore 7m = 28$   $\therefore m = \frac{28}{7} = 4$   
 $\therefore$  The two numbers are :  $3 \times 4$  and  $4 \times 4$  *i.e.* 12 and 16

**Remember** The proportion

- The proportion is the equality of two ratios or more.
- If  $\frac{a}{b} = \frac{c}{d}$ , then  $a, b, c$  and  $d$  are proportional quantities.
- If  $a, b, c$  and  $d$  are proportional quantities, then  $\frac{a}{b} = \frac{c}{d}$

**Remember** The properties of the proportion**Property 1**

If  $\frac{a}{b} = \frac{c}{d}$ , then  $a \times d = b \times c$

*i.e.* the product of the extremes = the product of the means.

**Example**

Find the fourth proportional of the quantities : 3, 4 and 27

**Solution**

Let the fourth proportional be  $x$   $\therefore$  The quantities : 3, 4, 27 and  $x$  are proportional

$$\therefore \frac{3}{4} = \frac{27}{x} \quad \therefore 3 \times x = 4 \times 27 \quad \therefore x = \frac{4 \times 27}{3} = 36 \quad \therefore \text{The fourth proportional} = 36$$

**Property 2**

If  $a \times d = b \times c$ , then  $\frac{a}{b} = \frac{c}{d}$

Also, each of the following proportions is correct:  $\frac{a}{c} = \frac{b}{d}$ ,  $\frac{d}{b} = \frac{c}{a}$ ,  $\frac{b}{a} = \frac{d}{c}$

**Example**

If  $\frac{x+3y}{2x-y} = \frac{4}{3}$ , then find the ratio  $x : y$

**Solution**

$$\therefore \frac{x+3y}{2x-y} = \frac{4}{3}$$

$$\therefore 13y = 5x$$

$$\therefore 3(x+3y) = 4(2x-y) \quad \therefore 3x+9y = 8x-4y$$

$$\therefore x : y = 13 : 5$$

**Property 3**

If  $\frac{a}{b} = \frac{c}{d}$ , then  $\frac{a}{c} = \frac{b}{d}$

i.e.  $\frac{\text{The antecedent of the first ratio}}{\text{The antecedent of the second ratio}} = \frac{\text{The consequent of the first ratio}}{\text{The consequent of the second ratio}}$

**For example:** If  $\frac{a}{4} = \frac{b}{3}$ , then  $\frac{a}{b} = \frac{4}{3}$  or  $\frac{b}{a} = \frac{3}{4}$

**Property 4**

If  $\frac{a}{b} = \frac{c}{d}$ , then  $a = cm$ ,  $b = dm$  where  $m$  is a constant  $\neq 0$

**Example**

If  $a : b = 3 : 5$ , then find the ratio  $20a - 7b : 15a + b$

**Solution**

$$\therefore \frac{a}{b} = \frac{3}{5}$$

$$\therefore a = 3m, b = 5m \text{ where } m \neq 0$$

Substituting by  $a$  and  $b$  in terms of  $m$ :

$$\therefore \frac{20a - 7b}{15a + b} = \frac{60m - 35m}{45m + 5m} = \frac{25m}{50m} = \frac{1}{2}$$

**Remark**

If  $a, b, c$  and  $d$  are proportional quantities and we assume that :  $\frac{a}{b} = \frac{c}{d} = m$ , then  $a = bm, c = dm$

**For example :** If  $\frac{a}{b} = \frac{c}{d} = \frac{3}{4}$ , then  $a = \frac{3}{4}b, c = \frac{3}{4}d$

• **Generally :** If  $a, b, c, d, e, f, \dots$  are proportional quantities and we assume that :  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots = m$ , then  $a = bm, c = dm, e = fm, \dots$

**Example**

If  $a, b, c$  and  $d$  are proportional quantities, **prove that :**

$$1 \quad \frac{2a+3c}{7a-5c} = \frac{2b+3d}{7b-5d}$$

$$2 \quad \frac{a+c}{b+d} = \frac{a^2+c^2}{ab+cd}$$

**Solution**

Let  $\frac{a}{b} = \frac{c}{d} = m \quad \therefore a = bm, c = dm$

$$1 \quad \text{L.H.S.} = \frac{2bm+3dm}{7bm-5dm} = \frac{m(2b+3d)}{m(7b-5d)} = \frac{2b+3d}{7b-5d} = \text{R.H.S}$$

$$2 \quad \therefore \frac{a+c}{b+d} = \frac{bm+dm}{b+d} = \frac{m(b+d)}{(b+d)} = m \quad (1)$$

$$\therefore \frac{a^2+c^2}{ab+cd} = \frac{(bm)^2+(dm)^2}{bm \times b + dm \times d} = \frac{b^2m^2+d^2m^2}{b^2m+d^2m} = \frac{m^2(b^2+d^2)}{m(b^2+d^2)} = m \quad (2)$$

From (1) and (2), we deduce that :  $\frac{a+c}{b+d} = \frac{a^2+c^2}{ab+cd}$

**Property 5**

If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$  and  $m_1, m_2, m_3, \dots$  are non-zero real numbers,

then  $\frac{m_1a+m_2c+m_3e+\dots}{m_1b+m_2d+m_3f+\dots}$  = one of the given ratios.

**Example**

If :  $\frac{a+3b}{x+5y} = \frac{3b+5c}{5y+7z} = \frac{5c+a}{7z+x}$ , **prove that :**  $\frac{a}{3b} = \frac{x}{5y}$

**Solution**

Multiplying the two terms of 2<sup>nd</sup> ratio by  $(-1)$  and adding the antecedents and consequents

of the three ratios :  $\therefore \frac{a+3b-3b-5c+5c+a}{x+5y-5y-7z+7z+x} = \frac{2a}{2x} = \frac{a}{x}$  = one of the given ratios. (1)

Multiplying the two terms of 3<sup>rd</sup> ratio by  $(-1)$  and adding the antecedents and consequents

of the three ratios :  $\therefore \frac{a+3b+3b+5c-5c-a}{x+5y+5y+7z-7z-x} = \frac{6b}{10y} = \frac{3b}{5y}$  = one of the given ratios (2)

From (1) and (2), we deduce that :  $\frac{a}{x} = \frac{3b}{5y} \quad \therefore \frac{a}{3b} = \frac{x}{5y}$

**Remember** The continued proportion

- The quantities  $a$ ,  $b$  and  $c$  are said to be in continued proportion if  $\frac{a}{b} = \frac{b}{c}$

$a$  is called the **first proportion**,  $c$  is called the **third proportion** and

$b$  is called the **middle proportion (proportional mean)**

$$\bullet \because \frac{a}{b} = \frac{b}{c} \qquad \therefore b^2 = ac \qquad \therefore b = \pm \sqrt{ac}$$

*i.e.*

The middle proportion between two quantities  $= \pm \sqrt{\text{the product of the two quantities}}$

**Notice that :**

The two quantities  $a$  and  $c$  should be either positive together or negative together.

$$\bullet \text{ If } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m, \text{ then } \begin{cases} c = dm \\ b = dm^2 \\ a = dm^3 \end{cases}$$

**Example**

If  $a$ ,  $b$ ,  $c$  and  $d$  are in continued proportion, then prove that :  $\frac{2a+3c}{2b+3d} = \frac{a-c}{b-d}$

**Solution**

$\because a, b, c, d$  are in continued proportion

$$\therefore \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = dm, b = dm^2, a = dm^3$$

$$\therefore \frac{2a+3c}{2b+3d} = \frac{2dm^3+3dm}{2dm^2+3d} = \frac{dm(2m^2+3)}{d(2m^2+3)} = m \quad (1)$$

$$\therefore \frac{a-c}{b-d} = \frac{dm^3-dm}{dm^2-d} = \frac{dm(m^2-1)}{d(m^2-1)} = m \quad (2)$$

From (1) and (2), we deduce that :  $\frac{2a+3c}{2b+3d} = \frac{a-c}{b-d}$

## Remember The direct variation and inverse variation

### Direct variation

- If  $y$  varies directly as  $x$  and is written as  $y \propto x$ , then :
  - 1  $y = m x$  (i.e.  $\frac{y}{x} = m$ )  
where  $m$  is a constant  $\neq 0$
  - 2  $\frac{y_1}{y_2} = \frac{x_1}{x_2}$
  - 3 The relation between  $x$  and  $y$  is represented graphically by a straight line passing through the origin point.
- To prove that  $y \propto x$ ,  
we prove that :  $y = m x$   
where  $m$  is a constant  $\neq 0$

**For example :**

If  $y = 5 x$ , then  $y \propto x$

### Inverse variation

- If  $y$  varies inversely as  $x$  and is written as  $y \propto \frac{1}{x}$ , then :
  - 1  $y = \frac{m}{x}$  (i.e.  $xy = m$ )  
where  $m$  is a constant  $\neq 0$
  - 2  $\frac{y_1}{y_2} = \frac{x_2}{x_1}$
  - 3 The relation between  $x$  and  $y$  is not a linear relation.
- To prove that  $y \propto \frac{1}{x}$ ,  
we prove that :  $xy = m$   
where  $m$  is a constant  $\neq 0$

**For example :**

If  $y = \frac{7}{x}$ , then  $xy = 7$ , and then  $y \propto \frac{1}{x}$

### Example on direct variation

- 1 If  $a \propto b$ ,  $a = 5$  when  $b = 2$ ,  
find :  $a$  when  $b = 3$
- 2 If  $a^2 + 4b^2 = 4ab$ , prove that :  $a \propto b$

#### Solution

- 1  $\because a \propto b \quad \therefore \frac{a_1}{a_2} = \frac{b_1}{b_2}$   
 $\therefore \frac{5}{a_2} = \frac{2}{3} \quad \therefore a_2 = 7.5$
- 2  $\because a^2 + 4b^2 = 4ab \quad \therefore a^2 - 4ab + 4b^2 = 0$   
 $\therefore (a - 2b)^2 = 0 \quad \therefore a - 2b = 0$   
 $\therefore a = 2b \quad \therefore a \propto b$

### Example on inverse variation

If  $x$  and  $y$  are two real variables where :  
 $x^2 y^2 + 25 = 10xy$   
 , prove that :  
 $x$  varies inversely as  $y$

#### Solution

$$\begin{aligned} \therefore x^2 y^2 - 10xy + 25 &= 0 \\ \therefore (xy - 5)^2 &= 0 & \therefore xy - 5 &= 0 \\ \therefore xy &= 5 & \therefore x &\propto \frac{1}{y} \end{aligned}$$

## Second Statistics

### Remember The resources of collecting data

#### Primary resources (field resources)

- These are the resources from which we get data directly.

##### Examples :

- \* Questionnaires and survey.
- \* Observing and measuring.
- \* The personal interview.

#### Secondary resources (historical resources)

- These are the resources from which we get data that previously collected.

##### Examples :

- \* Central agency for public mobilization and statistics.
- \* Mass-media.
- \* Internet.

### Remember The methods of collecting data

#### Method of mass population

##### Definition

It is based on collecting the data related to the phenomenon under study from all individuals of the statistical society.

##### Usages

- Elections
- Census
- Setting up a data base of all employees in an organization

##### Advantages

- Accuracy
- Inclusiveness
- Representing all the society individuals

##### Disadvantages

- Sometimes it needs long time , great effort and a great cost.

#### Method of sample

It is based on collecting the data related to the phenomenon under study from a representative sample of the society (Choosing a sample represented to the whole society)

- A sample of a patient's blood to make some clinical check up.
- A sample of some products of a factory of find out if it matches the standard specifications.

- Saving time , effort and money.
- It is the only method for collecting data about large unlimited societies.
- It is the only method for collecting data about some limited societies in which mass population method leads to a great loss in it.

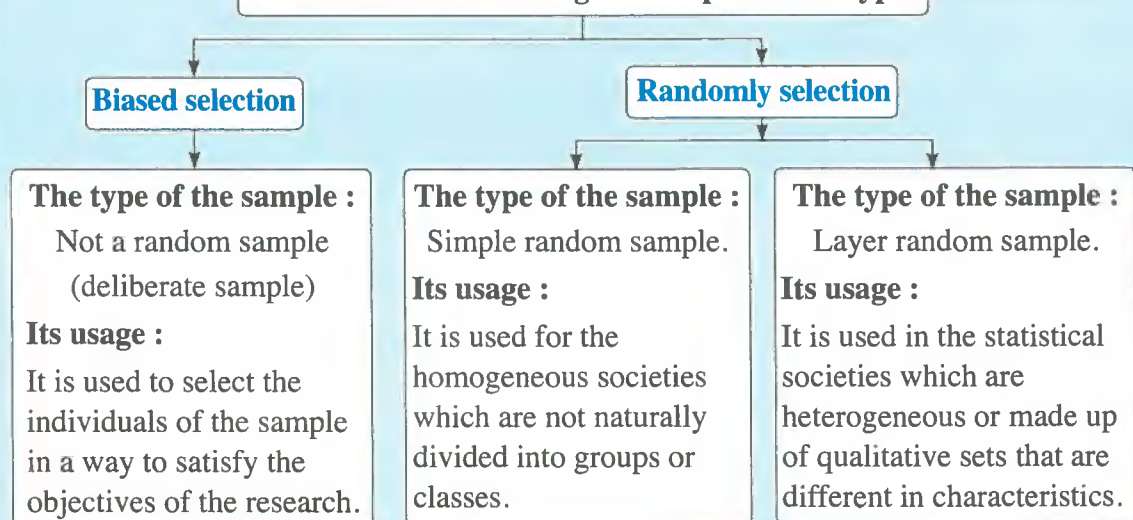
- The results sometimes are not accurate specially if the sample doesn't represent the statistical society authentically.

## Remember The concept of the sample and the methods of collecting it

### The sample :

It is a small part from a large society that looks like the society and represents it well.

### The methods of collecting the sample and its types



The number of individuals of the layer in the sample

$$= \frac{\text{the total number of individuals in the layer}}{\text{the total number of individuals in the society}} \times \text{the number of individuals of the sample}$$

«approximating the result to the nearest unit»

### Example

At a faculty , there are 4 000 university students in the first grade , 3 000 in the second grade , 2 000 in the third grade and 1 000 in the fourth grade. If we want to draw a layer sample of 500 students , where each layer is represented in this sample according to its size.

, calculate the number of students in each layer in the sample.

### Solution

The total number of students = 4 000 + 3 000 + 2 000 + 1 000 = 10 000 students.

The number of the individuals of the first layer in the sample =  $\frac{4\,000}{10\,000} \times 500 = 200$  students.

The number of the individuals of the second layer in the sample =  $\frac{3\,000}{10\,000} \times 500 = 150$  students.

The number of the individuals of the third layer in the sample =  $\frac{2\,000}{10\,000} \times 500 = 100$  students.

The number of the individuals of the fourth layer in the sample =  $\frac{1\,000}{10\,000} \times 500 = 50$  students.

**Remember** The dispersion and its measurements**The dispersion :**

It is a measure that expresses how much the sets are homogeneous.

**Dispersion measurements****1 The range (the simplest measure of dispersion)**

**It is the difference between the greatest value and the smallest value in the set.**

*i.e.* The range = the greatest value – the smallest value

**For example :**

The values of set X are : 55 , 53 , 57 , 56 and 54 , then the range =  $57 - 53 = 4$

The values of set Y are : 67 , 73 , 41 , 60 and 34 , then the range =  $73 - 34 = 39$

So the set Y is more divergent than the set X

**2 Standard deviation**

It is the most important , common and accurate measure of dispersion. We can calculate it by calculating the positive square root of the average of squares of deviations of the values from their mean.

**The standard deviation of  
a set of values**

$$\text{The standard deviation } \sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

Where :

$x$  denotes a value of the values ,

$\bar{x}$  denotes the mean of the values ,

$n$  denotes the number of the values ,

$\sum$  denotes the summation operation.

**The standard deviation of  
a frequency distribution**

$$\text{The standard deviation } \sigma = \sqrt{\frac{\sum (x - \bar{x})^2 k}{\sum k}}$$

Where :

$x$  represents the value or the centre of the set ,

$k$  represents the frequency of the value or the set ,

$\sum k$  is the sum of frequencies

and  $\bar{x}$  (the mean) =  $\frac{\sum (x \times k)}{\sum k}$

### Example on the standard deviation of a set of values

Calculate the standard deviation of the values : 55 , 53 , 57 , 56 and 54

#### Solution

① We find the mean of the values ( $\bar{x}$ ) =  $\frac{\sum x}{n}$   
 $= \frac{55 + 53 + 57 + 56 + 54}{5} = 55$

② We form the opposite table.

③ We calculate standard deviation by substituting in the law :

$$\text{The standard deviation } (\sigma) = \sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\frac{10}{5}} = \sqrt{2} \approx 1.4$$

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
55	$55 - 55 = 0$	0
53	$53 - 55 = -2$	4
57	$57 - 55 = 2$	4
56	$56 - 55 = 1$	1
54	$54 - 55 = -1$	1
<b>Total</b>		<b>10</b>

### Example on the standard deviation of a simple frequency distribution

The following table shows the distribution of wages of 20 persons in pounds :

The wage	20	25	30	35	40	45	Total
Number of persons	2	3	5	5	1	4	20

Find the standard deviation of the wages.

#### Solution

① We find the mean of the wages ( $\bar{x}$ )  
 by using the opposite table :

$$\therefore \text{The mean } (\bar{x}) = \frac{\sum (x \times k)}{\sum k}$$

$$= \frac{660}{20} = 33 \text{ pounds.}$$

The wage ( $x$ )	Number of persons ( $k$ )	$x \times k$
20	2	40
25	3	75
30	5	150
35	5	175
40	1	40
45	4	180
<b>Total</b>	<b>20</b>	<b>660</b>

② We form the opposite table :

$x$	$k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
20	2	$20 - 33 = -13$	169	338
25	3	$25 - 33 = -8$	64	192
30	5	$30 - 33 = -3$	9	45
35	5	$35 - 33 = 2$	4	20
40	1	$40 - 33 = 7$	49	49
45	4	$45 - 33 = 12$	144	576
<b>Total</b>	<b>20</b>			<b>1220</b>

③ We calculate the standard deviation from the law :

$$\text{The standard deviation } (\sigma) = \sqrt{\frac{\sum (x - \bar{x})^2 \times k}{\sum k}} = \sqrt{\frac{1220}{20}} = \sqrt{61} \approx 7.8 \text{ pounds.}$$

**Example on the standard deviation of a frequency distribution of sets**

The following is the frequency distribution of weekly incentives of 100 workers in a factory :

Incentives in pounds	35 –	45 –	55 –	65 –	75 –	85 –	Total
Number of workers	10	14	20	28	20	8	100

Find the standard deviation of this distribution.

**Solution**

1 We find the mean ( $\bar{X}$ )

by using the following table :

Sets	Centres of sets ( $X$ )	Frequency ( $k$ )	$X \times k$
35 –	40	10	400
45 –	50	14	700
55 –	60	20	1200
65 –	70	28	1960
75 –	80	20	1600
85 –	90	8	720
<b>Total</b>		<b>100</b>	<b>6580</b>

$$\therefore \text{The mean } (\bar{X}) = \frac{\sum (X \times k)}{\sum k} = \frac{6580}{100} = 65.8 \text{ pounds.}$$

2 We form the following table :

$X$	$k$	$X - \bar{X}$	$(X - \bar{X})^2$	$(X - \bar{X})^2 \times k$
40	10	$40 - 65.8 = -25.8$	665.64	6656.4
50	14	$50 - 65.8 = -15.8$	249.64	3494.96
60	20	$60 - 65.8 = -5.8$	33.64	672.8
70	28	$70 - 65.8 = 4.2$	17.64	493.92
80	20	$80 - 65.8 = 14.2$	201.64	4032.8
90	8	$90 - 65.8 = 24.2$	585.64	4685.12
<b>Total</b>	<b>100</b>			<b>20036</b>

3 We calculate the standard deviation by using the law :

$$\text{The standard deviation } (\sigma) = \sqrt{\frac{\sum (X - \bar{X})^2 \times k}{\sum k}} = \sqrt{\frac{20036}{100}} \approx 14.15 \text{ pounds.}$$

**Notice that**

- The values which are more homogeneous have less dispersion and their standard deviation is small.
- If the standard deviation equals zero that means the all values are equal , it is the perfect homogeneous case (the vanished dispersion)

# Final Examinations 2020

on Algebra and Statistics





## Model 1

*Answer the following questions :*

**1** Choose the correct answer from those given :

- 1** The point  $(-3, 4)$  lies in the ..... quadrant.  
 (a) first                      (b) second                      (c) third                      (d) fourth
- 2** The positive square root of mean of the squares of deviations of values from its arithmetic mean is called .....  
 (a) the range.                      (b) the arithmetic mean.  
 (c) the standard deviation.                      (d) the mode.
- 3** If  $3a = 4b$ , then  $a : b =$  .....  
 (a)  $3 : 4$                       (b)  $4 : 3$                       (c)  $3 : 7$                       (d)  $4 : 7$
- 4** If  $n(X) = 2$ ,  $n(Y^2) = 9$ , then  $n(X \times Y) =$  .....  
 (a) 6                      (b) 18                      (c) 11                      (d) 7
- 5** The range of the set of the values : 7, 3, 6, 9 and 5 is .....  
 (a) 3                      (b) 4                      (c) 6                      (d) 12
- 6** If  $y \propto x$  and  $y = 2$  when  $x = 8$ , then  $y = 3$  when  $x =$  .....  
 (a) 16                      (b) 12                      (c) 24                      (d) 6

**2** [a] If  $X \times Y = \{(2, 2), (2, 5), (2, 7)\}$

, find : **1**  $Y$                       **2**  $Y \times X$

[b] If  $a, b, c$  and  $d$  are proportional, prove that :  $\frac{a}{b-a} = \frac{c}{d-c}$

**3** [a] If  $X = \{2, 3, 5\}$ ,  $Y = \{4, 6, 8, 10\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $2a = b$ " for all  $a \in X, b \in Y$

- 1** Write  $R$  and represent it by an arrow diagram.
- 2** Show that  $R$  is a function.

[b] Find the number that if we add it to each term of the ratio  $7 : 11$  it becomes  $2 : 3$

- 4 [a] If  $X = \{1, 3, 5\}$  and  $R$  is a function on  $X$ , where  $R = \{(a, 3), (b, 1), (1, 5)\}$ , find :

1 The range of the function.

2 The value of  $a + b$

- [b] If  $y \propto \frac{1}{x}$  and  $y = 3$  when  $x = 2$

, find :

1 The relation between  $x$  and  $y$

2 The value of  $y$  when  $x = 1.5$

- 5 [a] Represent graphically the function  $f : f(x) = (x - 3)^2$ ,  $x \in [0, 6]$ , from the graph deduce the vertex of the curve, the minimum value of the function and the equation of the axis of symmetry.

- [b] Calculate the arithmetic mean and the standard deviation of the set of values :  
8, 9, 7, 6 and 5

## Model 2

*Answer the following questions :*

- 1 Choose the correct answer from those given :

- 1 The point  $(3, 4)$  lies in the ..... quadrant.

(a) first                      (b) second                      (c) third                      (d) fourth

- 2 ..... is one of the measures of the dispersion.

(a) The median                      (b) The arithmetic mean  
(c) The standard deviation                      (d) The mode

- 3 The third proportion of the two numbers 3 and 6 is .....

(a)  $\frac{1}{2}$                       (b) 9                      (c) 2                      (d) 12

- 4 If  $n(X) = 2$ ,  $n(Y \times X) = 6$ , then  $n(Y^2) = \dots\dots\dots$

(a) 4                      (b) 9                      (c) 16                      (d) 12

- 5 The range of the set of the values : 7, 3, 6, 9 and 5 is .....

(a) 3                      (b) 4                      (c) 6                      (d) 12

6 If  $x \propto y = 7$ , then  $y \propto$  .....

(a)  $\frac{1}{x}$

(b)  $x - 7$

(c)  $x$

(d)  $x + 7$

2 [a] If  $X = \{2, 5\}$ ,  $Y = \{1, 2\}$ ,  $Z = \{3\}$

, find : 1  $n(X \times Z)$

2  $(Y \cap X) \times Z$

[b] If  $b$  is the middle proportional between  $a$  and  $c$ , prove that :  $\frac{a-b}{a-c} = \frac{b}{b+c}$

3 [a] If  $X = \{1, 3, 4, 5\}$ ,  $Y = \{1, 2, 3, 4, 5, 6\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + b = 7$ " for all  $a \in X$ ,  $b \in Y$

1 Write  $R$  and represent it by an arrow diagram.

2 Show that  $R$  is a function.

[b] If  $5a = 3b$ , find the value of :  $\frac{7a+9b}{4a+2b}$

4 [a] If  $f(x) = 4x + b$  and  $f(3) = 15$ , find the value of :  $b$

[b] If  $y \propto x$ ,  $y = 6$  when  $x = 3$ , find :

1 The relation between  $x$  and  $y$

2 The value of  $y$  when  $x = 5$

5 [a] Represent graphically the function  $f : f(x) = 4 - x^2$ ,  $x \in [-3, 3]$ , from the graph deduce the vertex of the curve, the maximum value of the function and the equation of the axis of symmetry.

[b] The following frequency distribution shows the number of children of some families in a new city :

Number of children	0	1	2	3	4	Total
Number of families	6	15	40	25	14	100

Calculate the mean and the standard deviation of the number of children.

## Model for the merge students

*Answer the following questions :*

### 1 Complete :

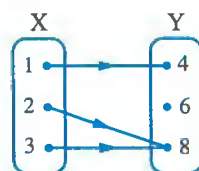
- 1 The point (5 , 3) lies in ..... quadrant.
- 2  $n(X) = X^3 + 8$  is called a polynomial of ..... degree.
- 3 The range of the set of the values : 4 , 14 , 25 and 34 is .....
- 4 If  $y = 2X$  , then  $y \propto$  .....
- 5 If  $X = \{2 , 4 , 6\}$  , then  $n(X^2) =$  .....
- 6 If  $(a , 3) = (6 , b)$  , then  $a + b =$  .....

### 2 Choose the correct answer from those given :

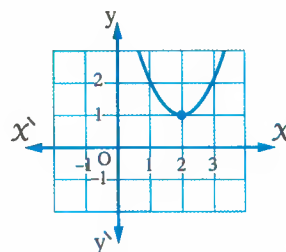
- 1 If  $Xy = 7$  , then  $y \propto$  .....  
 (a)  $\frac{1}{X}$                       (b)  $X - 7$                       (c)  $X$                       (d)  $X + 7$
- 2 If 2 , 3 , 6 and  $X$  are proportional , then  $X =$  .....  
 (a) 9                      (b) 18                      (c) 12                      (d) 3
- 3 If  $2a = 5b$  , then  $\frac{a}{b} =$  .....  
 (a)  $\frac{-5}{2}$                       (b)  $\frac{-2}{5}$                       (c)  $\frac{2}{5}$                       (d)  $\frac{5}{2}$
- 4 ..... is one of the measures of the dispersion.  
 (a) The arithmetic mean                      (b) The range  
 (c) The mode                      (d) The median
- 5 If  $n(X) = 5$  ,  $n(X \times Y) = 10$  , then  $n(Y) =$  .....  
 (a) 4                      (b) 3                      (c) 2                      (d) 1
- 6 If  $X = \{1\}$  , then  $X^2 =$  .....  
 (a) 1                      (b) (1 , 1)                      (c)  $\{(1 , 1)\}$                       (d)  $\{1\}$

**3 Put (✓) or (X) :**

- 1 If the function  $f = \{(1, 3), (2, 4), (3, 3)\}$   
 , then the domain of the function is  $\{1, 2, 3\}$  ( )
- 2 If  $y \propto X$  and  $y = 6$  when  $X = 3$  , then  $y = 2$  when  $X = 4$  ( )
- 3 If  $\sum (X - \bar{X})^2 = 36$  for a set of values whose number equals 9 , then  $\sigma = 4$  ( )
- 4 The intersection point of the straight line  $f(X) = X + 2$   
 with  $X$ -axis is the point  $(-2, 0)$  ( )
- 5 If  $f : X \longrightarrow Y$  , then  $X$  is called the domain of this function. ( )
- 6 The arrow diagram from  $X$  to  $Y$   
 is a function. ( )


**4 Join from column (A) to column (B) :**

(A)	(B)
1 If $(1, 4) \in \{2, X\} \times \{1, 4\}$ , then $X = \dots\dots\dots$	• 6
2 If the function $f$ where $f(X) = X - 4$ is represented graphically by a straight line passing through the point $(a, 2)$ , then $a = \dots\dots\dots$	• 1
3 $\frac{1}{2} = \frac{3}{6} = \frac{4}{8} = \frac{\dots\dots}{16}$	• 10
4 If $f(X) = 5$ , then $f(5) + f(-5) = \dots\dots\dots$	• $\pm 6$
5 The middle proportional of the two numbers 4 and 9 is $\dots\dots\dots$	• 2
6 In the opposite figure : The equation of the line of symmetry is $X = \dots\dots\dots$	• 8





1

Cairo Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 The simplest dispersion measure is .....

- (a) the arithmetic mean. (b) the median.  
(c) the range. (d) the mode.

2  $2x^2 \times 3x = \dots\dots\dots$

- (a)  $6x^3$  (b)  $5x^3$  (c)  $6x^2$  (d)  $5x^2$

3 If  $X = \{3\}$  ,  $n(Y) = 5$  , then  $n(X \times Y) = \dots\dots\dots$

- (a) 1 (b) 5 (c) 8 (d) 15

4 The simplest form of the expression :  $3x - 4y + 5x + 7y$  is .....

- (a)  $7x + 12y$  (b)  $11xy$  (c)  $10x + 9y$  (d)  $8x + 3y$

5 The relation which represents an inverse variation between the two variables  $y$  and  $x$  is .....

- (a)  $xy = 5$  (b)  $y = x + 3$  (c)  $\frac{x}{5} = \frac{y}{2}$  (d)  $y = 2x$

6 If  $\sqrt{x} = 4$  , then  $x = \dots\dots\dots$  where  $x \in \mathbb{Z}^+$

- (a) 2 (b) 4 (c) 8 (d) 16

2 [a] Graph the curve of the function  $f : f(x) = x^2$  where  $x \in [-3, 3]$   
 , from the graph find :

- 1 The maximum or the minimum value of the function.  
2 The equation of the axis of symmetry.

[b] Find the standard deviation to the set of the values : 15 , 19 , 20 , 21 , 25

3 [a] If  $X = \{3, 4\}$  ,  $Y = \{4, 5\}$  ,  $Z = \{5, 6\}$  , find :

- 1  $X \times Y$  2  $(X - Y) \times Z$

[b] If  $x, y, z$  and  $l$  are proportional quantities , prove that :  $\frac{y-x}{x} = \frac{l-z}{z}$

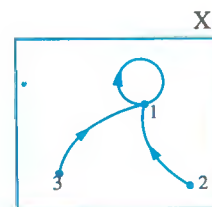
- 4 [a] Find the number which if added to both of terms of the ratio 3 : 5 , then it becomes 1 : 2

[b] In the opposite figure :

The arrow diagram represents  
the relation R on the set X

1 Write R

2 Is R a function ? If it's , find its range.



- 5 [a] If  $y \propto X$  and  $y = 20$  as  $X = 4$  , find :

1 The constant of variation between y and X

2 The value of X when  $y = 40$

- [b] If  $f(X) = 2X + k$  ,  $f(5) = 13$  , find the value of : k

2

Giza Governorate



Answer the following questions :

- 1 Choose the correct answer :

1 Double the number  $2^8$  is .....

(a)  $2^{10}$

(b)  $2^{16}$

(c)  $4^8$

(d)  $2^9$

2 If  $Xy = 3$  , then  $y \propto$  .....

(a) X

(b)  $3X$

(c)  $\frac{1}{X}$

(d)  $\frac{1}{3}X$

3 If  $X^2 + y^2 = 25$  ,  $(X + y)^2 = 49$  , then  $Xy =$  .....

(a) 6

(b) 10

(c) 12

(d) 24

4 If  $f(X) = 3$  , then  $f(3) + f(-3) =$  .....

(a) 0

(b) 1

(c) -6

(d) 6

5  $]-2, 5[ \cup \{-2, 5\} =$  .....

(a)  $[-2, 5]$

(b)  $[-2, 5[$

(c)  $]-2, 5]$

(d)  $]-2, 5[$

6 The range of the set of the values : 5 , 14 , 4 , 23 , 15 is .....

(a) 12

(b) 14

(c) 19

(d) 23

- 2 [a] If  $X = \{2, 5\}$  ,  $Y = \{1, 2\}$  ,  $Z = \{3\}$  , then find :

1  $n(X \times Z)$

2  $(Y \cap X) \times Z$

- [b] If  $f(X) = 4X + b$  ,  $f(2) = 10$  , then find the value of : b

- 3** [a] If  $X = \{2, 3, 5\}$ ,  $Y = \{4, 6, 8, 10\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a = \frac{b}{2}$ " for each  $a \in X$ ,  $b \in Y$ , write  $R$  and represent it by an arrow diagram. Is  $R$  a function? and why?
- [b] Find the number which if added to the two terms of the ratio  $7 : 11$ , it will be  $2 : 3$
- 4** [a] If  $2a = 3b = 3c$ , then find the numerical value of :  $\frac{6a + b + c}{4a + 6b + 6c}$
- [b] Calculate the standard deviation for the following values : 55, 53, 57, 56, 54
- 5** [a] If  $y \propto x$  and  $y = 6$  when  $x = 3$ , find :
- [1] The relation between  $x$ ,  $y$       [2] The value of  $y$  when  $x = 4$
- [b] Represent graphically the curve of the function  $f : f(x) = 4 - x^2$  where  $x \in [-3, 3]$  and from the graph deduce the vertex of the curve and the equation of the symmetry axis.

3

## Alexandria Governorate



Answer the following questions : (Calculators are allowed)

- 1** Choose the correct answer from those given :
- [1] If  $n(X) = 5$ ,  $n(X \times Y) = 10$ , then  $n(Y) = \dots\dots\dots$
- (a) 4                      (b) 3                      (c) 2                      (d) 1
- [2] If  $x = \frac{1}{\sqrt{3} + \sqrt{2}}$ ,  $y = \sqrt{3} + \sqrt{2}$ , then  $(x + y)^2 = \dots\dots\dots$
- (a) 12                      (b)  $2\sqrt{3}$                       (c)  $2\sqrt{2}$                       (d) zero
- [3] The arithmetic mean of the set of values : 8, 9, 7, 6 and 5 equals  $\dots\dots\dots$
- (a) 25                      (b) 7                      (c) 35                      (d) 5
- [4] For any set  $Y$ , then  $\emptyset \dots\dots\dots Y$
- (a)  $\in$                       (b)  $\notin$                       (c)  $\subset$                       (d)  $\not\subset$
- [5] The relation representing the direct variation between the two variables  $x$  and  $y$  is  $\dots\dots\dots$
- (a)  $xy = 5$                       (b)  $y = x + 3$                       (c)  $\frac{x}{3} = \frac{4}{y}$                       (d)  $\frac{x}{5} = \frac{y}{2}$
- [6]  $2^{100} = 2^{99} + \dots\dots\dots$
- (a) 2                      (b) 1                      (c)  $2^{99}$                       (d) 99

- 2** [a] If  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = 3x$ , mention the degree of  $f$ , then find :  $f(-2)$ ,  $f(\sqrt{3})$

- [b] If  $5a = 3b$ , then find the value of :  $\frac{7a + 9b}{4a + 2b}$

- 3** [a] If  $X = \{-1, 1, 2\}$ ,  $Y = \{2, 4, 6, 8\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $b = 2a + 4$ " for all  $a \in X, b \in Y$ , write  $R$  and represent it by an arrow diagram. Is  $R$  a function? Why?
- [b] If  $x^4 y^2 - 14x^2 y + 49 = 0$ , prove that:  $y \propto \frac{1}{x^2}$

- 4** [a] If  $(x - 2, 3) = (5, y + 1)$ , find the value of each of:  $x, y$
- [b] The following frequency distribution shows the number of children of some families in a new city:

Number of children	0	1	2	3	4
Number of families	8	16	50	20	6

Calculate the mean and standard deviation to the number of children.

- 5** [a] If  $a, b, c$  and  $d$  are in continued proportion, then prove that:  $\frac{a}{b+d} = \frac{c^3}{c^2d+d^3}$
- [b] Represent graphically the function  $f$  where  $f(x) = x^2 + 2x + 1$ , taking  $x \in [-4, 2]$  and from the drawing deduce:
- The coordinates of the vertex of the curve.
  - The equation of the symmetry axis.
  - The minimum or the maximum value of the function.

4

## El-Kalyoubia Governorate



Answer the following questions:

- 1** Choose the correct answer:
- 1**  $\sqrt[3]{x^6} = \sqrt{\dots}$
- (a)  $x^3$  (b)  $x^2$  (c)  $x$  (d)  $x^4$
- 2** If  $(x + 5, 8) = (1, 6y + x)$ , then  $y = \dots$
- (a) 5 (b) 6 (c) 2 (d) 12
- 3** The solution set of the equation:  $x^2 + 4 = 0$  in  $\mathbb{R}$  is  $\dots$
- (a)  $\{4\}$  (b)  $\{-2, 2\}$  (c)  $\{-2\}$  (d)  $\emptyset$
- 4** If  $xy = 7$ , then  $y \propto \dots$
- (a)  $\frac{1}{x}$  (b)  $x - 7$  (c)  $x$  (d)  $x + 7$
- 5** If  $x^2 - y^2 = 16$  and  $x + y = 8$ , then  $x - y = \dots$
- (a) 2 (b) 1 (c) 128 (d) 64
- 6** If  $\sum (x - \bar{x})^2 = 36$  to the set of 9 values, then  $\sigma = \dots$
- (a) 2 (b) 4 (c) 18 (d) 27

- 2 [a]** Represent graphically the function  $f$  where  $f(x) = (x-2)^2$ ,  $x \in [0, 4]$

**From the graph, deduce:**

- 1** The equation of the symmetry axis.
  - 2** The maximum (minimum) value of the function.
- [b]** If  $y \propto \frac{1}{x}$  and  $x = 2\frac{4}{5}$  when  $y = \frac{4}{7}$ , find the value of  $y$  when  $x = 3\frac{1}{5}$

- 3 [a]** If  $X = \{2, 3, 5\}$ ,  $Y = \{4, 6, 8, 10\}$  and  $R$  is a relation from  $X$  to  $Y$ , where " $a R b$ " means " $2a = b$ " for each  $a \in X, b \in Y$

- 1** Write  $R$  and represent it by an arrow diagram.
- 2** Is  $R$  a function?

- [b]** If  $a, b, c$  and  $d$  are proportional, **prove that:**  $\sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \frac{a+c}{b+d}$

- 4 [a]** If  $X = \{2, 4\}$ ,  $Y = \{4, 0\}$ ,  $Z = \{4, 5, -2\}$ , **find:**

- 1**  $(Z - Y) \times (X \cap Y)$
- 2**  $n(X^2)$

- [b]** If  $f(x) = 4x + b$ ,  $f(3) = 15$ , **find the value of:**  $b$

- 5 [a]** If  $\frac{a}{2x+y} = \frac{b}{3y-x} = \frac{c}{4x+5y}$ , **prove that:**  $\frac{a+2b}{7} = \frac{4b+c}{17}$

- [b]** Find the standard deviation for this distribution:

X	zero	1	2	3	4	5	Total
K	3	16	17	25	20	19	100

**5****El-Sharkia Governorate**

**Answer the following questions:** (Calculators are allowed)

- 1** Choose the correct answer from those given:

- 1** If the arithmetic mean of the quantities  $2x, 3, 4, 5$  equals  $4$ , then  $x = \dots\dots\dots$ 
  - (a) 1
  - (b) 2
  - (c) 3
  - (d) 4
- 2** If  $X \times Y = \{(1, 2), (3, 4)\}$ , then  $X \cap Y = \dots\dots\dots$ 
  - (a)  $\{1, 2\}$
  - (b)  $\{(3, 4)\}$
  - (c)  $\emptyset$
  - (d)  $\{1, 4\}$
- 3** If  $y = mx$  where  $m$  is a constant  $\neq$  zero, which of the following statements is false?
  - (a)  $y \propto x$
  - (b)  $x \propto y$
  - (c)  $x = \frac{1}{m}y$
  - (d)  $x \propto \frac{1}{y}$

4 If  $a, b, c, d$  are proportional quantities, then  $\frac{ad - bc}{a^2 + b^2 + c^2} = \dots\dots\dots$

- (a) zero (b) 1 (c) 2 (d) 3

5  $f : f(x) = (2a - 2)x^3 + 3x^2 + x + 2$  is a polynomial function from the second degree when  $a = \dots\dots\dots$

- (a) zero (b) 2 (c) 3 (d) 1

6 If the point  $(a - 5, 5 - a)$  lies in the fourth quadrant, then  $\dots\dots\dots$

- (a)  $a \geq 5$  (b)  $a \leq 5$  (c)  $a > 5$  (d)  $a < 5$

2 [a] If  $X = \{1, 2, 3\}$ ,  $Y = \{3, 4\}$ , find :

- 1  $X - Y$  2  $(Y \cap X) \times Y$  3  $n(Y^2)$

[b] If  $a, b, c$  and  $d$  are in continued proportional, prove that :  $\frac{b+d}{c^2 d + d^3} = \frac{a}{c^3}$

3 [a] If  $X = \{\frac{1}{2}, 1, \text{zero}, -\frac{1}{2}, -1\}$ ,  $Y = \{1, 2, \text{zero}, -1, -2\}$

and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a$  is the multiplicative inverse of  $b$ " for each  $a \in X$  and  $b \in Y$

Write  $R$  and represent it by an arrow diagram. Is  $R$  a function? and why?

[b] If  $y$  varies inversely as  $x^2$  where  $y = 9$  at  $x = \frac{2}{3}$

, find : 1 The relation between  $y$  and  $x$

2 The value of  $y$  when  $x = \frac{1}{2}$

4 [a] Represent graphically the quadratic function  $f$  where  $f(x) = (x - 3)^2 + 1$  taking  $x \in [0, 6]$  From the graph deduce :

- 1 The coordinates of the vertex of the curve.  
2 The minimum value of the function.  
3 The equation of the axis of symmetry of the curve.

[b] If  $\frac{x}{3} = \frac{y}{2} = \frac{z}{5}$ , find the value of :  $\frac{xy + yz}{x^2 + y^2}$

5 [a] Calculate the standard deviation for the values : 12, 13, 16, 18, 21

[b] If  $f(x) = ax + b$  and  $f(a) = b$

Find the value of the expression :  $ab^2 + 5$

6

El-Monofia Governorate



Answer the following questions : (Using calculator is permitted)

1 Choose the correct answer from those given :

- 1 The number 3 belongs to the solution set of the inequality .....  
 (a)  $x > 3$                       (b)  $x < 3$                       (c)  $-x \geq -3$                       (d)  $-x \geq 3$
- 2  $\left(\frac{-3}{4}\right)^{\text{zero}}$  .....  $\left(\frac{-3}{4}\right)^2$   
 (a)  $<$                       (b)  $>$                       (c)  $=$                       (d)  $\leq$
- 3 The number lying between 0.02 and 0.03 is .....  
 (a) 0.00025                      (b) 0.0025                      (c) 0.025                      (d) 0.25
- 4 If  $a < 5$ , then the point  $(2, a - 5)$  lies in the ..... quadrant.  
 (a) first                      (b) second                      (c) third                      (d) fourth
- 5 If  $\frac{a}{3} = \frac{b}{5}$ , then  $5a - 3b + 4 =$  .....  
 (a) 3                      (b) 4                      (c) 5                      (d) 6
- 6 If  $\sum (x - \bar{x})^2 = 48$  of a set of values and the number of these values is 12, then  $\sigma =$  .....  
 (a) 2                      (b) -2                      (c) -4                      (d) 4

2 [a] If  $X = \{-1, 1, 2\}$ ,  $Y = \{2, 4, 6, 8\}$ ,  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $b = 2a + 4$ " for each  $a \in X$  and  $b \in Y$

- 1 Write  $R$  and represent it by an arrow diagram.
- 2 Show that  $R$  is a function and write its range.

[b] If the straight line which represents the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  where  $f(x) = 6x - a$  cuts  $y$ -axis at the point  $(b, 3)$ , find the value of :  $2a - 5b$

3 [a] If  $X = \{1\}$ ,  $Y = \{2, 3\}$ ,  $Z = \{3, 4, 5\}$ , find each of the following :

- 1  $X \times Y$
- 2  $X \times (Y - Z)$
- 3  $n(Z^2)$

[b] If  $b$  is the middle proportional between  $a$  and  $c$ , prove that :  $\frac{a^2 + b^2}{b^2} = \frac{b^2 + c^2}{c^2}$

4 [a] If  $a : b : c = 2 : 3 : 5$  and  $a + b + c = 35$ , then find the value of each of :  $a$ ,  $b$  and  $c$

[b] If  $y = a + 7$ ,  $a \propto \frac{1}{x^2}$  and  $a = 3$  when  $x = 2$ , then find :

- 1 The relation between  $x$  and  $y$
- 2 The value of  $y$  when  $x = \sqrt{3}$

- 5 [a] Draw the curve of the function  $f : f(x) = x^2 - 4x$ , taking  $x \in [-1, 5]$

and from the graph find :

- 1 The coordinates of the vertex of the curve.
- 2 The equation of the line of symmetry.
- 3 The maximum or the minimum value of the function.

- [b] Find the standard deviation for the following set of values : 20 , 27 , 5 , 16 , 32

7

El-Gharbia Governorate



Answer the following questions : (Calculators are allowed)

- 1 Choose the correct answer :

- 1 The following functions are polynomial functions except the function  $f$  where  $f(x) = \dots\dots\dots$ 
  - (a)  $x + 3$
  - (b)  $\sqrt{2}x + 1$
  - (c)  $x\left(x + \frac{1}{x}\right)$
  - (d)  $x^2(x + 4)$
- 2 The solution set of the equation :  $(x - 5)^{\text{zero}} = 1$  in  $\mathbb{R}$  is  $\dots\dots\dots$ 
  - (a)  $\{5\}$
  - (b)  $\{5, -5\}$
  - (c)  $\mathbb{R}$
  - (d)  $\mathbb{R} - \{5\}$
- 3 If  $(a - 7, 26) = (-3, b^3 - 1)$ , then  $\sqrt{a^2 + b^2} = \dots\dots\dots$ 
  - (a) 5
  - (b) -5
  - (c)  $\pm 5$
  - (d)  $\pm 7$
- 4 The second proportional to the numbers 2 , ... , 8 is  $\dots\dots\dots$ 
  - (a) 4
  - (b) 6
  - (c)  $\pm 4$
  - (d)  $\pm 6$
- 5 The range of the set of the values : 7 , 3 , 6 , 9 , 5 is  $\dots\dots\dots$ 
  - (a) 3
  - (b) 4
  - (c) 6
  - (d) 12
- 6 If  $y \propto x$  and  $y = 2$  when  $x = 8$ , then  $y = 3$  when  $x = \dots\dots\dots$ 
  - (a) 16
  - (b) 12
  - (c) 24
  - (d) 6

- 2 [a] If  $X = \{-2, -3, 2\}$ ,  $Y = \{\frac{1}{8}, \frac{1}{27}, 8\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a^3 = b$ " for all  $a \in X, b \in Y$ , write  $R$ , and represent it by an arrow diagram. Is  $R$  a function or not with a reason ?

- [b] If  $x^4 y^2 - 14x^2 y + 49 = 0$ , then prove that :  $y \propto \frac{1}{x^2}$

- 3 [a] If  $a, b, c$  and  $d$  are proportional quantities, then prove that :  $\frac{a+b}{b} = \frac{c+d}{d}$

- [b] Represent graphically the curve of the function  $f : f(x) = 2 - x^2$ , taking  $x \in [-3, 3]$  and from the graph deduce the equation of the axis of symmetry, the maximum value or the minimum value of the function.

4 [a] If  $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$ , find  $Y^2$  and represent it by a Cartesian diagram.

[b] Find the positive number which if we add its square to each of the two terms of the ratio  $5 : 11$ , it becomes  $3 : 5$

5 [a] If the straight line representing the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  where  $f(x) = 6x - l$  cuts the y-axis at the point  $(m, 3)$ , find the value of each of :  $l$  and  $m$

[b] Calculate the arithmetic mean and the standard deviation for the following data : 23, 12, 17, 13, 15 rounding the result of the standard deviation to one decimal place.

8

## El-Dakahlia Governorate



Answer the following questions : (Calculator is permitted)

1 [a] Choose the correct answer from those given :

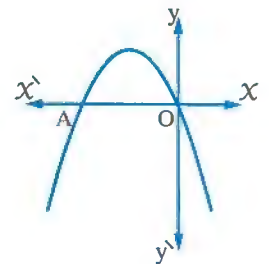
1 If  $5x = 9y$ , then  $\frac{3x}{2y} = \dots\dots\dots$

- (a)  $27 : 10$  (b)  $9 : 5$  (c)  $5 : 9$  (d)  $81 : 25$

2 In the opposite figure :

The curve of a quadratic function,  $A(-4, 0)$ , then the equation of the axis of symmetry is  $x = \dots\dots\dots$

- (a) 1 (b) -1  
(c) -2 (d) 0



3 The number added to each of the numbers 1, 3, 6 to be proportional is  $\dots\dots\dots$

- (a) 4 (b) 3 (c) 1 (d) 2

[b] If  $b$  is the middle proportional between  $a$  and  $c$ , prove that :  $\frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{2a}{c}$

2 [a] Choose the correct answer from those given :

1 If  $f(x+3) = x-3$ , then  $f(7) = \dots\dots\dots$

- (a) 4 (b) 1 (c) 7 (d) 10

2 If  $\sum (x - \bar{x})^2 = 36$  for 9 values, then the standard deviation =  $\dots\dots\dots$

- (a) 2 (b) 18 (c) 27 (d) 4

3 If  $f(x) = 3$ , then  $f(2) - f(7) = \dots\dots\dots$

- (a) 5 (b) -5 (c) 0 (d) -4

[b] If  $X = \{4, 5, 7\}$ ,  $R$  is a function on  $X$  and  $R = \{(a, 5), (b, 5), (4, 7)\}$ , find :

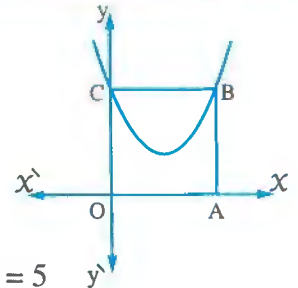
1 The value of  $3a + 3b$

2 The range of the function.

3 [a] If  $\frac{a}{4x+y} = \frac{b}{x-4y}$ , prove that :  $\frac{a+b}{5x-3y} = \frac{a-b}{3x+5y}$

[b] Calculate the standard deviation of values : 12 , 13 , 16 , 18 , 21

- 4 [a] The opposite figure represents the curve of the function  $f$  where  $f(x) = x^2 - (k-2)x - k + 4$ , the figure OABC is a square. Find the value of :  $k$



- [b] If  $y = 1 + b$ ,  $b$  varies inversely with the square of  $x$ ,  $x = 1$  at  $y = 5$ , find the relation between  $x$  and  $y$ , then find the value of  $y$  at  $x = 2$

- 5 [a] If  $f(x) = a + x^2$ ,  $l(x) = c$  are two polynomial functions where  $3f(2) + 3l(x) = 6$ , find the numerical value of :  $2f(0) + 2l(7)$  where  $a$  and  $c$  are constants.

- [b] If  $X = \{3, 5, 7\}$ ,  $Y = \{x : x \in \mathbb{N}, 10 < x < 30\}$  and the function  $f$  from  $X \rightarrow Y$  is  $f = \{(3, 9), (5, 15), (7, 21)\}$

1 Find the domain of  $f$

2 Write the rule of the function.

9

Ismailia Governorate



Answer the following questions : (Calculators are allowed)

- 1 Choose the correct answer from those given :

1 The expectation of the match of Ismaili club in mathematics is called .....

- (a) probability. (b) equations. (c) inequalities. (d) relations.

2 The third proportional of the quantities 2 , 3 and 6 is .....

- (a) 1 (b) 4 (c) 9 (d) 12

3 The number  $\frac{2x}{x-5}$  is a rational number if  $x \neq$  .....

- (a) zero (b)  $\frac{1}{5}$  (c)  $\frac{2}{5}$  (d) 5

4 If the point  $(b-4, 2-b)$  lies in the third quadrant, then  $b =$  .....

- (a) 2 (b) 3 (c) 4 (d) 6

5 If  $17x + 8 = 11$ , then  $17x + 11 =$  .....

- (a) 8 (b) 11 (c) 14 (d) 17

6 If a set of values are equal, then the dispersion of these values is .....

- (a)  $> \text{zero}$  (b)  $< \text{zero}$  (c)  $= 1$  (d)  $= \text{zero}$

2 [a] If  $X = \{2, 3\}$  ,  $Y = \{3, 4, 5\}$  , find :

1  $X \times Y$

2  $X^2$

3  $n(Y^2)$

[b] If  $3a = 4b$  , find the value of :  $\frac{2a+b}{5a-3b}$

3 [a] If  $y$  varies inversely with the square of  $X$  and  $y = 5$  when  $X = 3$  , find the value of  $y$  when  $X = 2$

[b] If the straight line which represents the function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(X) = 3X - a$  cuts the  $y$ -axis at the point  $(b, 5)$  , find the values of :  $a$  and  $b$

4 [a] If we add double the number  $X$  to each of the numbers 1 , 3 and 7 , it becomes proportional quantities. Find the value of :  $X$

[b] If  $X = \{-1, 1, 2\}$  ,  $Y = \{2, 4, 6, 8\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $b = 2a + 4$ " for all  $a \in X$  ,  $b \in Y$

1 Find the relation  $R$  and represent it by an arrow diagram.

2 Is  $R$  a function ? and why ?

5 [a] Represent graphically the curve of the function  $f : f(X) = 2 - X^2$  where  $X \in [-3, 3]$  , then from the graph find :

1 The coordinates of the vertex of the curve.

2 The equation of the axis of symmetry.

3 The maximum or minimum value of the function.

[b] Calculate the standard deviation of the values : 12 , 13 , 16 , 18 , and 21

10

Suez Governorate



Answer the following questions : (Calculators are allowed)

1 Choose the correct answer from those given :

1 If 2 , 3 , 6 and  $X$  are proportional , then  $X = \dots\dots\dots$

(a) 9

(b) 18

(c) 12

(d) 3

2 If  $3a = k = 12a^2$  , then  $k = \dots\dots\dots$

(a)  $4a^2$

(b)  $3a$

(c)  $4a$

(d)  $3a^2$

3 If  $X = \{1, 2\}$  ,  $Y = \{3, 4\}$  , then  $(3, 4) \in \dots\dots\dots$

(a)  $X \times Y$

(b)  $Y \times X$

(c)  $X^2$

(d)  $Y^2$

4 If  $(a, 5) = (6, b)$ , then  $a + b = \dots\dots\dots$

- (a) 5 (b) 11 (c) 6 (d) 1

5  $\frac{\text{Sum of the values}}{\text{Their number}} = \dots\dots\dots$

- (a) the range. (b) the standard deviation.  
(c) the arithmetic mean. (d) the mode.

6 If the point  $(2, y)$  lies on the  $X$ -axis, then  $y + 4 = \dots\dots\dots$

- (a) 5 (b) 4 (c) 2 (d) 3

2 [a] If  $4a = 3b$ , then find the value of :  $\frac{4a+b}{2a-b}$

[b] If  $X = \{0, 3, 4\}$ ,  $Y = \{1, 2, 3, 4, 5\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + b = 5$ " for all  $a \in X, b \in Y$

- 1 Find the relation  $R$   
2 Represent the relation  $R$  by an arrow diagram.  
3 Is  $R$  a function?

3 [a] If  $X \times Y = \{(2, 6), (2, 9), (3, 6), (3, 9)\}$ , find :

- 1  $X, Y$  2  $Y \times Y$

[b] Draw the curve of the function  $f : f(x) = 1 + x^2$  at the interval  $[-3, 3]$  and from the graph find :

- 1 The coordinates of the vertex of the curve.  
2 The equation of the axis of symmetry.  
3 The minimum value.

4 [a] If  $x, y, z, r$  are proportional quantities, then prove that :  $\frac{x^2 + 2z^2}{y^2 + 2r^2} = \frac{xz}{yr}$

[b] From the data of the following table, answer the following questions :

- 1 Show the kind of variation between  $y$  and  $x$   
2 Find the constant proportion.  
3 Find the value of  $y$  when  $x = 3$

$x$	2	4	6
$y$	6	3	2

5 [a] If  $f(x) = x^2 - 3x$ ,  $g(x) = x - 3$

- 1 Find :  $f(2) + g(2)$  2 Prove that :  $f(3) + g(3) = 0$

[b] Calculate the standard deviation for the values : 12, 13, 16, 18, 21

11

Port Said Governorate



Answer the following questions :

1 Choose the correct answer from those given :

- 1 If  $(3, 5) \in \{3, 6\} \times \{x, 8\}$ , then  $x = \dots\dots\dots$   
 (a) 8 (b) 6 (c) 5 (d) 3
- 2 The linear function given by the rule  $y = 2x - 1$  is represented graphically by a straight line intersecting the y-axis at the point  $\dots\dots\dots$   
 (a)  $(\frac{1}{2}, 0)$  (b)  $(0, -1)$  (c)  $(-1, 0)$  (d)  $(0, \frac{1}{2})$
- 3 The difference between the greatest value and the smallest value in a set of individuals is called  $\dots\dots\dots$   
 (a) the standard deviation. (b) the arithmetic mean.  
 (c) the median. (d) the range.
- 4 If the point  $(x - 4, 2 - x)$  where  $x \in \mathbb{Z}$  is located in the fourth quadrant, then  $x = \dots\dots\dots$   
 (a) 2 (b) 3 (c) 4 (d) 6
- 5 Which of the following tables represents the direct variation between  $x$  and  $y$  ?
- (a) 

$x$	$y$
2	9
4	18

 (b) 

$x$	$y$
3	20
5	12

 (c) 

$x$	$y$
3	6
-2	-9

 (d) 

$x$	$y$
10	9
5	18
- 6 If  $(x - 1, 11) = (8, y + 3)$ , then  $\sqrt{x + 2y} = \dots\dots\dots$   
 (a) 5 (b)  $\pm 5$  (c)  $\sqrt{17}$  (d) 25

2 [a] If  $X = \{1, 2\}$ ,  $Y = \{2, 5\}$ ,  $Z = \{4, 5\}$ , then find :

1  $n(X \times Z)$

2  $(X - Y) \cap Z$

[b] Represent graphically  $f : f(x) = x^2 + 2x + 1$ , consider  $x \in [-4, 2]$

From the graph deduce :

- 1 The coordinates of the vertex of the curve.  
 2 The minimum or the maximum value of the function.

3 [a] If  $f(x) = 4x + b$  and  $f(3) = 15$ , find the value of : b

[b] If  $y \propto \frac{1}{x}$  and  $y = 6$  when  $x = 2.5$ , find :

- 1 The relation between  $x, y$       2 The value of  $y$  when  $x = 5$

- 4 [a] If  $X = \{1, 2, 3\}$ ,  $Y = \{12, 21, 47, 52\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a$  is a digit from the digits of  $b$ " for each  $a \in X, b \in Y$

1 Write  $R$  and represent it by an arrow diagram.

2 Which of the following relations is correct and why?  $1 R 52, 2 R 21, 3 R 47$

- [b] If  $7, x$  and  $\frac{1}{y}$  are in continued proportion, then find the value of:  $x^4 y^2$

- 5 [a] If  $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ , then prove that:  $\frac{2y - z}{3x - 2y + z} = \frac{1}{2}$

- [b] Calculate the arithmetic mean and the standard deviation for the values:  
3, 6, 7, 9, 15

12

Damietta Governorate



Answer the following questions: (Calculators are allowed)

- 1 Choose the correct answer from the given ones:

1  $\sqrt{36} = \dots\dots\dots$

- (a) 6 (b) -6 (c)  $\pm 6$  (d) 18

- 2 The point  $(-2, 5)$  lies in the  $\dots\dots\dots$  quadrant.

- (a) first (b) second (c) third (d) fourth

- 3 The commonest measure of dispersions and the most accurate is  $\dots\dots\dots$

- (a) the median. (b) the arithmetic mean.  
(c) the mode. (d) the standard deviation.

4  $\mathbb{R} = \dots\dots\dots$

- (a)  $\mathbb{Q} \cap \mathbb{Q}$  (b)  $\mathbb{R}_+ \cap \mathbb{R}_-$  (c)  $\mathbb{R}_+ \cup \mathbb{R}_-$  (d)  $\mathbb{Q} \cup \mathbb{Q}$

- 5 If  $(x - 3, 2^y) = (2, 32)$ , then  $(x, y) = \dots\dots\dots$

- (a)  $(5, 2)$  (b)  $(2, 5)$  (c)  $(5, 5)$  (d)  $(2, 2)$

- 6 If  $xy = 8$ , then  $y \propto \dots\dots\dots$

- (a)  $x - 8$  (b)  $\frac{1}{x}$  (c)  $x$  (d)  $x + 8$

- 2 [a] If  $X = \{2, 5\}$ ,  $Y = \{1, 2\}$ ,  $Z = \{3\}$ , find:

- 1  $n(X \times Y)$  2  $(X - Y) \times Z$  3  $Y^2$

- [b] If  $b$  is the middle proportional between  $a$  and  $c$ , prove that:  $\frac{a - b}{a - c} = \frac{b}{b + c}$

- 3 [a] If  $X = \{1, 3, 4, 5\}$  ,  $Y = \{1, 2, 3, 4, 5, 6\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + b = 7$ " for all  $a \in X, b \in Y$

1 Write  $R$

2 Show giving reasons that  $R$  is a function and find its range.

- [b] If  $\frac{21x - y}{7x - z} = \frac{y}{z}$  , then prove that :  $y \propto z$

- 4 [a] Calculate the standard deviation for the values : 12 , 13 , 16 , 18 , 21

[b] If  $y \propto x$  ,  $y = 6$  when  $x = 3$  , find :

1 The relation between  $x, y$

2 The value of  $y$  when  $x = 5$

- 5 [a] If  $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$  , then prove that :  $\sqrt{3x^2 + 3y^2 + z^2} = 2x + y$

[b] Represent graphically the function  $f : f(x) = x^2 + 3$  ,  $x \in [-2, 2]$

From the graph deduce :

1 The equation of symmetry line.

2 The minimum value of the function.

### 13 Kafr El-Sheikh Governorate



Answer the following questions : (Calculator is allowed)

- 1 [a] Choose the correct answer from the given ones :

1 The third proportional of the numbers 4 , 12 , ..... , 48 is .....

(a) 7 (b) 32 (c) 16 (d) 36

2  $\emptyset$  .....  $\{1, 2\}$

(a)  $\in$  (b)  $\notin$  (c)  $\not\subset$  (d)  $\subset$

3 The range of the set of the values : 7 , 3 , 6 , 9 and 5 equals .....

(a) 3 (b) 6 (c) 4 (d) 12

- [b] Represent graphically the function  $f : f(x) = (x - 2)^2$  , where  $x \in [-1, 5]$  , then from the graph deduce the vertex of the curve , the equation of the symmetry axis and the minimum value of the function.

- 2 [a] Choose the correct answer from the given ones :

1  $(\sqrt{7} + \sqrt{5})(\sqrt{7} - \sqrt{5}) = \dots\dots\dots$

(a) 2 (b) 12 (c) 35 (d) -2

2  $|-5| + |5| = \dots\dots\dots$

- (a) zero (b) 25 (c) 10 (d) -10

3 If  $(X - 2, 3) = (5, X + y)$ , then  $X - y = \dots\dots\dots$

- (a) 7 (b) 3 (c) -11 (d) 11

[b] If  $y$  is the middle proportional between  $X$  and  $z$ , prove that :  $\frac{X-y}{X-z} = \frac{y}{y+z}$

3 [a] If  $X = \{1, 3, 4, 5\}$ ,  $Y = \{1, 2, 3, 4, 5\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $b = 6 - a$ " for all  $a \in X, b \in Y$

1 Write  $R$  and represent it by an arrow diagram.

2 Show that  $R$  is a function and find its range.

[b] If  $3X = 2y$ , find the value of the ratio :  $\frac{3X+2y}{6y-X}$

4 [a] If  $X = \{2, -1\}$ ,  $Y = \{4, 0\}$ ,  $Z = \{4, 5, -2\}$ , find :

- 1  $X \times Y$  2  $(Y \cap Z) \times X$  3  $n(Y^2)$

[b] If  $f(X) = 2X + a$  and  $f(2) = 1$ , find the value of :  $a$

5 [a] If  $y$  changes inversely with  $X^2$  and  $y = 2$  when  $X = 4$

1 Find the relation between  $y$  and  $X$

2 Deduce the value of  $y$  when  $X = 16$

[b] Calculate the arithmetic mean and the standard deviation of the set of values : 8, 9, 7, 6 and 5

14

El-Beheira Governorate



Answer the following questions : (Calculator is permitted)

1 Choose the correct answer from the given ones :

1 The solution set in  $\mathbb{R}$  for the equation  $X^2 + 9 = 0$  is  $\dots\dots\dots$

- (a)  $\{-3\}$  (b)  $\{3\}$  (c)  $\{-3, 3\}$  (d)  $\emptyset$

2 If the point  $(k-4, 2-k)$  where  $k \in \mathbb{Z}$  is located in the third quadrant, then  $k = \dots\dots\dots$

- (a) 2 (b) 3 (c) 4 (d) 6

3 The multiplicative inverse of the number  $\frac{\sqrt{3}}{6}$  is  $\dots\dots\dots$

- (a)  $-\frac{\sqrt{3}}{6}$  (b)  $6\sqrt{3}$  (c)  $2\sqrt{3}$  (d)  $-2\sqrt{3}$

- 4 If  $7, x, \frac{1}{y}$  are in continued proportion, then  $x^2 y = \dots\dots\dots$   
 (a) 7 (b)  $\frac{1}{7}$  (c) 14 (d) 49
- 5 If  $a + 3b = 7$ ,  $c = 3$ , then the value of  $a + 3(b + c) = \dots\dots\dots$   
 (a) 10 (b) 16 (c) 21 (d) 30
- 6 The difference between the greatest value and the smallest value in a set of values is called  $\dots\dots\dots$   
 (a) the arithmetic mean. (b) the median.  
 (c) the range. (d) the standard deviation.

2 [a] If  $X = \{1\}$ ,  $Y = \{2, 3\}$ ,  $Z = \{2, 5, 6\}$ , find ;

- 1  $X \times (Y \cap Z)$  2  $n(Z^2)$

[b] Find the positive number which if its square is added to each of the two terms of the ratio  $5 : 11$ , it becomes  $3 : 5$

3 [a] If the point  $(a, 3)$  is located on the straight line which represents the function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = 4x - 5$ , find the value of : a

[b] If  $\frac{a+b}{3} = \frac{b+c}{6} = \frac{c+a}{5}$ , then prove that :  $\frac{a+b+c}{a} = 7$

4 [a] If  $X = \{1, 3, 5\}$  and  $R$  is a relation on  $X$  where " $a R b$ " means " $a + b = 6$ " for each  $a \in X, b \in X$

- 1 Write  $R$   
 2 Show that  $R$  is a function and find its range.

[b] Calculate the standard deviation for the values : 17, 22, 20, 23, 18

5 [a] If  $y \propto x$ ,  $y = 6$  when  $x = 3$ , find ;

- 1 The relation between  $y, x$   
 2 The value of  $y$  when  $x = 5$

[b] Represent graphically the quadratic function  $f$  where  $f(x) = x^2 - 3$ , where  $x \in [-3, 3]$  and from the graph deduce :

- 1 The equation of the axis of symmetry.  
 2 The minimum value of the function.

15

## El-Fayoum Governorate



Answer the following questions : (Using calculators is allowed)

**1 Choose the correct answer :**

- 1** The positive square root to the average of squares deviations of values from the mean is called the .....  
 (a) median. (b) mode. (c) range. (d) standard deviation.
- 2** If  $f(3X) = 6$ , then  $f(-2) = \dots\dots\dots$   
 (a) -12 (b) -3 (c) 6 (d) -18
- 3**  $[-5, 3] - ]-5, 3[ = \dots\dots\dots$   
 (a)  $\{-5, 3\}$  (b)  $] -5, 3]$  (c)  $[-5, 3[$  (d)  $\emptyset$
- 4** The fifth of the number  $5^{10}$  equals .....  
 (a)  $5^2$  (b)  $5^9$  (c)  $5^5$  (d)  $5^8$
- 5** If  $\frac{X}{2} = \frac{y}{3} = \frac{z}{5}$ , then each ratio equals .....  
 (a)  $\frac{X+y+z}{30}$  (b)  $\frac{X+2y-z}{3}$  (c)  $\frac{X-y+z}{10}$  (d)  $\frac{X-y}{5}$
- 6** If  $X$  is an odd number, then the odd number of the following is .....  
 (a)  $X-1$  (b)  $X+1$  (c)  $X+2$  (d)  $X+3$

- 2 [a]** If  $3a = 2b$ , then find the value of the ratio :  $\frac{3a-b}{a+2b}$

**[b]** If  $f(X) = aX + 5$  and  $f(-3) = 8$ , then find the value of :  $a$

- 3 [a]** If  $X, y, z$  are in continued proportion, prove that :  $\frac{X^2 + y^2}{y^2 + z^2} = \frac{X}{z}$

**[b]** If  $X = \{-1, 1, 2\}$ ,  $Y = \{2, 4, 6, 8\}$  and  $R$  is a relation from  $X$  to  $Y$  where "a R b" means " $b = 2a + 4$ " for each  $a \in X, b \in Y$ , write  $R$  and represent it by an arrow diagram, show that  $R$  is a function from  $X$  to  $Y$ , why?

- 4 [a]** If  $y \propto X$  and  $y = 20$  when  $X = 7$ , then find the relation between  $X$  and  $y$ , then find the value of  $y$  when  $X = 14$

**[b]** If  $(5 - 2X, y^3) = (1, 27)$ , then find the value of :  $\sqrt{3X + y}$

- 5 [a]** Represent graphically the function  $f : f(X) = X^2 - 2$  where  $X \in [-3, 3]$ , and from the drawing deduce the coordinates of the vertex of the curve and the minimum value of the function.

**[b]** Find the standard deviation of the values : 7, 16, 13, 5, 9

16

## Beni Suef Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

- 1 The point  $(-4, -2)$  lies in the ..... quadrant.  
 (a) first (b) second (c) third (d) fourth
- 2 If  $X$  represents a negative number, then the positive number from the following is .....  
 (a)  $2X$  (b)  $3X^2$  (c)  $4X^3$  (d)  $6X^5$
- 3 If  $Xy = 1$ , then  $y$  varies with .....  
 (a)  $\frac{1}{X}$  (b)  $X - 1$  (c)  $X$  (d)  $X + 1$
- 4 The simplest and easiest method of measuring dispersion is .....  
 (a) the median. (b) the mean.  
 (c) the standard deviation. (d) the range.
- 5 If  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$  where  $k \in \mathbb{R}$ , then  $\frac{ace}{bdf} =$  .....  
 (a)  $k^3$  (b)  $k^2$  (c)  $k$  (d)  $3$
- 6 If  $3X = 2y$ , then  $\frac{2X}{3y} =$  .....  
 (a)  $\frac{1}{2}$  (b)  $\frac{2}{3}$  (c)  $\frac{9}{4}$  (d)  $\frac{4}{9}$

2 [a] Find the number that if we add it to each term of the ratio  $7 : 11$ , it becomes  $2 : 3$

[b] If  $X = \{1, 2, 3\}$ ,  $Y = \{1, 3, 4, 9\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $b = a^2$ " for all  $a \in X, b \in Y$ , write  $R$  and represent it by an arrow diagram and show whether  $R$  is a function or not.

3 [a] If  $\frac{X}{2} = \frac{y}{3} = \frac{z}{4} = \frac{3X - 2y + 5z}{5k}$ , find the numerical value of :  $k$

[b] Represent graphically the function  $f : f(X) = 2 - X^2$ ,  $X \in [-2, 2]$ , from the graph deduce the vertex point of the curve and the maximum value of the function.

4 [a] If  $y$  varies directly with  $X$  and  $y = 3$  when  $X = 15$ , find the relation between  $y$  and  $X$ , then find the value of  $X$  when  $y = 100$

[b] If  $X = \{1, 2\}$ ,  $Y = \{3, 4, 5\}$ , find :

1  $X \times Y$

2  $Y \times X$

3  $X^2$

- 5 [a] If  $f(x) = 3x + k$ ,  $g(x) = k$  where  $f$  and  $g$  are polynomial functions, find the value of  $k$  if:  $f(3) + g(5) = 15$

- [b] Calculate the standard deviation of the set of values: 12, 13, 16, 18, 21

17

El-Menia Governorate



Answer the following questions: (Calculators are allowed)

- 1 Choose the correct answer from those given:

1  $\sqrt{5} + \sqrt{20} = \dots\dots\dots$

- (a)  $\sqrt{25}$  (b)  $5\sqrt{5}$  (c)  $9\sqrt{5}$  (d)  $3\sqrt{5}$

- 2 If three times a number = 45, then  $\frac{1}{5}$  this number =  $\dots\dots\dots$

- (a) 15 (b) 5 (c) 3 (d) 9

3  $5^2 \times 5^{-2} = \dots\dots\dots$

- (a) 5 (b) 1 (c) zero (d) -5

- 4 If  $n(X) = 3$ ,  $n(X \times Y) = 12$ , then  $n(Y) = \dots\dots\dots$

- (a) 4 (b) 9 (c) 15 (d) 36

- 5 The relation which represents direct variation between the two variables  $X$  and  $y$  is  $\dots\dots\dots$

- (a)  $xy = 5$  (b)  $y = x + 3$  (c)  $\frac{x}{3} = \frac{5}{y}$  (d)  $\frac{x}{5} = \frac{y}{3}$

- 6 The range is the  $\dots\dots\dots$  measure of dispersions.

- (a) simplest (b) greatest (c) difficult (d) otherwise

- 2 [a] If  $X = \{1, 2, 3\}$ ,  $Y = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{5}\}$  and  $R$  is a relation from  $X$  to  $Y$

where " $a R b$ " means " $a$  is the multiplicative inverse of  $b$ " for all  $a \in X$ ,  $b \in Y$ , write  $R$  and represent it by an arrow diagram. Is  $R$  a function? Why?

- [b] If  $b$  is the middle proportional between  $a$  and  $c$ , prove that:  $\frac{a+b}{a-c} = \frac{b}{b-c}$

- 3 [a] If  $2y = 3x$ , find the value of:  $\frac{3x+2y}{6y-x}$

- [b] If  $X = \{3, 4\}$ ,  $Y = \{4, 5\}$ ,  $Z = \{6, 5\}$ , find:

1  $X \times (Y \cap Z)$

2  $(X - Y) \times Z$

- 4 [a] If  $y \propto \frac{1}{x}$  and  $y = 3$  when  $x = 2$ , find:

- 1 The relation between  $x$  and  $y$

- 2 The value of  $x$  when  $y = 4$

- [b] Calculate the standard deviation for the values: 12, 13, 16, 18, 21

- 5 [a] Mention the degree of the function  $f : f(x) = 3 - 2x^3$ , then find :  $f(0)$  and  $f(-2)$
- [b] Represent graphically the function  $f : f(x) = x^2 + 2x + 1$ , consider  $x \in [-4, 2]$  and from the drawing deduce :
- 1 The equation of the symmetry axis.
  - 2 The maximum or the minimum value of the function.

18

Assiut Governorate



Answer the following questions : (Calculator is allowed)

- 1 Choose the correct answer :
- 1  $x^5 + x^2 = \dots\dots\dots$  (where  $x \neq 0$ )  
 (a)  $x^7$  (b)  $x^3$  (c)  $x^{10}$  (d)  $x^5$
  - 2 If  $X = \{1\}$  ,  $Y = \{3\}$  , then  $n(X \times Y) = \dots\dots\dots$   
 (a)  $\{(1, 3)\}$  (b)  $\{(3, 1)\}$  (c) 3 (d) 1
  - 3 The multiplicative inverse of the number 0.25 is  $\dots\dots\dots$   
 (a) 4 (b)  $-0.25$  (c)  $\frac{1}{4}$  (d)  $-0.5$
  - 4 The middle proportional between 4 , 16 is  $\dots\dots\dots$   
 (a)  $-8$  (b) 8 (c)  $\pm 8$  (d) 64
  - 5  $0.12 + 0.3 = \dots\dots\dots$   
 (a) 0.42 (b) 0.15 (c) 0.24 (d) 0.36
  - 6 The range of the set of the values : 4 , 14 , 25 and 34 equals  $\dots\dots\dots$   
 (a) 4 (b) 30 (c) 38 (d) 34

- 2 [a] If  $X = \{6, 7\}$  ,  $Y = \{3, 7\}$  , find :
- 1  $(X \cap Y) \times X$
  - 2  $n(Y^2)$
- [b] If  $\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$  , then prove that :  $\frac{3c-b}{a+b} = \frac{9}{5}$
- 3 [a] If  $X = \{-1, 2, 3\}$  ,  $Y = \{1, 4, 6, 9\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a^2 = b$ " for each of  $a \in X$  ,  $b \in Y$  , write  $R$  and represent it by an arrow diagram and show that  $R$  is a function from  $X$  to  $Y$  and find its range.
- [b] If  $y \propto \frac{1}{x}$  and  $y = 3$  when  $x = 4$
- 1 Find the relation between  $y$  and  $x$
  - 2 Find the value of  $y$  when  $x = \frac{3}{4}$
- 4 [a] Find the positive number which if its square is added to each of the two terms of the ratio  $7 : 11$  , it becomes  $2 : 3$

- [b]** Represent graphically the function  $f : f(x) = x^2 - 4$ ,  $x \in [-3, 3]$ , from the graph deduce the vertex of the curve, the maximum value or the minimum value of the function and the equation of the axis of symmetry.

**5 [a]** If  $f(x) = x^2 - 2$ ,  $g(x) = 3$ , find :  $f(\sqrt{2}) + g(5)$

- [b]** Calculate the arithmetic mean and the standard deviation of the set of the values : 11, 12, 15, 17, 20

19

Souhag Governorate



Answer the following questions : (Calculators are allowed)

- 1** Choose the correct answer :

**1**  $2^8 + 2^8 + 2^8 + 2^8 = \dots\dots\dots$

- (a)  $2^{32}$  (b)  $8^8$  (c)  $2^{10}$  (d)  $4^{12}$

**2** If  $n(X) = 2$ ,  $n(Y^2) = 9$ , then  $n(X \times Y) = \dots\dots\dots$

- (a) 6 (b) 18 (c) 11 (d) 7

**3** If  $\sqrt{3}x - 1 = 2$  ( $x \in \mathbb{R}$ ), then  $x = \dots\dots\dots$

- (a) 3 (b)  $3\sqrt{3}$  (c) -3 (d)  $\sqrt{3}$

**4** If 8, 6,  $x$ , 12 are proportional quantities, then  $x = \dots\dots\dots$

- (a) 4 (b) 16 (c) 5 (d) 25

**5** If the median of the values :  $a + 3$ ,  $a + 2$ ,  $a + 4$  ( $a \in \mathbb{Z}^+$ ) is 8, then  $a = \dots\dots\dots$

- (a) 2 (b) 5 (c) 3 (d) 4

**6**  $\dots\dots\dots$  is a measure for dispersion.

- (a) The median (b) The mode (c) The range (d) The mean

**2 [a]** If  $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$

, find : **1**  $X, Y$

**2**  $Y \times X$

**[b]** If  $\frac{x}{y} = \frac{2}{3}$ , then find the value of :  $\frac{3x + 2y}{6y - x}$

**3 [a]** If  $X = \{0, 1, 2, 3\}$ ,  $Y = \{2, 3, 4, 5, 6\}$ ,  $R$  is a relation from  $X$  to  $Y$  where "a R b" means " $a + b = 5$ " for each  $a \in X, b \in Y$

**1** Write  $R$  and represent it by an arrow diagram.

**2** Show that  $R$  is a function from  $X$  to  $Y$  and find its range.

**[b]** Find the number that if we add it to the two terms of the ratio 7 : 11, the result will be 2 : 3

4 [a] If the straight line  $y = 4x - 5$  passes through the point  $(a, 3)$ , find the value of : a

[b] If  $y \propto x$  and  $y = 6$  when  $x = 3$ , find :

1 The relation between  $x$  and  $y$

2 The value of  $y$  when  $x = 5$

5 [a] Represent graphically the function  $f : f(x) = x^2 - 4x + 4, x \in [-1, 5]$ , from the graph deduce :

1 The vertex of the curve.

2 The equation of the axis of symmetry.

[b] Calculate the mean and the standard deviation of the values : 12 , 13 , 16 , 18 , 21

20

Qena Governorate



Answer the following questions : (Calculators are permitted)

1 Choose the correct answer :

1 If  $xy = 5$ , then  $y \propto$  .....

(a)  $x^{-1}$

(b)  $x$

(c)  $5x$

(d)  $x^2$

2  $\sqrt{3} + \sqrt{3} + \sqrt{3} =$  .....

(a) 3

(b) 9

(c)  $3\sqrt{3}$

(d) 27

3 The middle proportional between 3 , 12 is .....

(a) 6

(b) -6

(c)  $\pm 6$

(d) 9

4 The point  $(-2, 3)$  lies in the ..... quadrant.

(a) first

(b) second

(c) third

(d) fourth

5 All of the following are polynomial functions except .....

(a)  $f_1(x) = x^3 + x^2 + 3$

(b)  $f_2(x) = x^3 + \frac{1}{x} + 7$

(c)  $f_3(x) = 5 - x^2$

(d)  $f_4(x) = x^2(x-3)^2$

6 The range of the values : 51 , 24 , 43 , 55 , 28 is .....

(a) 55

(b) 24

(c) 21

(d) 31

2 [a] If  $X = \{1, 3, 4, 5\}$  ,  $Y = \{1, 2, 3, 4, 5, 6, 7\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + b = 7$ ", write  $R$  and represent it by an arrow diagram. Is  $R$  a function ? Why ? and if it's a function , find its range.

[b] If  $b$  is the middle proportional between  $a$  and  $c$  , prove that :  $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

3 [a] If  $f(x) = x^2 - 3x$  ,  $g(x) = x - 3$

1 Find :  $f(\sqrt{2}) + 3g(\sqrt{2})$

2 Prove that :  $f(3) = g(3)$

[b] Find the number which if added to each of the two terms of the ratio  $7 : 11$  , it becomes  $2 : 3$

4 [a] If  $5a = 3b$ , find :  $\frac{7a+9b}{4a+2b}$

[b] The following table shows the frequency distribution for the ages of 10 students :

Ages in years	5	8	9	10	12	Total
Number of students	1	2	3	3	1	10

Calculate the standard deviation to age in years.

5 [a] If  $y \propto x$  and  $y = 40$  when  $x = 14$ , find  $x$  when  $y = 80$

[b] Represent graphically  $f : f(x) = 2x^2 - 3$ ,  $x \in [-2, 2]$  From the graph find :

- 1 The vertex of the curve.
- 2 The equation of the axis of symmetry.
- 3 The maximum or minimum value of the function.

21

Luxor Governorate



Answer the following questions :

1 Choose the correct answer :

- 1 The sum of the factors of the number 15 equals .....  
 (a) 3 (b) 4 (c) 15 (d) 24
- 2 If  $f(x) = 4x + a$  and  $f(2) = 15$ , then  $a =$  .....  
 (a) 2 (b) 4 (c) 7 (d) 15
- 3 The smallest expression in value when  $x = 7$  is .....  
 (a)  $\frac{6}{x}$  (b)  $\frac{6}{x+1}$  (c)  $\frac{6}{x-1}$  (d)  $\frac{x}{6}$
- 4 The third proportional of the two numbers - 6, 12 is .....  
 (a) - 24 (b) 6 (c) 18 (d) 72
- 5 If  $3x - 1 = 1 - 3x$ , then  $x =$  .....  
 (a) zero (b)  $\frac{1}{3}$  (c) - 1 (d) 3
- 6 Which of the following values for the number  $x$  makes the range of the values  $x, 15, 20, 24$  equal 14 ?  
 (a) 30 (b) 25 (c) 19 (d) 10

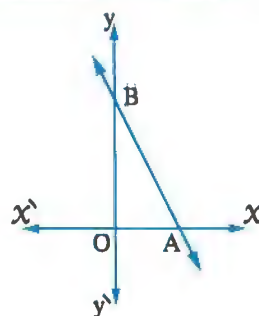
2 [a] If  $f = \{(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)\}$ , write :

- 1 The domain of  $f$
- 2 The range of  $f$
- 3 The rule of  $f$

[b] Two integers, the ratio between them is 2 : 3, if 7 is subtracted from each of them, the ratio becomes 1 : 2 Find the two numbers.

- 3 [a]** If  $X = \{-2, 2, 5\}$ ,  $Y = \{3, 7, \ell\}$  and  $R$  is a function from  $X$  to  $Y$  where " $a R b$ " means " $b = a^2 - 1$ " for each  $a \in X, b \in Y$
- 1** Find the value of  $\ell$  **2** Write  $R$
- 3** Represent the function by an arrow diagram.
- [b]** If  $y = a - 9$ ,  $y \propto \frac{1}{x^2}$  and  $a = 18$  when  $x = \frac{2}{3}$ , find the relation between  $x, y$  and find the value of  $y$  when  $x = 1$

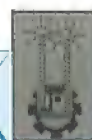
- 4** [a] The opposite figure represents the function  $f$  where  $f(x) = 4 - 2x$   
Find the coordinates of A , B and the area of  $\triangle AOB$



- [b]** If  $\frac{x}{7} = \frac{y}{3}$ , prove that :  $(2x - 3y)$ ,  $(x + 2y)$ , 10, 26 are proportional.

- 5** [a] Calculate the standard deviation of the values : 72 , 53 , 61 , 70 , 59
- [b] Graph the function  $f : f(x) = 1 - 4x + x^2$  where  $x \in [0, 4]$  and from the graph find :
- 1 The vertex of the curve.
  - 2 The equation of the axis of symmetry.
  - 3 The maximum or the minimum value of the function.

## Aswan Governorate



**Answer the following questions : (Calculator is allowed)**

- 1 Choose the correct answer :**
- 1** If  $X = \{1, 2\}$  ,  $Y = \{0\}$  , then  $n(X \times Y) = \dots\dots\dots$
- (a) 0 (b) 1 (c) 2 (d) 3
- 2**  $(\sqrt{5} - 2)(\sqrt{5} + 2) = \dots\dots\dots$
- (a) 5 (b) 3 (c) 2 (d) 1
- 3** The range of the set of the values : 16 , 32 , 5 , 27 and 20 is  $\dots\dots\dots$
- (a) 27 (b) 20 (c) 16 (d) 13
- 4** The third proportional for the numbers 8 , 6 , ... and 12 is  $\dots\dots\dots$
- (a) 24 (b) 20 (c) 16 (d) 8

5 If  $x = 3$ ,  $y = 5$ , then  $y^x = \dots\dots\dots$

- (a) 135                      (b) 125                      (c) 115                      (d) 95

6 If  $5x = 12$ , then  $10x = \dots\dots\dots$

- (a) 12                      (b) 22                      (c) 24                      (d) 34

2 [a] If  $X \times Y = \{(2, 2), (2, 5), (2, 7)\}$

, find : 1 Y

2  $Y \times X$

[b] If  $b$  is the middle proportional between  $a$  and  $c$ , prove that :  $\frac{a-b}{a-c} = \frac{b}{b+c}$

3 [a] If  $X = \{2, 3, 5\}$ ,  $Y = \{4, 6, 8, 10\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $2a = b$ " for each  $a \in X, b \in Y$

1 Write  $R$  and represent it by an arrow diagram.

2 Is  $R$  a function ?

[b] If  $y$  varies inversely as  $x$  and  $y = 2$  when  $x = 4$

, find the relation between  $y$  and  $x$ , then find the value of  $y$  when  $x = 16$

4 [a] If the point  $(a, 3)$  is located on the straight line which represents the function

$f: \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = 4x - 5$ , find the value of :  $a$

[b] If  $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a-b+5c}{3x}$ , find the value of :  $x$

5 [a] Represent graphically the function  $f: f(x) = (x-3)^2$ , taking  $x \in [0, 6]$  and from the graph deduce the coordinates of the vertex point of the curve, the maximum or minimum value of the function and the equation of the axis of symmetry.

[b] The following frequency distribution shows the number of children of some families in a new city :

Number of children	Zero	1	2	3	4
Number of families	8	16	50	20	6

Calculate the mean and the standard deviation of the number of children.

23

New Valley Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 If  $\sqrt[3]{x} = \sqrt{16}$  , then  $x = \dots\dots\dots$

(a) 4

(b) 8

(c) 16

(d) 64

2 If 2 ,  $x$  , 4 and 6 are proportional , then  $x = \dots\dots\dots$

(a) 1

(b) 3

(c) 5

(d) 8

3 If  $y = 2x$  , then  $y \propto \dots\dots\dots$

(a)  $\frac{1}{x}$ (b)  $x$ (c)  $x + 2$ (d)  $x - 2$ 

4  $2^{x-5} = 1$  where  $x \in \dots\dots\dots$

(a) 5

(b)  $\mathbb{R} - \{5\}$ (c)  $\mathbb{R}$ (d)  $\{5\}$ 

5 The middle proportional between 3 and  $\frac{1}{3}$  is  $\dots\dots\dots$

(a)  $\pm 1$ 

(b) 9

(c)  $\frac{1}{9}$ (d)  $\pm 9$ 

6 If  $\Sigma (x - \bar{x})^2 = 36$  for a set of values whose number equals 9 , then the standard deviation =  $\dots\dots\dots$

(a) 2

(b) 3

(c) 4

(d) 6

2 [a] If  $X = \{2, 3\}$  ,  $Y = \{3, 4, 5\}$  , then find :

1  $X \times Y$  and represent it by an arrow diagram.

2  $n(X \times Y)$

[b] If  $x^2 y^2 - 14xy + 49 = 0$  , then prove that :  $y \propto \frac{1}{x}$

3 [a] Find the negative number which if its square is added to each of the two terms of the ratio 7 : 11 , it becomes 4 : 5

[b] If  $X = \{2, 4, 8\}$  and  $R$  is a relation on  $X$  where " $a R b$ " means " $a$  is double  $b$ " for each  $a \in X$  ,  $b \in X$  , write  $R$  Is  $R$  a function ? and why ?

4 [a] If  $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - 5b + 3c}{x}$  , then find the value of each of :

1  $x$

2  $\frac{a+b+c}{b}$

[b] If the function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  ,  $f(x) = 2x - 3$  , then find the value of  $k$  if :

1  $f(k) = 5$

2  $(2, k) \in f$

- 5 [a] The following frequency distribution shows the numbers of children of some families in a new city :

Number of children ( $x$ )	3	5	7	9	11
Number of families ( $k$ )	3	12	21	10	4

Calculate the mean and the standard deviation to the number of children.

- [b] Represent graphically the curve of the function  $f$  where  $f(x) = (x+1)^2$ ,  $x \in [-3, 1]$  and from the drawing deduce :

- 1 The coordinates of the vertex of the curve.
- 2 The equation of the symmetry axis.
- 3 The minimum value of the function.

## 24 South Sinai Governorate



Answer the following questions :

- 1 Choose the correct answer :

- 1 The degree of the polynomial function  $f : f(x) = x^4 - 2x^2 + 5$  is .....  
 (a) fourth. (b) third. (c) second. (d) first.
- 2 The fourth proportional of 3 , 6 , 6 is .....  
 (a) 9 (b) 12 (c) 6 (d) 1
- 3 If  $n(X) = 5$  and  $n(X \times Y) = 15$  , then  $n(Y) =$  .....  
 (a) 20 (b) 10 (c) 3 (d) 2
- 4 The arithmetic mean of the values : 3 , 4 , 6 , 7 equals .....  
 (a) 40 (b) 20 (c) 10 (d) 5
- 5 If  $y^2 + 4x^2 = 4xy$  , then .....  
 (a)  $y \propto x$  (b)  $y \propto x^2$  (c)  $y \propto \frac{1}{x}$  (d)  $y \propto \frac{1}{x^2}$
- 6 If  $x$  is an odd number , then the next odd number is .....  
 (a)  $x^2$  (b)  $x^2 + x$  (c)  $x + 6$  (d)  $x + 2$

- 2 If  $X = \{2, 3, 4\}$  and  $Y = \{y : y \in \mathbb{N}, 2 \leq y < 9\}$  where  $\mathbb{N}$  is the set of natural numbers and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a = \frac{1}{2} b$ ",  $a \in X$  and  $b \in Y$  , write  $R$  Is  $R$  a function from  $X$  to  $Y$  ? Then find the range.

- 3 [a] Find the number which if we added it to each of the two terms of the ratio 7 : 11 , it becomes 2 : 3

- [b] If  $y \propto x$  and  $y = 14$  when  $x = 42$ , find the relation between  $x$  and  $y$ , then find the value of  $y$  when  $x = 60$

- 4 [a] Represent graphically the function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = 2x - 3$   
 [b] If  $b$  is the middle proportional between  $a$  and  $c$ , prove that :  $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

- 5 [a] If  $(x^3, y + 1) = (27, \sqrt[3]{125})$ , find the value of each of :  $x$  and  $y$   
 [b] Calculate the arithmetic mean and the standard deviation for the values :  
 20, 17, 22, 23, 18

## 25 North Sinai Governorate



Answer the following questions :

- 1 Choose the correct answer from those given :
- 1 If  $f(x) = 5$ , then  $f(5) + f(-5) = \dots\dots\dots$   
 (a) zero (b) 5 (c) -5 (d) 10
  - 2 If  $(x - 2, 3) = (5, 3)$ , then  $x = \dots\dots\dots$   
 (a) 5 (b) 3 (c) 7 (d) 8
  - 3 If  $f$  is an odd number, then the next odd number is  $\dots\dots\dots$   
 (a)  $f^2$  (b)  $f + 6$  (c)  $f + 2$  (d)  $f^2 + 1$
  - 4 The fourth proportional of the quantities 4, 8, 8 equals  $\dots\dots\dots$   
 (a) 4 (b) 8 (c) 12 (d) 16
  - 5 The sum of the two square roots of  $2\frac{1}{4}$  equals  $\dots\dots\dots$   
 (a)  $1\frac{1}{2}$  (b) zero (c)  $\frac{1}{2}$  (d)  $\sqrt{2}$
  - 6 The difference between the greatest value and the smallest value of a set of individuals is called  $\dots\dots\dots$   
 (a) the range. (b) the arithmetic mean.  
 (c) the median. (d) the standard deviation.
- 2 [a] If  $X = \{1, 2, 3\}$ ,  $Y = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{5}\}$  and  $R$  is a relation from  $X$  to  $Y$  where "a R b" means "a is the multiplicative inverse of b" for all  $a \in X, b \in Y$ , write  $R$  and represent it by an arrow diagram. Is  $R$  a function or not?  
 [b] If  $y \propto \frac{1}{x}$  and  $y = 3$  when  $x = 2$
- 1 Find the relation between  $x$  and  $y$
  - 2 Find the value of  $y$  when  $x = 1.5$

- 3 [a]** If  $f(x) = 5x + 4$  is represented graphically by a straight line passing through the point  $(3, b)$ , then find the value of :  $b$

**[b]** If  $\frac{x}{y} = \frac{3}{4}$ , find the value of :  $\frac{3x+y}{x+5y}$

- 4 [a]** If  $X \times Y = \{(1, 2), (4, 2), (5, 2)\}$ , then find :  $X, Y, Y^2$

**[b]** If  $b$  is the middle proportional between  $a$  and  $c$ , prove that :  $\frac{5c^2 - 2b^2}{5b^2 - 2a^2} = \frac{c}{a}$

- 5 [a]** Calculate the standard deviation to the following data : 12, 13, 16, 18 and 21

**[b]** Represent graphically the function  $f : f(x) = 2 - x^2$ ,  $x \in [-3, 3]$

From the graph deduce :

- 1** The coordinates of the vertex point of the curve.
- 2** The equation of the axis of symmetry.
- 3** The maximum or minimum value of the function.

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## Red Sea Governorate



Answer the following questions : (Calculators are allowed)

- 1 Choose the correct answer from those given :**

**1** If the point  $(a - 3, 5)$  lies on  $y$ -axis, then  $a =$  .....

- (a) 5 (b) 3 (c) 2 (d) 0

**2** If 2, 3, 6,  $x$  are proportional quantities, then  $x =$  .....

- (a) 9 (b) 18 (c) 12 (d) 3

**3** The range of the set of the values : 3, 5, 6, 7, 9 equals .....

- (a) 3 (b) 4 (c) 6 (d) 12

**4** If  $f(x) = 3$ , then  $f(5) + f(-5) =$  .....

- (a) -1 (b) 0 (c) 1 (d) 6

**5** If  $x - y = 5$ ,  $x + y = 1$ , then  $x^2 - y^2 =$  .....

- (a)  $\frac{1}{25}$  (b) 1 (c) 5 (d) 25

**6** If  $xy = 7$ , then  $y \propto$  .....

- (a)  $\frac{1}{x}$  (b)  $x - 7$  (c)  $x + 7$  (d)  $x$

- 2 [a]** If  $X \times Y = \{(1, 1), (1, 5), (1, 7)\}$ , find :

- 1**  $X$  **2**  $n(Y)$  **3**  $Y \times X$

**[b]** If  $b$  is the middle proportional between  $a$  and  $c$ , prove that :  $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

- 3** [a] If  $f(x) = 4x + a$  ,  $f(2) = 15$  , find the value of : a
- [b] If  $X = \{1, 2, 3\}$  ,  $Y = \{1, 2, 3, 4, 5\}$  and R is a relation from X to Y where "a R b" means " $a + b = 5$ " for each  $a \in X, b \in Y$
- 1** Write R and represent it by a Cartesian diagram.
- 2** Is R a function or not ?
- 
- 4** [a] If  $\frac{x}{y} = \frac{2}{3}$  , find the value of :  $\frac{3x + 2y}{6y - x}$
- [b] If  $y \propto x$  and  $y = 2$  when  $x = 6$  , find :
- 1** The relation between y and x
- 2** The value of y when  $x = 15$
- 
- 5** [a] Represent graphically the curve of the function f where  $f(x) = 4 - x^2$  and  $x \in [-3, 3]$  , from the graph deduce :
- 1** The coordinates of the vertex of the curve.
- 2** The equation of the axis of symmetry of the curve.
- [b] Find the standard deviation for the values : 12 , 13 , 16 , 18 , 26

27

## Matrouh Governorate



Answer the following questions : (Calculators are allowed)

- 1** Choose the correct answer from those given :
- 1** If a , b , 2 and 3 are proportional , then  $\frac{a}{b} = \dots\dots\dots$
- (a)  $\frac{2}{3}$  (b)  $\frac{3}{2}$  (c)  $\frac{3}{4}$  (d)  $\frac{4}{3}$
- 2**  $[1, 4] - ]1, 4[ = \dots\dots\dots$
- (a)  $\{0\}$  (b)  $\{1, 4\}$  (c)  $[1, 4[$  (d)  $\emptyset$
- 3** If  $(2, 5) \in \{3, 2\} \times \{1, x\}$  , then  $x = \dots\dots\dots$
- (a) 2 (b) 3 (c) 1 (d) 5
- 4** If  $(x - 1, 2^y) = (1, 8)$  , then  $(x, y) = \dots\dots\dots$
- (a) (2, 3) (b) (3, 2) (c) (0, 3) (d) (0, -3)
- 5** The point (3, -4) lies in the ..... quadrant.
- (a) first (b) second (c) third (d) fourth
- 6** If  $\sum (x - \bar{x})^2 = 36$  for a set of values whose number is 9 , then  $\sigma = \dots\dots\dots$
- (a) 2 (b) 4 (c) 18 (d) 27

- 2** [a] If  $X = \{1, 2, 3\}$ ,  $Y = \{1, 3, 6, 9, 12\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a = \frac{1}{3} b$ " for all  $a \in X, b \in Y$ , write  $R$  and show whether it is a function or not, and if it is a function, write the range.
- [b] If  $\frac{a}{b} = \frac{2}{5}$ , find the value of:  $\frac{2a-2b}{3a+2b}$

- 3** [a] If  $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$ , find: **1**  $X, Y$  **2**  $Y^2$
- [b] If  $\frac{x}{2a+b} = \frac{y}{2b-c} = \frac{z}{2c-a}$ , prove that:  $\frac{2x+y}{4a+4b-c} = \frac{2x+2y+z}{3a+6b}$

- 4** [a] If the point  $(a, 3)$  is located on the straight line which represents the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  where  $f(x) = 4x - 5$ , find the value of:  $a$
- [b] The following frequency distribution shows the number of children of some families in a new city:

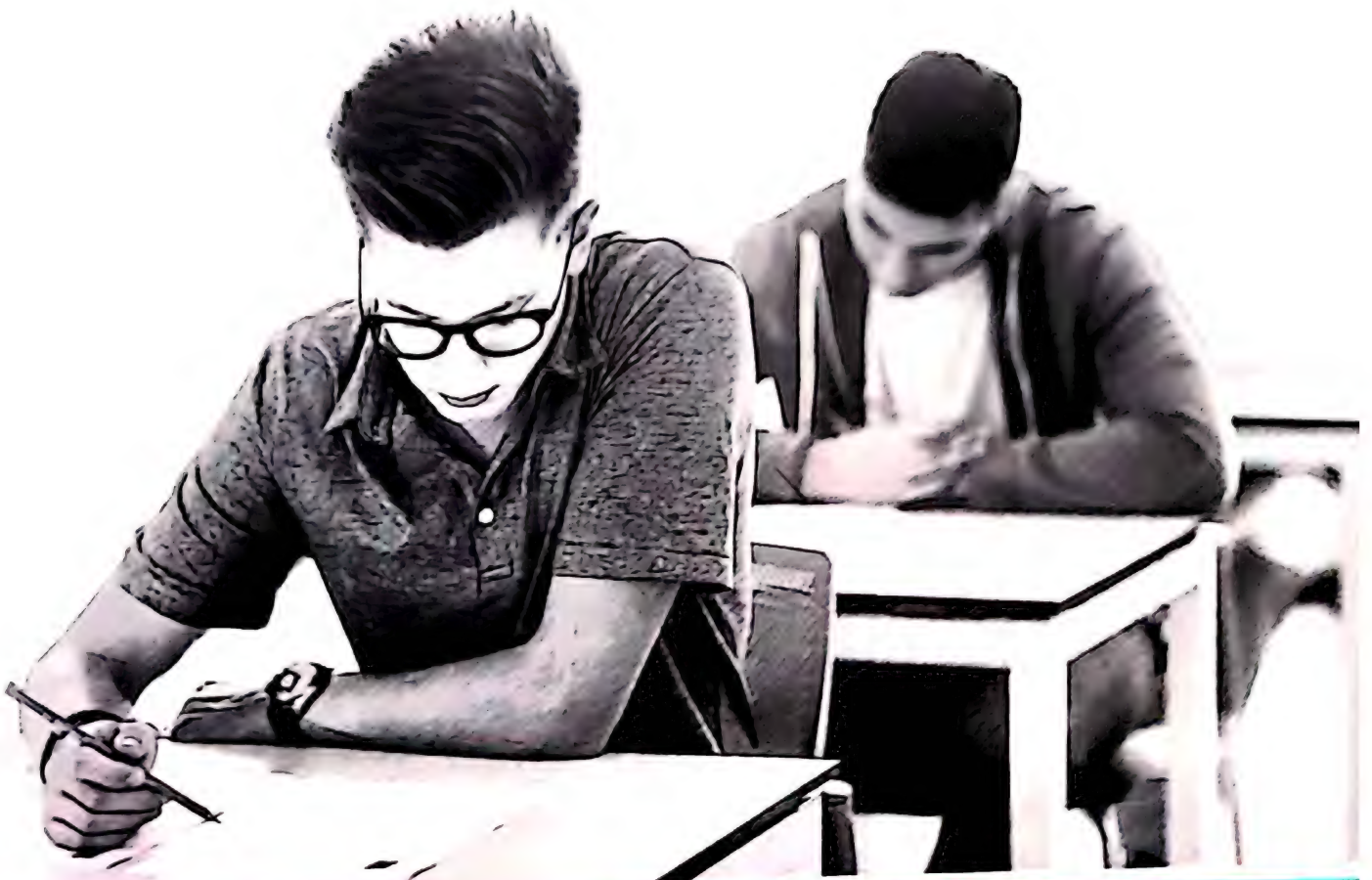
Number of children	Zero	1	2	3	4
Number of families	8	16	50	20	6

Calculate the mean and the standard deviation to the number of children.

- 5** [a] If  $y$  varies inversely as  $x$  and  $y = 10$  when  $x = 3$ , find the relation between  $x$  and  $y$ , then find the value of  $y$  when  $x = 5$
- [b] Represent graphically the function  $f: f(x) = (x-3)^2$ ,  $x \in [0, 6]$   
From the graph deduce the vertex of the curve, the maximum or minimum value of the function.

# Final Examinations 2021

on Algebra and Statistics



1

## Cairo Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 If  $(a + 3, b - 1) = (-2, 4)$ , then  $a + b = \dots\dots\dots$

- (a) 0 (b) 2 (c) 5 (d) 10

2 If  $X - y = 5$ , then  $6X - 6y = \dots\dots\dots$

- (a) 30 (b) 11 (c) 1 (d) -1

3 If  $X, 3, 4$  and  $6$  are proportional, then  $X = \dots\dots\dots$

- (a) 0 (b) 1 (c) 2 (d) 3

4  $\{3\} \cup ]3, 5] = \dots\dots\dots$

- (a)  $\emptyset$  (b)  $\{3\}$  (c)  $]3, 5]$  (d)  $[3, 5]$

5 The positive square root of mean of the squares of deviations of the values from their arithmetic mean is called  $\dots\dots\dots$

- (a) the range. (b) the standard deviation.  
(c) the median. (d) the mean.

6 If  $X^2 = 25$ , where  $X \in \mathbb{Z}$ , then  $X = \dots\dots\dots$

- (a) 5 (b) -5 (c)  $\pm 5$  (d) -25

2 [a] If  $X = \{2\}$ ,  $Y = \{3, 4, 5\}$ , find :

- 1  $X \times Y$  2  $n(Y^2)$  3  $X^2$

[b] If  $\frac{a}{b} = \frac{3}{5}$ , then find the value of :  $\frac{7a + 9b}{4a + 2b}$  in the simplest form.

3 [a] If  $y \propto \frac{1}{x}$  and  $y = 3$ , when  $x = 2$ , find :

- 1 The relation between  $y$  and  $x$  2 The value of  $y$  when  $x = 1.5$

[b] If  $X = \{1, 3, 4, 5\}$ ,  $Y = \{2, 3, 4, 5, 6\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + b = 7$ " for all  $a \in X, b \in Y$  write  $R$  and represent it by an arrow diagram. Is  $R$  a function? Why?

4 [a] The following frequency distribution shows the ages of 10 children :

Ages in years	5	8	9	10	12	Total
Number of children	1	2	3	3	1	10

Calculate the standard deviation to ages in years.

[b] Graph the curve of the function  $f : f(x) = x^2 + 2x - 4$ , where  $x \in [-4, 2]$

From the graph find :

1 The vertex of the curve.

2 The equation of the axis of symmetry.

5 [a] If  $b$  is the middle proportional between  $a$  and  $c$ , prove that :  $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

[b] If  $f(x) = x^2 - 2x$ ,  $g(x) = x - 2$

1 Prove that :  $f(2) = g(2)$

2 If  $g(k) = 7$ , find : the value of  $k$

2

Giza Governorate



Answer the following questions :

1 Choose the correct answer :

1 If  $x \in \mathbb{R}$  and  $1 < x < 3$ , then  $(3x - 1) \in \dots\dots\dots$

(a)  $]2, 8[$  (b)  $[2, 8]$  (c)  $]2, 8]$  (d)  $\{2, 8\}$

2 The range of the set of the values : 7, 3, 6, 5, 9 is .....

(a) 3 (b) 4 (c) 6 (d) 12

3 Half of the number  $4^{20} = \dots\dots\dots$

(a)  $2^{20}$  (b)  $2^{39}$  (c)  $2^{29}$  (d)  $4^{19}$

4 If  $X, Y$  are two non empty sets and  $n(X^2) = 4$ ,  $n(X \times Y) = 6$ , then  $n(Y^2) = \dots\dots\dots$

(a) 4 (b) 9 (c) 16 (d) 12

5 If  $a \times \frac{b}{3} = \frac{a}{3}$ , then  $b = \dots\dots\dots$

(a)  $-a$  (b) 1 (c)  $\frac{a}{3}$  (d)  $a$

6 If  $xy = 7$ , then  $y \propto \dots\dots\dots$

(a)  $\frac{1}{x}$  (b)  $x - 7$  (c)  $x$  (d)  $x + 7$

2 [a] If  $(x + 3, 9) = (5, y^2)$ , then find : the value of each of  $x$  and  $y$

[b] If  $y \propto \frac{1}{x}$  and  $y = 4$  when  $x = 2$ , then find :

1 The relation between  $x$  and  $y$

2 The value of  $y$  when  $x = 8$

3 [a] If  $X = \{0, 1, 2, 3, 4, 5, 6\}$  and  $R$  is a relation on  $X$  "where  $a R b$ " means " $a$  double  $b$ " for all  $a \in X, b \in X$

1 Write  $R$  as a set of ordered pairs and show if it is a function or not.

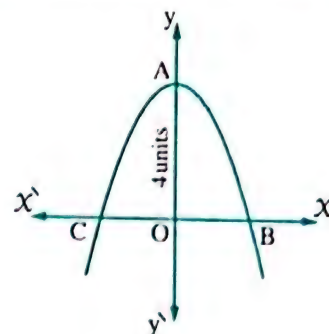
2 Is  $2 R 4$ ?

3 Find the value of  $x$  if  $6 R x$

[b] If  $b$  is the middle proportional between  $a$  and  $c$ , then prove that :  $\frac{a-b}{a-c} = \frac{b}{b+c}$

- 4 [a] The opposite figure represents the curve of the function  $f : f(x) = m - x^2$ . If  $OA = 4$  units, then find :

- 1 The value of  $m$
- 2 The coordinates of the two points B and C
- 3 The area of the triangle whose vertices are A, B, C



- [b] If the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  where  $f(x) = 2x + a$  and  $f(3) = 9$ , then find :

- 1 The value of  $a$
- 2 The coordinates of the intersection point of the straight line representing the function with  $x$ -axis

- 5 [a] If  $\frac{x}{2} = \frac{y}{3} = \frac{z}{4} = \frac{2x - y + 5z}{3m}$ , then find : the value of  $m$

- [b] Find the standard deviation of the values : 4, 8, 12, 10, 6

3

Alexandria Governorate



Answer the following questions : (Calculators are allowed)

- 1 Choose the correct answer from those given :

- 1 If  $(3, 5) \in \{3, 6\} \times \{x, 8\}$ , then  $x = \dots\dots\dots$

- (a) 8 (b) 6 (c) 5 (d) 3

- 2 A quarter of the number  $2^8$  is  $\dots\dots\dots$

- (a)  $2^6$  (b)  $2^{10}$  (c)  $\left(\frac{1}{2}\right)^8$  (d)  $\left(\frac{1}{2}\right)^6$

- 3 If  $\frac{3a}{5b} = \frac{1}{2}$ , then  $\frac{a}{b} = \dots\dots\dots$

- (a)  $\frac{6}{5}$  (b)  $\frac{5}{6}$  (c)  $\frac{2}{3}$  (d)  $\frac{3}{2}$

- 4 If  $x$  is an odd number, then the next odd number directly is  $\dots\dots\dots$

- (a)  $x^2$  (b)  $x^2 + 1$  (c)  $x + 1$  (d)  $x + 2$

- 5  $\frac{\text{Sum of the values}}{\text{Their number}}$  is  $\dots\dots\dots$

- (a) the range. (b) the standard deviation.  
(c) the mode. (d) the arithmetic mean.

- 6 If  $3 > x > 1$ ,  $x \in \mathbb{R}$ , then  $(3x - 1) \in \dots\dots\dots$

- (a)  $\{2, 8\}$  (b)  $]2, 8[$  (c)  $[2, 8]$  (d)  $[2, 8[$

2 [a] If  $X = \{2, 5\}$ ,  $Y = \{1, 2\}$ ,  $Z = \{3\}$

, find : 1  $n(X \times Z)$

2  $(Y \cap X) \times Z$

[b] Find the number which if its square is added to each of the two terms of the ratio 5 : 11 it becomes 3 : 5

3 [a] If  $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$ , find the value of :  $\frac{2y - z}{3x - 2y + z}$

[b] If  $X = \{1, 3, 4, 5\}$ ,  $Y = \{1, 2, 3, 4, 5, 6\}$ , and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + b = 7$ " for all  $a \in X$ ,  $b \in Y$ , write the relation  $R$  and represent it by an arrow diagram. Is  $R$  a function ? and why ?

4 [a] If  $y$  varies inversely with  $x$ ,  $y = 2$  when  $x = 4$ , find :

1 The relation between  $y$  and  $x$

2 The value of  $y$  when  $x = 16$

[b] The following frequency distribution shows the ages of 20 persons :

Ages in years	15	20	22	23	25	30	Total
Number of persons	2	3	5	5	1	4	20

Calculate the standard deviation to ages.

5 [a] Represent graphically the function  $f$  where  $f(x) = 4 - x^2$  taking  $x \in [-3, 3]$  and from the drawing deduce :

1 The coordinates of the vertex of the curve.

2 The maximum or the minimum value of the function.

3 The equation of the symmetry axis

[b] If  $f(x) = 5x - a$ ,  $r(x) = x - 2a$  and  $f(1) + r(3) = -7$ , find : the value of  $a$

4

El-Kalyoubia Governorate



Answer the following questions :

1 Choose the correct answer from the given answers :

1 If the point  $(5, b - 7)$  lies on the  $x$ -axis, then  $b = \dots\dots\dots$

(a) 2

(b) 5

(c) 7

(d) 12

2 If  $f(x) = 7$ , then  $f(7) + f(-7) = \dots\dots\dots$

(a) 7

(b) -7

(c) -14

(d) 14

3 If  $\sqrt[3]{-27} = -\sqrt{x}$ , then  $x = \dots\dots\dots$

- (a) 9 (b) -9 (c) 3 (d) -3

4 If  $\frac{a}{3} = \frac{b}{4}$ , then  $8a - 6b + 4 = \dots\dots\dots$

- (a) 3 (b) 4 (c) 5 (d) 6

5 If  $X = \{2\}$ , then  $X^2 = \dots\dots\dots$

- (a) 4 (b)  $\{4\}$  (c)  $(2, 2)$  (d)  $\{(2, 2)\}$

6 The positive square root of the average of squares of deviations of the values from their mean is called  $\dots\dots\dots$

- (a) the mean. (b) the range.  
(c) the standard deviation. (d) the mode.

2 [a] If  $y$  varies inversely as  $x$  and  $y = 3$  as  $x = 2$

1 Find the relation between  $x$  and  $y$

2 Find the value of  $y$  when  $x = \frac{3}{2}$

[b] If  $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3x}$ , find : the value of  $x$

3 [a] If  $X = \{1, 3, 5\}$ ,  $Y = \{2, 3, 4, 5, 6\}$  and  $R$  is a relation from  $X$  to  $Y$ , where " $a R b$ " means " $a + b = 7$ " for each  $a \in X, b \in Y$

1 Write  $R$  and represent it by an arrow diagram.

2 Is  $R$  a function? and why?

[b] If  $b$  is the middle proportional between  $a$  and  $c$ , prove that :  $\frac{a-c}{a-b} = \frac{b+c}{b}$

4 [a] If  $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$

Find : 1  $X, Y$

2  $Y \times X$

[b] Represent graphically the function  $f : f(x) = 2 - x^2$  since  $x \in [-3, 3]$

and find from the drawing deduce :

- 1 The coordinates of the vertex of the curve. 2 The maximum value of the function.  
3 The equation of the symmetry axis.

5 [a] If  $X = \{1, 3, 5\}$  and  $R$  is a function on  $X$  where  $R = \{(a, 3), (b, 1), (1, 5)\}$ , find :

1 The range of the function.

2 The numerical value of  $a + b$

[b] Find the mean and the standard deviation for the following frequency distribution :

Set	Zero -	2 -	4 -	6 -	8 - 10	Total
Frequency	1	3	6	5	5	20



Answer the following questions : (Calculators are allowed)

1 Choose the correct answer from those given :

- 1 If  $(X + 2, y) = (2, 3)$ , then  $X^5 y + 1 = \dots\dots\dots$   
 (a) 3 (b) 2 (c) zero (d) 1
- 2 If  $a \in X^2$  where  $X = \{x : 5 < x < 7, x \in \mathbb{N}\}$ , then  $a$  is  $\dots\dots\dots$   
 (a) 36 (b)  $\{36\}$  (c)  $(6, 6)$  (d)  $[5, 7]$
- 3 If  $y$  varies directly as  $X$ , then  $\dots\dots\dots$   
 (a)  $X$  varies inversely as  $y$  (b)  $X$  varies directly as  $y$   
 (c)  $y = X + 5$  (d)  $\frac{X}{3} = \frac{2}{y}$
- 4 If  $\frac{a}{b} = \frac{c}{d} = \frac{h}{m}$ , then  $\frac{a+c+h}{b+d+m} = \dots\dots\dots$   
 (a)  $\frac{a}{b} + \frac{c}{d} + \frac{h}{m}$  (b)  $\frac{c}{h}$  (c)  $\frac{c}{a}$  (d)  $\frac{c}{d}$
- 5 If  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(X) = X^{k-2} + 3$  and  $f(2) = 11$ , then  $k = \dots\dots\dots$   
 (a) 5 (b) 3 (c) 2 (d)  $-3$
- 6 If the range of the values  $6 + k, 6 - k, 6 + 5k, 6 - 2k$  is 14 where  $k \in \mathbb{N}$ , then  $k = \dots\dots\dots$   
 (a) 1 (b) 2 (c) 3 (d) 4

2 [a] If  $X = \{2, 3, 4\}$ ,  $Y = \{3, 4, 5\}$ ,  $Z = \{3, 4\}$ , find :

- 1  $(Y \cap X) - Z$  2  $X \times Z$  3  $n(X \cup Z)$

[b] If  $a, b, c$  and  $d$  are in continued proportion, prove that :  $\frac{2a^3 + 3b^3}{2a + 3d} = a^2$

3 [a] If  $X = \{3, 4, 5\}$ ,  $Y = \{1, 2, 3\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means that " $a + b$  is a prime number" for all  $a \in X$  and  $b \in Y$ , write  $R$  and represent it by an arrow diagram. Is  $R$  a function or not? And why?

[b] If  $y$  varies inversely as  $\sqrt{x}$ , and  $y = 3$  when  $x = 9$ , find  $x$  at  $y = 4$

4 [a] Represent graphically the curve of the function  $f : f(x) = (x - 2)^2 + 1$  where  $x \in [\text{zero}, 4]$  From the graph find :

- 1 The equation of the axis of symmetry of the curve.  
 2 The minimum value of the function.

[b] If  $\frac{a}{4} = \frac{b}{3}$ , find the value of :  $\frac{ab + a^2}{ab - b^2}$

- 5** [a] Find the arithmetic mean and the standard deviation of the values : 6 , 3 , 9 , 2 , 5
- [b] If the function  $f : f(x) = x^{m-3} + x^{4-m}$  is a polynomial function where  $m \in \mathbb{N}$
- Find : **1** The value of  $f(1)$  **2** The value of  $m$

**6****El-Monofia Governorate**

Answer the following questions : (Using calculator is permitted)

- 1** Choose the correct answer :
- 1** If  $\frac{5}{4} + \frac{5}{x} = \frac{5}{2}$  , then  $x = \dots\dots\dots$
- (a) 2 (b) 4 (c) 5 (d)  $\frac{5}{2}$
- 2** If  $x + y = xy = 5$  , then  $x^2 y + xy^2 = \dots\dots\dots$
- (a) 10 (b) 15 (c) 20 (d) 25
- 3** If  $1 < x < 3$  ,  $x \in \mathbb{R}$  , then  $(3x - 1) \in \dots\dots\dots$
- (a)  $[2, 8[$  (b)  $[2, 8]$  (c)  $]2, 8[$  (d)  $\{2, 8\}$
- 4** If  $\frac{a+2b}{a-b} = \frac{2}{3}$  , then  $\frac{b}{a} = \dots\dots\dots$
- (a)  $\frac{1}{8}$  (b) 8 (c)  $-\frac{1}{8}$  (d) -8
- 5** Which of the following values of the number  $x$  makes the range of the set of the values  $x, 15, 20, 24$  equal to 14 ?
- (a) 30 (b) 25 (c) 19 (d) 10
- 6** If  $x \in \mathbb{R}_-$  , then the point  $(-x, \sqrt[3]{x})$  lies in the ..... quadrant.
- (a) first (b) second (c) third (d) fourth

- 2** [a] If  $X = \{4, 3\}$  ,  $Y = \{5, 4\}$  and  $Z = \{5, 6\}$  , find :

**1**  $X \times (Y \cap Z)$  **2**  $(X - Y) \times Z$  **3**  $n(Z^2)$

[b] If  $a, b, c$  and  $d$  are in continued proportion , prove that :  $\frac{ab - cd}{b^2 - c^2} = \frac{a + c}{b}$

- 3** [a] If  $X = \{-2, -1, 1, 2\}$  ,  $Y = \{8, \frac{1}{3}, -1, 1, -8\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $b = a^3$ " for each  $a \in X, b \in Y$
- 1** Write  $R$  and represent it by an arrow diagram.
- 2** Show that  $R$  is a function and find its range.
- [b] If the straight line that represents the function  $f$  where  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = ax + b$  cuts  $y$ -axis at the point  $(0, 3)$  and  $f(2) = 7$  , find : the values of  $a$  and  $b$

- 4** [a] Find the number that if its square is added to the terms of the ratio  $7 : 11$ , then it will become  $4 : 5$
- [b] If  $y$  varies inversely as  $X^2$  and  $X = 3$  when  $y = 4$ , find :
- 1 The relation between  $X$  and  $y$
  - 2 The value of  $X$  when  $y = 9$
- 5** [a] Draw the curve of the function  $f$  where  $f(X) = 1 - X^2$  taking  $X \in [-3, 3]$  and from the graph find :
- 1 The coordinates of the vertex of the curve.
  - 2 The equation of the axis of symmetry.
  - 3 The area of the triangle whose vertices are the intersection points of the curve with the two axes.

- [b] The following frequency distribution shows the number of children of some families in a new city :

Number of children	zero	1	2	3	4
Number of families	8	16	50	20	6

Calculate the mean and the standard deviation of the number of children.

7

## El-Gharbia Governorate



Answer the following questions : (Calculators are allowed)

- 1** Choose the correct answer :

- 1 The function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(X) = aX + b$  represents a linear function on condition  $a \in \dots\dots\dots$ 
  - (a)  $\mathbb{R}$
  - (b)  $\mathbb{R}_+$
  - (c)  $\mathbb{R} - \{0\}$
  - (d)  $\mathbb{R}_-$
- 2 The fourth proportional of the numbers : 4 , 12 , 16 is .....
  - (a) 24
  - (b)  $\pm 24$
  - (c) 48
  - (d)  $\pm 48$
- 3 If the weekly wages in pounds of a set of workers in a factory are 170 , 180 , 180 , 230 and 240 , then the median of wages equals .....
  - (a) 200
  - (b) 70
  - (c) 180
  - (d) 205
- 4 If  $X^2 + y^2 = 6$  ,  $XY = 5$  , then  $(X + y)^2 = \dots\dots\dots$ 
  - (a) 16
  - (b)  $\pm 16$
  - (c) 11
  - (d)  $\pm 11$
- 5 The relation which represents the direct variation between  $y$  and  $X$  is .....
  - (a)  $XY = 5$
  - (b)  $y = 3 - X$
  - (c)  $\frac{X}{3} = \frac{y}{5}$
  - (d)  $\frac{X}{3} = \frac{4}{y}$

- 6 If  $X = \{1, 3, 5\}$  and  $R$  is a function on  $X$  where  $R = \{(a, 3), (b, 1), (1, 5)\}$ , then the numerical value of  $a + b = \dots\dots\dots$

(a) 4 (b) 6 (c) 8 (d) other.

- 2 [a] If  $X = \{-1, \text{zero}, 2, 3\}$ ,  $Y = \{1, \text{zero}, \frac{1}{2}, \frac{1}{3}\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means "The number  $a$  is the multiplicative inverse of the number  $b$ " for each  $a \in X, b \in Y$ , write  $R$ , and represent it by an arrow diagram and show if  $R$  is a function or not? And why?

- [b] From the data of the following table answer the following questions :

$x$	2	4	6
$y$	6	3	2

- 1 Show the kind of variation between  $x$  and  $y$   
 2 Find the constant proportional.  
 3 Find the value of  $y$  when  $x = 3$

- 3 [a] If  $a, b, c$  and  $d$  are in continued proportion, prove that :  $\frac{a}{b+d} = \frac{c^3}{c^2 d + d^3}$

- [b] If  $X = \{6\}$ ,  $Y = \{2, 3\}$  and  $Z = \{2, 5, 6\}$ , find :

- 1  $n(X^2)$  2  $(Z - Y) \times (X \cap Z)$

- 4 [a] Two integers the ratio between them is  $2 : 3$ , if you add to the first 7 and subtract from the second 12, the ratio between them becomes  $5 : 3$ , find the two integers.

- [b] If the function  $f : f(x) = 3x - 6$  represents a straight line passing through the point  $(a, 2a)$ , find the value of  $a$ , and find the intersection point of the straight line with  $y$ -axis.

- 5 [a] Calculate the standard deviation for the following data :

16, 32, 5, 20, 27 rounding the result to one decimal place.

- [b] Represent graphically the function  $f : f(x) = (x - 2)^2$ , taking  $x \in [-1, 5]$  and from the graph deduce :

- 1 The equation of the axis of symmetry.  
 2 The maximum value or the minimum value of the function.



Answer the following questions : (Calculator is permitted)

1 [a] Choose the correct answer :

1 The point  $(X - 3, 2 - X)$  lies in the fourth quadrant , then  $X = \dots\dots\dots$

- (a) 4 (b) 3 (c) 2 (d) 1

2 If  $d(X) = cX + 8$  ,  $d(2) = 0$  , then  $c = \dots\dots\dots$

- (a) 8 (b) 6 (c) 4 (d) - 4

3 If  $a, 2, 4, b$  are in continued proportion , then  $a + b = \dots\dots\dots$

- (a) 2 (b) 4 (c) 6 (d) 9

[b] If  $b$  is the middle proportion between  $a$  and  $c$  , prove that :  $\frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{c}{a}$

2 [a] Choose the correct answer :

1 If  $y \propto X$  ,  $y \propto \frac{1}{d}$  , then  $y \propto \dots\dots\dots$

- (a)  $Xd$  (b)  $\frac{d}{X}$  (c)  $\frac{X}{d}$  (d)  $X^2d$

2 The standard deviation of the values  $5, 5, 5, 5$  equals  $\dots\dots\dots$

- (a) zero (b) 5 (c) 6 (d) 2

3 The function  $d : d(X) = X^2 - (X - 3)^2$  is of the  $\dots\dots\dots$  degree.

- (a) zero (b) first (c) second (d) third

[b] If  $(-1, 2)$  is the point of the vertex of the curve of the function  $d : d(X) = aX^2 - 6X + c$  , find : the value of  $c$

3 [a] If  $3a = 4b = 6c$  , find :  $a : b : c$  , and the value of :  $\frac{3a + 2b}{a + 4c}$

[b] If  $X = \{-2, -1, 0, 1, 2\}$  ,  $R$  is a relation on the set  $X$  where " $a R b$ " means " $a$  is the additive inverse of the number  $b$ " for every  $a \in X$  and  $b \in X$  , state  $R$  , then represent it by an arrow diagram , and mention giving reasons if  $R$  represents a function or not.

4 [a] If  $X = z + 8$  where  $z$  varies inversely as  $y$  and  $z = 2$  when  $y = 3$  , find the relation between  $y$  and  $X$  , then find :  $y$  when  $X = 3$

[b] If  $d(X) = 2X + 5$  ,  $r(X) = X - 6$  , prove that :  $d(2) + 3r(3) = 0$

5 [a] Calculate the mean and the standard deviation of the following data :  $5, 7, 8, 9, 6$

[b] If  $(X - 2, 3^{y-1}) = (3, 1)$  , find :  $X, y$

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## Ismailia Governorate



Answer the following questions : (Calculators are allowed)

1 Choose the correct answer from those given answers :

1  $\sqrt{36} + \sqrt{16} = \dots\dots\dots$

(a) 10

(b) 24

(c) 52

(d) 100

2 The middle proportional between 3 , 27 is .....

(a) 9

(b) - 9

(c)  $\pm 9$

(d) 1

3 If  $f(x) = 2$  , then  $f(2) + f(-2) = \dots\dots\dots$

(a) zero

(b) 4

(c) - 4

(d) 1

4 The positive number which twice its square equals 50 is .....

(a) 5

(b) 10

(c) 25

(d) 100

5 If  $x + y = xy = 5$  , then  $x^2y + y^2x = \dots\dots\dots$

(a) 10

(b) 15

(c) 20

(d) 25

6 The simplest and easiest method of measuring dispersion is .....

(a) the range.

(b) the standard deviation.

(c) the arithmetic mean.

(d) the mode.

2 [a] If  $X = \{2, 3, 5\}$  ,  $Y = \{4, 6, 8, 10\}$  and R is a relation from X to Y where "a R b" means " $2a = b$ " for all  $a \in X, b \in Y$

1 Write R and represent it by an arrow diagram.

2 Is the relation R a function ? Why ? and if it's a function , find its range.

[b] The ratio between two integers is 3 : 7 If 5 is subtracted from each of them , then the ratio becomes 1 : 3 , find the two integers.

3 [a] As Yousef was reading a book , he found out after 3 hours 50 pages remained , after 6 hours 20 pages remained. If there was a relation between the time (t) and the number of pages (y) Is a linear relation.

1 Represent the relation between (t) and (y) , then find the algebraic relation between them.

2 How much time did Yousef takes to finish reading the book ?

3 How many pages left when Yousef started reading ?

[b] If  $x, y, z$  and  $l$  are proportional quantities , prove that :  $\frac{y-x}{x} = \frac{l-z}{z}$

- 4 [a] If  $y \propto X$  and  $y = 40$  at  $X = 14$ , find the relation between  $X$  and  $y$ , then find the value of  $X$  when  $y = 80$

[b] If  $X \times Y = \{(1, 2), (1, 3), (2, 2), (2, 3)\}$

, find : 1  $X \cup Y$

2  $n(Y^2)$

- 5 [a] Represent graphically the function  $f : f(X) = (X - 2)^2$ , taking  $X \in [-1, 5]$

And from the graph find :

1 The coordinates of the vertex of the curve. 2 The equation of the line of symmetry.

3 The maximum or the minimum value of the function.

- [b] Find the standard deviation for the following set of values : 13, 14, 17, 19, 22

10

Suez Governorate



Answer the following questions : (Calculators are allowed)

- 1 Choose the correct answer from those given :

1 If  $(2, 3) \in \{2, 5\} \times \{X, 6\}$ , then  $X = \dots\dots\dots$

(a) 6

(b) 5

(c) 3

(d) 2

2  $(\sqrt{5} - 3)(\sqrt{5} + 3) = \dots\dots\dots$

(a) 8

(b) 2

(c) 4

(d) -4

3 The positive square root of the mean of the squares of deviations of the values from their arithmetic mean is called .....

(a) the range.

(b) the arithmetic mean.

(c) the standard deviation.

(d) the mode.

4 If the number  $\frac{3}{b} + 1 = 4$ , then  $b = \dots\dots\dots$  where  $b \neq 0$

(a) 1

(b) 2

(c) 3

(d) 4

5  $\mathbb{Z} \cup \mathbb{N} = \dots\dots\dots$

(a)  $\emptyset$

(b)  $\mathbb{Z}$

(c)  $\mathbb{N}$

(d)  $\mathbb{R}$

6 If  $\frac{a}{b} = \frac{c}{d} = m$  (where  $m \in \mathbb{R}^*$ ), then  $\frac{ac}{bd} = \dots\dots\dots$

(a)  $m$

(b)  $m^2$

(c)  $2m$

(d)  $2m^2$

- 2 [a] If  $X = \{1, 2, 3\}$ ,  $Y = \{1, 4, 6, 9\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a = \sqrt{b}$ " for all  $a \in X, b \in Y$

1 Find the relation  $R$

2 Represent the relation  $R$  by an arrow diagram.

3 Is  $R$  a function? Why?

[b] If  $b$  is the middle proportional between  $a$  and  $c$ , prove that :  $\frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{c}{a}$

3 [a] If  $(2x, 4) = (8, y + 1)$ , find :  $\sqrt{x^2 + y^2}$

[b] If  $y \propto x$  and  $y = 2$  when  $x = 8$ , find :

1 The relation between  $y$  and  $x$

2 The value of  $y$  when  $x = 12$

4 [a] Draw the curve of the function  $f : f(x) = x^2 + 1$ , taking  $x \in [-2, 2]$  and from the graph find :

1 The coordinates of the vertex of the curve. 2 The equation of the axis of symmetry.

3 The minimum value.

[b] If  $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3x}$ , find :  $x$

5 [a] If  $X = \{1, 3, 5\}$  and  $R$  is a function on  $X$  where  $R = \{(a, 3), (b, 1), (1, 5)\}$ , find :

1 The range of the function. 2 The numerical value of the expression  $a + b$

[b] Calculate the standard deviation for the values : 8, 9, 7, 6, 5

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Port Said Governorate



Answer the following questions :

1 Choose the correct answer from those given :

1  $[1, 3] - \{0, 1\} = \dots\dots\dots$

(a)  $]1, 3[$  (b)  $]1, 3]$  (c)  $[1, 3[$  (d)  $\{3\}$

2 If  $2^x = 2^6$ , then  $x = \dots\dots\dots$

(a) 3 (b) 4 (c) 6 (d) 64

3 20% from 10 pounds =  $\dots\dots\dots$  pounds.

(a) 2 (b) 2.5 (c) 5 (d) 20

4 If  $n(X) = 3$ ,  $n(X \times Y) = 12$ , then  $n(Y) = \dots\dots\dots$

(a) 4 (b) 9 (c) 15 (d) 36

5 If  $3a = 4b$ , then  $a : b = \dots\dots\dots$

(a) 3 : 4 (b) 4 : 7 (c) 3 : 7 (d) 4 : 3

6 The range of the set of the values 7, 3, 6, 9 and 5 equals  $\dots\dots\dots$

(a) 3 (b) 4 (c) 6 (d) 12

- 2** [a] If  $X = \{2, 3, 4\}$ ,  $Y = \{2, 3, 4, 5, 6, 7, 8\}$ ,  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a = \frac{1}{2} b$ " for each  $a \in X$ ,  $b \in Y$ , write  $R$  and represent it by an arrow diagram. Show that  $R$  is a function from  $X$  to  $Y$  and find its range.
- [b] If  $f(X) = 4X + b$  and  $f(3) = 15$ , find : the value of  $b$
- 3** [a] If  $f(X) = X^2 - 3X$ ,  $g(X) = X - 3$
- 1** Find :  $f(\sqrt{2}) + 3g(\sqrt{2})$  **2** Prove that :  $f(3) = g(3) = 0$
- [b] Represent graphically the quadratic function  $f$  where  $f(X) = X^2$ ,  $X \in \mathbb{R}$ , consider  $X \in [-3, 3]$ , from the graph deduce the vertex of the curve, the minimum value of the function, the equation of the axis of symmetry.
- 4** [a] If  $b$  is the middle proportional between  $a$  and  $c$ , prove that :  $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$
- [b] If  $y \propto X$ , where  $y = 14$  when  $X = 42$ , then find :
- 1** The relation between  $X$  and  $y$  **2** The value of  $y$  when  $X = 60$
- 5** [a] Calculate the standard deviation for the values : 16, 32, 5, 20, 27
- [b] If the height of a right constant cylinder (constant volume) is  $(h)$  varies inversely as the square of its radius length  $r$  and  $h = 27$  cm. when  $r = 10.5$  cm., find  $h$  when  $r = 15.75$  cm.

12

Damietta Governorate



Answer the following questions : (Calculators are allowed)

- 1** Choose the correct answer from the given ones :
- 1** If  $n(X) = 3$ ,  $n(Y^2) = 4$ , then  $n(X \times Y) = \dots\dots\dots$
- (a) 6 (b) 12 (c) 18 (d) 36
- 2** The range of the set of the values 7, 4, 6, 9 and 5 equals  $\dots\dots\dots$
- (a) 3 (b) 4 (c) 5 (d) 6
- 3** If  $\frac{y}{X} = 5$ , then  $y \propto \dots\dots\dots$
- (a)  $X$  (b)  $\frac{1}{X}$  (c)  $X - 5$  (d)  $X + 5$
- 4** If  $\frac{3}{4} + \frac{3}{X} = \frac{3}{2}$ , then  $X = \dots\dots\dots$
- (a)  $\frac{3}{2}$  (b) 2 (c) 3 (d) 4
- 5** The third proportional of the two numbers 3 and 6 is  $\dots\dots\dots$
- (a)  $\frac{1}{2}$  (b) 2 (c) 9 (d) 12

6 The solution set of the equation  $(X - 1)^2 = 9$  in  $\mathbb{R}$  is .....

- (a)  $\{4\}$  (b)  $\{-2\}$  (c)  $\{4, -2\}$  (d)  $\{3\}$

2 [a] If  $X = \{1, 9, 6\}$ ,  $Y = \{3, 4, 5, 6\}$ ,  $Z = \{4\}$ , then find :  $(X - Y) \times Z$

[b] If  $b$  is the middle proportional between  $a$  and  $c$ , prove that :  $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

3 [a] If  $y \propto \frac{1}{x}$  and  $y = 3$  when  $x = 2$

1 Find the relation between  $x$  and  $y$

2 Find the value of  $y$  when  $x = 1.5$

[b] If  $\frac{y}{x-z} = \frac{x}{y} = \frac{x+y}{z}$ , prove that :

1 Each ratio is equal to 2 (unless  $x + y = 0$ )

2  $3y = 2z$

4 [a] If  $(x^3, y + 1) = (8, 3)$ , find the value of :  $\sqrt[3]{x + 3y}$

[b] If  $X = \{-1, 0, 2, 3\}$ ,  $Y = \{0, 1, 4, 6, 9\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ ", means " $a^2 = b$ " for each  $a \in X, b \in Y$

1 Write  $R$  and represent it by an arrow diagram

2 Show that  $R$  is a function from  $X$  to  $Y$  and find its range.

5 [a] Calculate the arithmetic mean and the standard deviation of the set of values :  
72, 53, 61, 70, 59

[b] Represent graphically the function  $f : f(x) = x^2 - 2, x \in [-3, 3]$

From the graph deduce : 1 The vertex of the curve.

2 The equation of the axis of symmetry.

13

Kafr El-Sheikh Governorate



Answer the following questions : (Calculators are permitted)

1 Choose the correct answer from those given :

1 If  $X = ]-\infty, 0[$ , then  $X^c = \dots\dots\dots$

- (a)  $\mathbb{R}_+$  (b)  $[0, \infty[$  (c)  $] -\infty, 0]$  (d)  $\mathbb{R}_-$

2 The function  $f : f(x) = (x - 2)^2 - x^2$  is of the ..... degree.

- (a) first (b) second (c) third (d) fourth

3 If  $\sum (x - \bar{x})^2 = 36$  of a set of values and the number of these values = 9 ,  
then  $\sigma = \dots\dots\dots$

- (a) 2 (b) 18 (c) 27 (d) 4

4 The middle proportional between  $3x^3$  and  $27x$  is  $\dots\dots\dots$

- (a)  $9x^2$  (b)  $\pm 9x^4$  (c)  $\pm 9x^2$  (d)  $9x^4$

5 If  $y^2 + 4x^2 = 4xy$  , then  $\dots\dots\dots$

- (a)  $y \propto x$  (b)  $y \propto x^2$  (c)  $y \propto \frac{1}{x}$  (d)  $y \propto \frac{1}{x^2}$

6 If  $\{2\} \times \{x, y\} = \{(2, 4), (2, 3)\}$  , then  $x - y = \dots\dots\dots$

- (a) 1 (b) -1 (c)  $\pm 1$  (d) zero

2 [a] If  $X = \{-1, 1, 2\}$  ,  $Y = \{2, 4, 6, 8\}$  and R is a relation from X to Y , where  
"a R b" means "b = 2a + 4" for each  $a \in X$  ,  $b \in Y$  , write R and represent it by an  
arrow diagram , and show if R is a function or not ? If R is a function mention its range.

[b] If :  $\frac{21x - y}{7x - z} = \frac{y}{z}$  , prove that :  $y \propto z$

3 [a] Represent graphically the function  $f : f(x) = x^2 - 2x$  ,  $x \in [-2, 4]$   
and from the graph deduce :

- 1 The equation of the line of symmetry.  
2 The maximum or the minimum value of the function.

[b] If a , b , c and d are in continued proportion , prove that :  $\frac{a}{b+d} = \frac{c^3}{c^2d + d^3}$

4 [a] If  $\frac{x+y}{3} = \frac{y+z}{8} = \frac{z+x}{6}$  , prove that :  $\frac{x+y+z}{2x+3y+3z} = \frac{17}{50}$

[b] If the point (a , 4) is one of the points of the function  $g : \mathbb{R} \longrightarrow \mathbb{R}$   
where  $g(x) = 2x + b$  , then find the value of :  $6a + 3b$

5 [a] The following table represents the daily wages of a set of workers in a factory :

Set of wages	20 -	30 -	40 -	50 -	60 -	70 -
Number of workers	10	12	8	6	3	1

Find the mean and the standard deviation of the wages.

[b] If the straight line which represents the function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  ,  $f(x) = ax + b$  cuts  
from the positive part of y-axis 3 length units and passes through the point (1 , 5) ,  
find : the value of each of a , b

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El-Beheira Governorate



Answer the following questions : (Calculator is permitted)

1 Choose the correct answer from the given ones :

1 If  $3^x = 9^2$ , then  $x = \dots\dots\dots$

- (a) 3 (b) 4 (c) 6 (d) 64

2 The range of the set of the values 7 , 3 , 6 , 8 and 5 equals .....

- (a) 3 (b) 8 (c) 11 (d) 5

3 If the point  $(x - 4, 2 - x)$  where  $x \in \mathbb{Z}$  is located in the third quadrant , then  $x = \dots\dots\dots$

- (a) 2 (b) 3 (c) 4 (d) 6

4 The relation which represents the direct variation between the two variables  $x$  and  $y$  is .....

- (a)  $xy = 7$  (b)  $y = x + 5$  (c)  $\frac{x}{3} = \frac{7}{y}$  (d)  $\frac{x}{2} = \frac{y}{5}$

5 The solution set of the equation  $x^2 - 25 = 0$  in  $\mathbb{R}$  is .....

- (a)  $\{5, -5\}$  (b)  $[-5, 5]$  (c) 5 (d) -5

6 If  $(3, 5) \in \{3, 6\} \times \{y, 8\}$ , then  $y = \dots\dots\dots$

- (a) 8 (b) 6 (c) 5 (d) 3

2 [a] If  $X = \{2, 3, 5\}$ ,  $Y = \{4, 6, 8, 10\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $2a = b$ " for all  $a \in X, b \in Y$

1 Write  $R$

2 Show that  $R$  is a function and find its range.

[b] If  $b$  is the middle proportional between  $a$  and  $c$ , then prove that :  $\frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{c}{a}$

3 [a] If  $y \propto \frac{1}{x}$  and  $y = 9$  when  $x = 2$ , find :

1 The relation between  $y$  and  $x$

2 The value of  $y$  when  $x = 3$

[b] If  $f(x) = 5x + a$  and  $f(2) = 12$ , find : the value of  $a$

4 [a] If  $X = \{3, 4\}$ ,  $Y = \{4, 5\}$ ,  $Z = \{6, 5\}$ , find :

1  $(X - Y) \times Z$

2  $n(X \times Y)$

[b] Find the number that if subtracted thrice of it from each of the two terms of the ratio  $\frac{49}{69}$  the ratio becomes  $\frac{2}{3}$

- 5 [a]** Calculate the mean and the standard deviation of the following data :

8 , 13 , 20 , 16 , 18 , 21

- [b]** Represent graphically the function  $f$  where  $f(x) = 3 - x^2$ , where  $x \in [-3, 3]$  and from the graph deduce :

- 1** The equation of the symmetry axis.      **2** The maximum value of the function.

**15**

**El-Fayoum Governorate**



**Answer the following questions : (Using calculators is allowed)**

- 1** Choose the correct answer :

- 1** If  $(x + 1, \sqrt[3]{27}) = (-1, y)$ , then the point  $(x, y)$  lies in the ..... quadrant.  
 (a) first                      (b) second                      (c) third                      (d) fourth
- 2** If  $\frac{3}{4} + \frac{3}{x} = \frac{3}{2}$ , then  $x =$  .....  
 (a) 2                      (b) 4                      (c) 3                      (d)  $\frac{3}{2}$
- 3** Twice of the number  $2^8$  is .....  
 (a)  $2^{10}$                       (b)  $2^{16}$                       (c)  $2^4$                       (d)  $2^9$
- 4** If  $xy = 12$ , then  $y$  varies directly as .....  
 (a)  $\frac{1}{x}$                       (b)  $x - 12$                       (c)  $x$                       (d)  $x + 12$
- 5** Omar bought 4 notebooks and 3 pens, he paid 50 pounds for them. If the price of a pen is twice the price of a notebook, then the price of a notebook is ..... pounds.  
 (a) 4                      (b) 5                      (c) 10                      (d) 20
- 6** If the range of the set of the values 7,  $x$ , 8, 9 and 5 is 6, then  $x =$  .....  
 (a) 3                      (b) 4                      (c) 6                      (d) 12

- 2 [a]** If  $X = \{2, 5\}$ ,  $Y = \{1, 2\}$ ,  $Z = \{3\}$ , find :

- 1**  $n(X \times Y)$                       **2**  $(Y \cap X) \times Z$

- [b]** If  $a = 2b$ , find the value of :  $\frac{8a + 5b}{7a - 2b}$

- 3 [a]** If  $X = \{1, 2, 3\}$ ,  $Y = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\}$  and  $R : X \longrightarrow Y$ , where " $a R b$ " means " $a$  is the multiplicative inverse of  $b$ " for all  $a \in X, b \in Y$

- 1** Write  $R$  and represent it by an arrow diagram  
**2** Is  $R$  a function ? Write its range.

- [b]** If  $f(x) = 4x + a$ ,  $f\left(\frac{1}{4}\right) = 12$ , find : the value of  $a$

- 4 [a] If  $a, b, c$  and  $d$  are in continued proportion, prove that :  $\frac{a}{b+d} = \frac{c^3}{c^2d+d^3}$

[b] If  $y$  varies inversely as  $X$ , and  $y = 3$  when  $X = 2$

1 Find the relation between  $X$  and  $y$

2 Find the value of  $y$  when  $X = 3$

- 5 [a] Graph the function  $f$  where  $f(X) = 4 - X^2, X \in [-3, 3]$ , from the graph determine :

1 The coordinates of the vertex of the curve.

2 The equation of the symmetry axis of this function.

- [b] Calculate the mean and the standard deviation of the following data :  
3, 6, 7, 9, 15

16

## Beni Suef Governorate



Answer the following questions : (Calculator is allowed)

- 1 Choose the correct answer from those given :

1 The middle proportional between  $a$  and  $c$  equals .....

(a)  $\sqrt{a+c}$

(b)  $\frac{a+c}{2}$

(c)  $\pm\sqrt{ac}$

(d)  $ac$

2 The difference between the greatest value and the smallest value of a set of data is called .....

(a) the range.

(b) the arithmetic mean.

(c) the mode.

(d) the standard deviation.

3  $|\sqrt[3]{-8}| = \dots\dots\dots$

(a) 4

(b) 2

(c) -2

(d)  $\pm 2$

4  $\frac{7}{X}$  is a rational number if  $X \neq \dots\dots\dots$

(a) 7

(b) -7

(c) 1

(d) zero

5 If the point  $(a, 3-a)$  lies on the  $X$ -axis, then  $a = \dots\dots\dots$

(a) zero

(b) 3

(c) -3

(d) 5

6 If  $-X > 3$ , then  $X \in \dots\dots\dots$

(a)  $\{-3\}$

(b)  $]3, \infty[$

(c)  $] -\infty, 3[$

(d)  $] -\infty, -3[$

- 2 [a] If  $X = \{2, 5\}$ ,  $Y = \{3, 2\}$ ,  $Z = \{3\}$ , find :

1  $X \times Z$

2  $Y^2$

3  $(X \cap Y) \times Z$

[b] Find the positive number which if we add its square to each of the two terms of the ratio 5 : 11 it becomes 3 : 5

3 [a] If  $f(x) = x^2 - \sqrt{2}x$  ,  $g(x) = x + 1$

1 Find :  $f(3) + 3g(\sqrt{2})$

2 Prove that :  $f(\sqrt{2}) = g(-1)$

[b] If  $y$  varies inversely with  $x$  and  $y = 3$  when  $x = 2$  , find :

1 The relation between  $x$  and  $y$

2 The value of  $y$  when  $x = 1.5$

4 [a] If  $X = \{1, 2, 3\}$  ,  $Y = \{6, 7, 8\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + b = 8$ " for all  $a \in X$  and  $b \in Y$  , write  $R$  and represent it by an arrow diagram. Is  $R$  a function ? Why ?

[b] If  $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$  , prove that :  $\frac{2y - z}{3x - 2y + z} = \frac{1}{2}$

5 [a] Calculate the arithmetic mean and the standard deviation of the following values :  
7 , 16 , 13 , 9 , 5

[b] Represent graphically the function  $f : f(x) = x^2 - 2x$  where  $x \in [-1, 3]$  and from the drawing deduce the equation of the axis of symmetry and the maximum or minimum value of the function.

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El-Menia Governorate



Answer the following questions : (Calculators are allowed)

1 Choose the correct answer from the given ones :

1  $(\sqrt{7} - \sqrt{5})(\sqrt{7} + \sqrt{5}) = \dots\dots\dots$

(a) 2

(b) 12

(c)  $2\sqrt{7}$

(d)  $-2\sqrt{5}$

2 If  $xy = 3$  , then  $y \propto \dots\dots\dots$

(a)  $x$

(b)  $x - 3$

(c)  $\frac{1}{x}$

(d)  $x + 3$

3  $[1, 3] - \{0, 1\} = \dots\dots\dots$

(a)  $]1, 3[$

(b)  $]1, 3]$

(c)  $[1, 3[$

(d)  $\{3\}$

4 The arithmetic mean of the set of values 8 , 9 , 7 , 6 and 5 equals .....

(a) 5

(b) 2

(c) 3

(d) 7

5 20% of 10 pounds = ..... pounds.

(a) 2

(b) 2.5

(c) 5

(d) 20

6 If the point  $(x - 4, 2 - x)$  where  $x \in \mathbb{Z}$  is located in the third quadrant , then  $x = \dots\dots\dots$

(a) 2

(b) 3

(c) 4

(d) 6

**2** [a] Find the standard deviation of the values : 6 , 8 , 10 , 12 and 14

[b] If  $X = \{2, 5\}$  ,  $Y = \{1, 2\}$  ,  $Z = \{3\}$  , find :

**1**  $n(X \times Z)$

**2**  $(Y \cap X) \times Z$

**3** [a] If  $y \propto \frac{1}{x}$  and  $y = 3$  when  $x = 2$  , find :

**1** The relation between  $x$  and  $y$

**2** The value of  $y$  when  $x = 1.5$

[b] If  $X = \{1, 3, 4, 5\}$  ,  $Y = \{1, 2, 3, 4, 5, 6\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + b = 7$ " for each  $a \in X$  ,  $b \in Y$  , write  $R$  and represent it by an arrow diagram. Show if  $R$  is a function from  $X$  to  $Y$  or not. Give the reason.

**4** [a] If  $\frac{x}{y} = \frac{2}{3}$  , find the value of the ratio :  $\frac{3x + 2y}{6y - x}$

[b] If the function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = 4x - a$  is represented graphically by a straight line intersecting the  $x$ -axis at the point  $(2, b)$  , find :  $a, b$

**5** [a] If  $b$  is the middle proportional between  $a$  and  $c$  , prove that :  $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

[b] Represent graphically the following function and from the drawing deduce the coordinates of the curve , and the equation of the symmetry axis and the minimum or the maximum value of the function  $f : f(x) = x^2 - 2$  , where  $x \in [-3, 3]$

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Assiut Governorate



**Answer the following questions : (Calculator is allowed)**

**1** Choose the correct answer :

**1**  $[2, 5] \cup \{2\} = \dots\dots\dots$

(a)  $]2, 5[$

(b)  $[2, 5[$

(c)  $] - \infty, \infty[$

(d)  $[2, 5]$

**2**  $\sqrt{10^2 - 8^2} = \dots\dots\dots$

(a) 8

(b) 6

(c) 4

(d) 2

**3** The solution set of the equation :  $x(x - 1) = 0$  in  $\mathbb{R}$  is  $\dots\dots\dots$

(a)  $\{0\}$

(b)  $\{1\}$

(c)  $\{0, 1\}$

(d)  $\emptyset$

**4** If  $3a = 8b$  , then  $a : b = \dots\dots\dots$

(a)  $-8 : 3$

(b)  $8 : 3$

(c)  $3 : 8$

(d)  $-3 : 8$

**5** If  $xy = 5$  , then  $y \propto \dots\dots\dots$

(a)  $\frac{1}{x}$

(b)  $x - 5$

(c)  $x$

(d)  $\frac{1}{y}$

6 If a regular die is thrown once, then the probability of appearance of an odd number is .....

(a) zero

(b)  $\emptyset$ 

(c) 1

(d)  $\frac{1}{2}$ 

2 [a] If  $X = \{1, 5, 6\}$ ,  $Y = \{2, 4, 5\}$ , find  $X \times Y$  and represent it by an arrow diagram.

[b] Represent graphically the quadratic function  $f : f(x) = x^2 - 1$ ,  $x \in [-2, 2]$ , from the graph deduce :

1 The equation of the axis of symmetry.

2 The maximum value or the minimum value of the function.

3 [a] If  $f(x) = 4x + m$ ,  $f(3) = 15$ , find : the value of  $m$

[b] If  $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a - b + 5c}{3x}$ , then find : the value of  $x$

4 [a] If  $y \propto x$ ,  $y = 3$  when  $x = 2$ , find :

1 The relation between  $y$ ,  $x$

2 The value of  $y$  when  $x = \frac{1}{3}$

[b] If  $b$  is the middle proportional between  $a$  and  $c$ , then prove that :  $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

5 [a] If  $X = \{1, 3, 5\}$ ,  $Y = \{2, 3, 4, 6\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + b = 7$ " for each  $a \in X$ ,  $b \in Y$

1 Write  $R$  and represent it by an arrow diagram.

2 Show if  $R$  is a function or not. If  $R$  is a function, find its range.

[b] Calculate the mean and the standard deviation for the values : 8, 9, 7, 6, 5

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Souhag Governorate



Answer the following questions : (Calculators are allowed)

1 Choose the correct answer :

1 The simplest dispersion measure is .....

(a) the mean.

(b) the median.

(c) the range.

(d) the mode.

2 20% from 100 pounds = ..... pounds.

(a) 5

(b) 10

(c) 15

(d) 20

3  $[3, 7] - \{3, 7\} = \dots\dots\dots$

(a)  $[3, 7[$ (b)  $]3, 7]$ (c)  $]3, 7[$ (d)  $[3, 7]$

- 4 The solution set of the equation :  $x^2 - 9 = 0$  in  $\mathbb{R}$  is .....
- (a)  $\{-3\}$  (b)  $\{3\}$  (c)  $\{-3, 3\}$  (d)  $\emptyset$
- 5 If  $n(X) = 5$  ,  $n(X \times Y) = 10$  , then  $n(Y) = \dots\dots\dots$
- (a) 4 (b) 3 (c) 2 (d) 1
- 6 The relation representing the direct variation between the two variables  $y$  and  $x$  is .....
- (a)  $xy = 5$  (b)  $y = x + 3$  (c)  $\frac{x}{3} = \frac{4}{y}$  (d)  $\frac{x}{5} = \frac{y}{2}$

2 [a] If  $\frac{x}{y} = \frac{3}{4}$  , find the value of :  $\frac{3x+y}{x+5y}$

[b] If  $X = \{1, 2, 3\}$  ,  $Y = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{5}\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a$  is the multiplicative inverse of  $b$ " for all  $a \in X$  ,  $b \in Y$  , write  $R$  and represent it by an arrow diagram. Is  $R$  a function ? Why ?

3 [a] If  $X = \{4, 5, 7\}$  ,  $R$  is a function on  $X$  and  $R = \{(a, 5), (b, 5), (4, 7)\}$  , find :

- 1 The value of  $a + b$       2 The range of the function.

[b] Represent graphically the function  $f : f(x) = 2 - x^2$  ,  $x \in [-3, 3]$  , from the graph deduce :

- 1 The coordinates of the vertex point of the curve.  
2 The equation of the axis of symmetry.  
3 The maximum value of the function.

4 [a] If  $b$  is the middle proportional between  $a$  and  $c$  , prove that :  $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

[b] From the data of the following table , answer the following questions :

$x$	2	4	6
$y$	6	3	2

- 1 Show the kind of variation between  $y$  and  $x$   
2 Find the constant proportion.  
3 Find the value of  $y$  when  $x = 2\frac{2}{5}$

5 [a] If the point  $(a, 3)$  is located on the straight line which represents the function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = 4x - 5$  , find : the value of  $a$

[b] Find the standard deviation of the set of the values : 15 , 19 , 20 , 21 , 25



Answer the following questions : (Calculators are permitted)

1 Choose the correct answer from those given :

1 The ordered pair  $(x^2, y^2)$ , where  $x \neq 0, y \neq 0$  lies in the ..... quadrant.

- (a) first (b) second (c) third (d) fourth

2 The positive square root of mean of the squares of deviations of the values from their arithmetic mean is called .....

- (a) the range. (b) the median.  
(c) the standard deviation. (d) the mode.

3 If  $x$  and  $x + 17$  are two prime numbers, then  $x =$  .....

- (a) 1 (b) 2 (c) 3 (d) 5

4 If  $xy = 5$ , then  $y \propto$  .....

- (a)  $x$  (b)  $\frac{1}{x}$  (c)  $x^2$  (d)  $\frac{1}{x^2}$

5 If  $X = \{3\}$ , then  $n(X^2) =$  .....

- (a) 1 (b) 9 (c)  $\{(3, 3)\}$  (d) 3

6 The ratio between the area of a square of side length  $l$  and the area of a square of side length  $3l$  equals .....

- (a) 1 : 3 (b) 3 : 1 (c) 1 : 9 (d) 9 : 1

2 [a] If  $X = \{1, 2, 3\}$ ,  $Y = \{0, 1, 2, 3, 4\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $b - a = 1$ " for all  $a \in X, b \in Y$ , write  $R$  and represent it by an arrow diagram. Show that  $R$  is a function and write its range.

[b] If  $\frac{a}{4} = \frac{b}{5} = \frac{c}{3}$ , prove that :  $\frac{a-b+c}{a+b-c} = \frac{1}{3}$

3 [a] If  $y \propto x$ ,  $y = \frac{5}{6}$  when  $x = \frac{1}{6}$ , write the relation between  $y$  and  $x$ , then find the value of  $x$  when  $y = 15$

[b] If the point  $(a, -a)$  lies on the straight line that represents the function  $f : f(x) = x - 6$ , find : the value of  $a$

4 [a] If  $y$  is the middle proportional between  $x$  and  $z$ , prove that :  $\frac{xz}{y(y+z)} = \frac{x}{x+y}$

[b] If  $X = \{2, 3\}$ ,  $Y = \{5\}$ ,  $Z = \{4, 5\}$ , find :

1  $(X - Y) \times Z$

2  $X \times (Y \cap Z)$

- 5 [a] Represent graphically the function  $f : f(x) = (x-3)^2$ ,  $x \in [0, 6]$ , from the graph find :

- 1 The vertex of the curve. 2 The maximum or minimum value of the function.

- [b] Calculate the arithmetic mean and the standard deviation for the following data :  
73 , 54 , 62 , 71 , 60

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Luxor Governorate



Answer the following questions :

- 1 Choose the correct answer :

1  $\frac{1}{3}$  of the number  $3^4 = \dots\dots\dots$

- (a) 3 (b)  $3^2$  (c)  $3^3$  (d)  $2^3$

2 If  $n(X^2) = 4$ ,  $n(X \times Y) = 6$ , then  $n(Y^2) = \dots\dots\dots$

- (a) 12 (b) 9 (c) 6 (d) 3

3  $4, 6 \cap \{4, 6\} = \dots\dots\dots$

- (a)  $\{5\}$  (b)  $[4, 6]$  (c)  $\{4, 6\}$  (d)  $\emptyset$

4 If  $x, y, z$  are in continued proportion, then  $x = \dots\dots\dots$

- (a)  $\pm\sqrt{yz}$  (b)  $yz$  (c)  $\frac{y^2}{z}$  (d)  $\frac{y}{z}$

5  $\sqrt[3]{64} = \sqrt{\dots\dots\dots}$

- (a) 2 (b) 16 (c) 8 (d) 4

6 If all the values are equal, then  $\dots\dots\dots$

- (a)  $x - \bar{x} > 0$  (b)  $x - \bar{x} < 0$  (c)  $\bar{x} = 0$  (d)  $\sigma = 0$

- 2 [a] If  $X = \{2, 1\}$ ,  $Y = \{4, 0\}$ ,  $Z = \{4, 5, -2\}$ , find :

- 1  $X \times Y$  2  $(Y \cap Z) \times X$  3  $n(Y^2)$

- [b] Find the number which if subtracted from the first term of the ratio 15 : 13 and added to the second term, then it becomes 3 : 4

- 3 [a] If  $f(x) = 2x + a$ ,  $g(x) = x^2 + a$  and if  $f(2) + g(-4) = 30$ , find : the value of a

[b] If  $a, b, c$  and  $d$  are proportional quantities, prove that :  $\frac{a+c}{b+d} = \frac{a^2+c^2}{ab+cd}$

- 4 [a] If  $X = \{0, 1, 2, \frac{1}{2}\}$  and  $R$  is a relation on  $X$  where " $a R b$ " means " $a$  is the multiplicative inverse of  $b$ " for each  $a \in X, b \in X$ , write  $R$  and represent it by an arrow diagram. Is  $R$  a function or not ?

[b] If  $y \propto x^3$  and  $y = 64$  when  $x = 2$ , find :

- 1 The relation between  $x$  and  $y$       2 The value of  $y$  when  $x = \frac{1}{2}$

5 [a] Calculate the mean and the standard deviation for the values : 22 , 20 , 20 , 20 , 18

[b] Represent graphically the function  $f : f(x) = x^2 - 4x + 5$  where  $x \in [0, 4]$ , then from the graph find :

- 1 The equation of the axis of symmetry.  
2 The maximum or the minimum value of the function.

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Aswan Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer :

1 If  $n(X^2) = 9$  ,  $n(X \times Y) = 6$  , then  $n(Y) = \dots\dots\dots$

- (a) 2                                      (b) 3                                      (c) 4                                      (d) 6

2 If  $xy = 3$  , then  $y \propto \dots\dots\dots$

- (a)  $3x$                                       (b)  $\frac{3}{x}$                                       (c)  $\frac{1}{x}$                                       (d)  $\frac{x}{3}$

3  $[2, 5] - \{2, 5\} = \dots\dots\dots$

- (a)  $[1, 6]$                                       (b)  $\emptyset$                                       (c)  $]2, 5[$                                       (d)  $\{0\}$

4  $\sqrt{50} - \sqrt{8} = \dots\dots\dots$

- (a)  $\sqrt{200}$                                       (b)  $\sqrt{98}$                                       (c)  $\sqrt{42}$                                       (d)  $\sqrt{18}$

5 If  $\sum (x - \bar{x})^2 = 48$  of a set of values and the number of these values = 12 , then  $\sigma = \dots\dots\dots$

- (a) -2                                      (b) 2                                      (c) 4                                      (d) 6

6 If  $x - y = 5$  ,  $x + y = \frac{1}{5}$  , then  $x^2 - y^2 = \dots\dots\dots$

- (a)  $\frac{1}{25}$                                       (b) 1                                      (c) 5                                      (d) 25

2 [a] If  $X = \{1, 3, 4, 5\}$  ,  $Y = \{1, 2, 3, 4, 5, 6\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + b = 7$ " for each  $a \in X$  ,  $b \in Y$

- 1 Write  $R$  and represent it by an arrow diagram.  
2 Is  $R$  a function ? and why ?

[b] If  $y \propto x$  and  $y = 6$  when  $x = 3$  , find :

- 1 The relation between  $x$  and  $y$       2 The value of  $y$  when  $x = 5$

- 3** [a] Represent graphically the function  $f : f(x) = 4 - x^2$ , taking  $x \in [-3, 3]$  and from the graph deduce : the coordinates of the vertex point of the curve , the maximum value of the function and the equation of line of symmetry.
- [b] Find the positive number which its square is added to the antecedent of the ratio  $29 : 46$  and subtracted its square from its consequent the ratio becomes  $3 : 2$

- 4** [a] If the straight line which represents the function  $f : \mathbb{R} \longrightarrow \mathbb{R}$ ,  $f(x) = 6x - a$  intersects the y-axis at the point  $(b, 2)$ , find : the value of each of a and b

- [b] The following frequency distribution shows the marks of a number of students in an exam :

Marks	0	1	2	3	4	5	6
Number of students	3	4	6	9	5	3	4

Find the standard deviation of the marks.

- 5** [a] If  $X = \{1, 3, 5\}$  and R is a function on X where  $R = \{(a, 3), (b, 1), (1, 5)\}$ , find :
- 1** The range of the function. **2** The value of  $a + b$

- [b] If a , b , c and d are proportional quantities , prove that :  $\frac{a}{b-a} = \frac{c}{b-c}$



**Answer the following questions : (Calculator is allowed)**

- 1** Choose the correct answer from those given answers :
- 1** The next in the pattern :  $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}$  is .....  
 (a)  $\sqrt{50}$  (b)  $\sqrt{75}$  (c)  $\sqrt{60}$  (d)  $\sqrt{90}$
- 2** The point  $(-3, 4)$  lies in the ..... quadrant.  
 (a) first (b) second (c) third (d) fourth
- 3** If y varies inversely with x , and  $x = \sqrt{3}$  when  $y = \frac{2}{\sqrt{3}}$  , then the constant proportional equals .....  
 (a)  $\frac{1}{2}$  (b)  $\frac{2}{3}$  (c) 2 (d) 6
- 4** If the point  $(a, 3)$  is located on the straight line which represents the function  $f : \mathbb{R} \longrightarrow \mathbb{R}$  where  $f(x) = 4x - 5$  , then a = .....  
 (a) 2 (b) 3 (c) 4 (d) 5
- 5** ..... is one of the measures of the dispersions.  
 (a) The median (b) The arithmetic mean  
 (c) The standard deviation (d) The mode

6 If  $(X + 1)^2$  is one of the linear factors of the expression  $(X^2 - 1)^2$ , then the other factor is .....

- (a)  $(X - 1)^2$  (b)  $(X - 1)$  (c)  $(X^2 + 1)$  (d)  $(X^2 - 1)$

2 [a] If  $X = \{2, 3, 5\}$ ,  $Y = \{4, 6, 8, 10\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $2a = b$ " for all  $a \in X, b \in Y$

1 Write  $R$  and represent it by an arrow diagram. 2 Show that  $R$  is a function.

[b] If  $\frac{X}{y} = \frac{2}{3}$ , find the value of the ratio :  $\frac{3X + 2y}{6y - X}$

3 [a] If  $X \times Y = \{(1, 1), (1, 3), (1, 5)\}$ , then find :

- 1  $X, Y$  2  $Y \times X$  3  $Y^2$

[b] If  $\frac{21X - y}{7X - z} = \frac{y}{z}$ , then prove that :  $y \propto z$

4 [a] If  $f(X) = 4X + b$  and  $\frac{1}{3} f(3) = 5$ , find : the value of  $b$

[b] If  $a, b, c$  and  $d$  are in continued proportion, then prove that :  $\frac{a^2 - 3c^2}{b^2 - 3d^2} = \frac{b}{d}$

5 [a] Calculate the standard deviation for the values : 12, 13, 16, 18, 21

[b] Represent graphically the function  $f : f(X) = (X - 3)^2$ ,  $X \in [0, 6]$ , from the graph deduce the vertex of the curve, the minimum value of the function, the equation of the axis of symmetry.

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South Sinai Governorate



Answer the following questions :

1 Choose the correct answer from the given answers :

1 If  $(2, 3) \in \{2, 5\} \times \{X, 4\}$ , then  $X = \dots\dots\dots$

- (a) 2 (b) 3 (c) 4 (d) 5

2 If  $Xy = 5$ , then  $y \propto \dots\dots\dots$

- (a)  $\frac{1}{X}$  (b)  $X$  (c)  $X - 5$  (d)  $X + 5$

3 ..... is one of the measures of the dispersions.

- (a) The arithmetic mean (b) The median  
(c) The mode (d) The standard deviation

4 The mean of the values 1, 2, 3, 4 and 5 equals .....

- (a) 5 (b) 4 (c) 3 (d) 2

5  $\sqrt[3]{x^6} = \sqrt{\dots}$

(a)  $x^4$

(b)  $x^3$

(c)  $x^2$

(d)  $x$

6 If  $\frac{5}{4} + \frac{5}{a} = \frac{5}{2}$ , then  $a = \dots$

(a)  $\frac{5}{2}$

(b)  $-\frac{5}{2}$

(c) 4

(d) -4

2 [a] If  $X = \{1\}$ ,  $Y = \{2, 3\}$ ,  $Z = \{2, 5, 6\}$ ,

find : 1  $X \times (Y \cap Z)$

2  $n(X \times Y)$

3  $Z - Y$

[b] Represent graphically the function  $f : f(x) = x^2 - 4$ ,  $x \in [-3, 3]$ , from the graph deduce the vertex of the curve, the minimum value of the function.

3 If  $X = \{1, 3, 4, 5\}$ ,  $Y = \{1, 2, 3, 4, 5, 6\}$  and  $R$  is a relation from  $X$  to  $Y$  where " $a R b$ " means " $a + b = 7$ " for all  $a \in X, b \in Y$ , write  $R$ , and represent it by an arrow diagram and also by a Cartesian diagram. Is  $R$  a function? and why?

4 [a] If  $y \propto x$ ,  $y = 6$  when  $x = 3$ , find the value of  $y$  when  $x = 5$

[b] Find the positive number which if we add its square to each of the two terms of the ratio  $5 : 11$  it becomes  $3 : 5$

5 [a] If  $a, b, c$  and  $d$  are in continued proportion, then prove that :  $\frac{c^2 - d^2}{a - c} = \frac{b d}{a}$

[b] The following frequency distribution shows the ages of 10 children :

Ages in years	5	8	9	10	12	Total
Number of children	1	2	3	3	1	10

Calculate the standard deviation to the ages in years.

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North Sinai Governorate



Answer the following questions : (Calculators are allowed)

1 Choose the correct answer from those given :

1  $\sqrt{16} + \sqrt[3]{-8} = \dots$

(a) -4

(b) -2

(c) 2

(d) 4

2 If  $(9, 4) \in \{9, 7\} \times \{x, 5\}$ , then  $x = \dots$

(a) 9

(b) 4

(c) 7

(d) 5

3 If  $x^2 - y^2 = 12$ ,  $x + y = 4$ , then  $x - y = \dots$

(a) -3

(b) 3

(c) 4

(d) 12

- 4 The fourth proportional of the quantities 2 , 3 , 6 equals .....
- (a) 9 (b) 3 (c) 12 (d) 18
- 5 If  $\frac{3}{4} + \frac{3}{x} = \frac{3}{2}$  , then  $x =$  .....
- (a) 2 (b) 3 (c)  $\frac{3}{2}$  (d) 4
- 6 The range of the set of the values 3 , 5 , 6 , 7 , 9 equals .....
- (a) 3 (b) 4 (c) 6 (d) 12

2 [a] If  $X = \{2, 3, 4\}$  ,  $Y = \{2, 3, 4, 5, 6, 7, 8\}$  and R is a relation from X to Y where "a R b" means " $a = \frac{1}{2} b$ " for all  $a \in X, b \in Y$

- 1 Write R and represent it by an arrow diagram.
- 2 Show that R is a function from X to Y and find its range.

[b] If  $y \propto x$  and  $y = 2$  when  $x = 8$  , find : the value of y when  $x = 12$

3 [a] If  $f(x) = 4x + b$  ,  $f(3) = 15$  , find : the value of b

[b] If  $\frac{x}{y} = \frac{2}{3}$  , then find the value of :  $\frac{3x + 2y}{6y - x}$

4 [a] If  $(6, b - 3) = (2 - a, -1)$  , find : the value of  $a + b$

[b] If b is the middle proportional between a and c , prove that :  $\frac{a^2 + b^2}{b^2 + c^2} = \frac{a}{c}$

5 [a] Calculate the arithmetic mean and the standard deviation of the set of the values : 23 , 12 , 17 , 13 , 15

[b] Graph the function  $f : f(x) = 4 - x^2$  where  $x \in [-3, 3]$  and from the graph find :

- 1 The vertex of the curve. 2 The equation of the axis of symmetry.
- 3 The maximum value of the function.

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Red Sea Governorate



**Answer the following questions : (Calculators are allowed)**

1 Choose the correct answer from those given :

- 1 The range of the set of the values 7 , 3 , 6 , 9 , 5 equals .....
- (a) 3 (b) 4 (c) 6 (d) 12
- 2 If  $x = 3$  ,  $y = 5$  , then  $y^x =$  .....
- (a) 243 (b) 125 (c) 15 (d) 8

3 The relation which represents the direct variation between the two variables  $X$  and  $y$  is .....

- (a)  $xy = 5$  (b)  $y = x + 3$  (c)  $\frac{x}{3} = \frac{4}{y}$  (d)  $\frac{x}{5} = \frac{y}{2}$

4 If  $x - y = 5$  ,  $x + y = 1$  , then  $x^2 - y^2 = \dots\dots\dots$

- (a) 5 (b) 4 (c) 25 (d)  $\frac{1}{25}$

5 If  $n(x^2) = 9$  , then  $n(x) = \dots\dots\dots$

- (a) 1 (b) 3 (c) 6 (d) 9

6  $[3, 5] - ]3, 5[ = \dots\dots\dots$

- (a)  $[3, 5[$  (b)  $\{3, 5\}$  (c)  $\{3\}$  (d)  $\{5\}$

2 [a] If  $X \times Y = \{(2, 2), (2, 5), (2, 7)\}$  , find :

- 1  $Y$  2  $X^2$

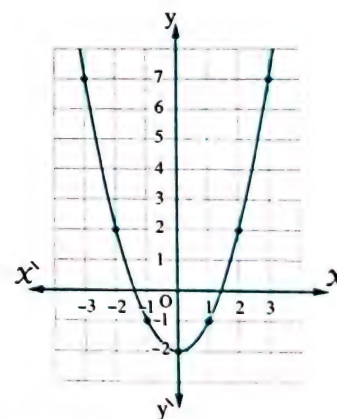
[b] If  $5a = 3b$  , find the value of :  $\frac{7a + 9b}{4a + 2b}$

3 [a] If  $y \propto \frac{1}{x}$  and  $y = 3$  when  $x = 2$  , then find :

- 1 The relation between  $x$  and  $y$  2 The value of  $y$  when  $x = 1.5$

[b] The opposite figure represents the function  $f : f(x) = x^2 - 2$  , find :

- 1 The point of the vertex of the curve.  
2 The equation of the line of symmetry.  
3 The maximum or minimum value of the function.



4 [a] If  $X = \{-2, -1, 0, 1, 2\}$  ,  $R$  is a relation on  $X$  where " $a R b$ " means

" $a$  is the additive inverse of  $b$ " for each  $a \in X$  ,  $b \in X$  , write  $R$  and represent it by an arrow diagram.

[b] If  $b$  is the middle proportion between  $a$  and  $c$  , prove that :  $\frac{a-b}{a-c} = \frac{b}{b+c}$

5 [a] Represent graphically  $f : f(x) = x - 3$  , then find the points of intersection with  $x$ -axis and  $y$ -axis.

[b] Calculate the standard deviation for the values : 8 , 9 , 7 , 6 , 5



Answer the following questions : (Calculators are allowed)

1 Choose the correct answer from those given :

1 The relation which represents the inverse variation between the two variables  $X$  and  $y$  is .....

- (a)  $y = 5X$       (b)  $y = \frac{1}{5}X$       (c)  $y = \frac{5}{X}$       (d)  $y = X + 5$

2 If  $2^X = 4^3$ , then  $X = \dots\dots\dots$

- (a) 3      (b) 4      (c) 6      (d) 46

3  $[1, 5] - \{0, 1\} = \dots\dots\dots$

- (a)  $]1, 5[$       (b)  $]1, 5]$       (c)  $[1, 5[$       (d)  $\{5\}$

4 If the arithmetic mean of the set of the values  $a, 5, 8, 7, 6$  equals 6, then  $a = \dots\dots\dots$

- (a) 4      (b) 6      (c) 8      (d) 30

5 If  $\frac{3}{4} + \frac{3}{X} = \frac{3}{2}$ , then  $X = \dots\dots\dots$

- (a) 2      (b) 4      (c) 3      (d)  $\frac{3}{2}$

6 The linear function  $f : f(X) = 2X - 1$  is represented by a straight line cutting  $y$ -axis at the point .....

- (a)  $(0, 1)$       (b)  $(0, -1)$       (c)  $(1, 0)$       (d)  $(-1, 0)$

2 [a] If  $X = \{13, 14, 43, 84\}$ , and  $R$  is a relation on  $X$  such that " $a R b$ " means

"the two numbers  $a$  and  $b$  have the same unit digit" for all  $a \in X, b \in X$ , write  $R$  and represent it by an arrow diagram. Is  $R$  a function? Why?

[b] If  $y$  varies as  $X$  and  $y = 10$  when  $X = 7$ , find  $X$  when  $y = 20$

3 [a] If  $X = \{3, 4\}$ ,  $Y = \{4, 5\}$ ,  $Z = \{6, 5\}$ , then find :

- 1  $X \times (Y \cap Z)$       2  $(X - Y) \times Z$

[b] If  $\frac{X+y}{l+m} = \frac{y+z}{m+n} = \frac{z+X}{n+l}$ , then prove that :  $\frac{X}{l} = \frac{y-X}{m-l}$

4 [a] If  $f : \mathbb{R} \longrightarrow \mathbb{R}$  is represented by a straight line cutting  $X$ -axis at  $(3, b)$  where  $f(X) = 2X - a$ , find : " $3a + 5b$ "

- [b] The following is the frequency distribution for a number of defective units which are found in 100 boxes of manufactured units :

Number of defective units	zero	1	2	3	4	5
Number of boxes	3	16	17	25	20	19

Find the standard deviation to the defective units.

- 5 [a] If  $\frac{a}{b} = \frac{2}{5}$  , then find the value of :  $\frac{b-a}{b+a}$

- [b] Represent graphically the function  $f : f(x) = x^2 - 3$  ,  $x \in [-3, 3]$  and from the drawing deduce the coordinates of the vertex of the curve , the equation of the symmetry axis and the minimum or the maximum value of the function.

**Second**

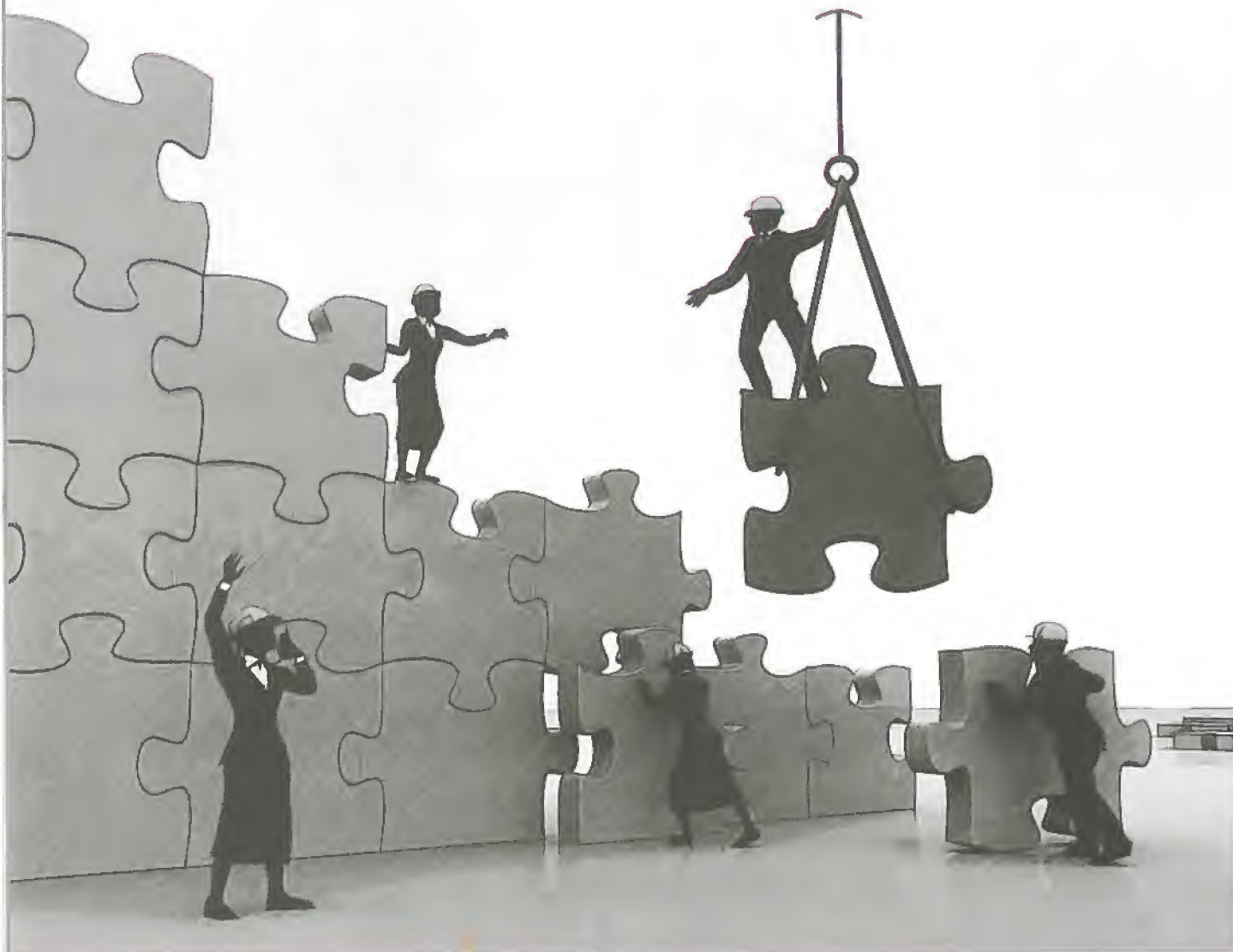
# Trigonometry and Geometry



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  - School book examinations.  
(2 model examinations + model for the merge students)
  - 27 governorates' examinations

# Quizzes

on Trigonometry and  
Geometry



## Quiz

1

on lesson 1 – unit 4



1 Choose the correct answer from those given :

1 In the opposite figure :

ABC is a triangle in which :  $m(\angle A) = 90^\circ$

,  $AB = 5$  cm. and  $BC = 13$  cm. , then  $\tan B = \dots\dots\dots$

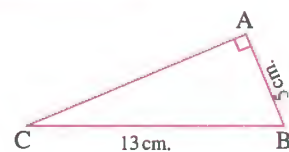
- (a)  $\frac{5}{13}$  (b) 2.4 (c)  $\frac{13}{5}$  (d)  $\frac{25}{12}$

2 If  $\triangle ABC$  is a right-angled triangle at B and  $\cos A = 0.6$  , then  $\sin A = \dots\dots\dots$

- (a)  $\frac{4}{5}$  (b)  $\frac{3}{5}$  (c)  $\frac{5}{4}$  (d)  $\frac{5}{3}$

3 For any two acute angles A and B , if  $m(\angle A) + m(\angle B) = 90^\circ$  , then  $\dots\dots\dots$

- (a)  $\sin A = \cos B$  (b)  $\sin A = \sin B$   
(c)  $\tan A = \tan B$  (d)  $\cos A = \cos B$



2 ABCD is a trapezium in which :  $\overline{AD} \parallel \overline{BC}$  ,  $m(\angle B) = 90^\circ$

If  $AB = 4$  cm. ,  $AD = 7$  cm. ,  $BC = 10$  cm.

, prove that :  $\cos(\angle DCB) - \tan(\angle ACB) = \frac{1}{5}$

## Quiz

2

till lesson 2 – unit 4



1 Choose the correct answer from those given :

1 If  $\sin X = \frac{\sqrt{3}}{2}$  where X is the measure of an acute angle , then  $X = \dots\dots\dots$

- (a)  $30^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $90^\circ$

2 If  $\triangle ABC$  is a right-angled triangle at B ,  $\sin C = \frac{3}{5}$  and  $AB = 6$  cm. , then  $AC = \dots\dots\dots$  cm.

- (a) 5 (b) 10 (c) 6 (d) 3

3 If  $m(\angle A) = 75^\circ$  ,  $\sin B = \cos A$  , where  $\angle B$  is an acute angle , then  $m(\angle B) = \dots\dots\dots$

- (a)  $45^\circ$  (b)  $75^\circ$  (c)  $15^\circ$  (d)  $105^\circ$

2 [a] Without using the calculator , find the value of X that satisfies :

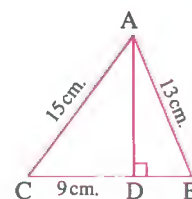
$\sqrt{3} \tan X = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$  (where X is the measure of an acute angle)

[b] In the opposite figure :

$\overline{AD} \perp \overline{BC}$  ,  $AB = 13$  cm. ,  $AC = 15$  cm. ,  $CD = 9$  cm.

Find in the simplest form the value of the expression :

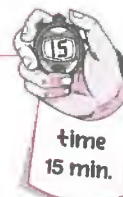
$$\frac{\tan(\angle CAD) + \tan(\angle BAD)}{\tan(\angle CAD) - \tan(\angle BAD)}$$



## Quiz

3

Till lesson 1 – unit 5



1 Choose the correct answer from those given :

1 The distance between the point  $(2, -4)$  and the  $X$ -axis is ..... length unit.

- (a) 2 (b)  $-4$  (c)  $2\sqrt{5}$  (d) 4

2 If  $\cos 3X = \frac{1}{2}$  where  $3X$  is the measure of an acute angle, then  $X = \dots\dots\dots$

- (a)  $10^\circ$  (b)  $20^\circ$  (c)  $60^\circ$  (d)  $90^\circ$

3 If the distance between the two points  $(k, 0)$ ,  $(0, 3)$  is 5 length units, then  $k = \dots\dots\dots$

- (a) 2 (b) 4 (c)  $-4$  (d)  $\pm 4$

2 [a] If the point  $A(8, 9)$  lies on the circle whose centre is  $M(2, 1)$ , find the area of this circle ( $\pi = 3.14$ )

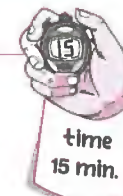
[b]  $XYZ$  is a right-angled triangle at  $Y$ , in which  $XY = 5$  cm.,  $XZ = 13$  cm.

Find the value of :  $\cos X \cos Z - \sin X \sin Z$

## Quiz

4

Till lesson 2 – unit 5



1 Choose the correct answer from those given :

1 If  $A(1, 2)$ ,  $B(3, -4)$ , then the midpoint of  $\overline{AB}$  is .....

- (a)  $(2, 4)$  (b)  $(2, -1)$  (c)  $(2, 1)$  (d)  $(-2, 1)$

2  $ABC$  is a triangle in which  $AB = BC = 5$  cm.,  $AC = 5\sqrt{2}$  cm., then  $\tan(\angle C) = \dots\dots\dots$

- (a)  $\frac{1}{\sqrt{2}}$  (b)  $\frac{1}{2}$  (c) 1 (d)  $\frac{\sqrt{3}}{2}$

3 If the origin point is the midpoint of  $\overline{AB}$  where  $A(2, -3)$ , then  $B = \dots\dots\dots$

- (a)  $(-3, 2)$  (b)  $(-2, 3)$  (c)  $(-2, -3)$  (d)  $(2, 3)$

2 [a] If  $A(-1, 3)$ ,  $B(4, 3)$ ,  $C(7, 7)$

, prove that :  $\triangle ABC$  is an isosceles triangle and calculate its area.

[b] If  $\tan 2X = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$  where  $2X$  is the measure of an acute angle, then find  $X$

## Quiz

5

Till lesson 3 – unit 5



## 1 Choose the correct answer from those given :

- 1 The straight line which passes through the two points (0, 0) and (2, 5) is parallel to the straight line whose slope equals .....

(a)  $\frac{5}{2}$  (b)  $\frac{2}{5}$  (c)  $-\frac{5}{2}$  (d)  $-\frac{2}{5}$

- 2 If ABCD is a square where A (-1, -4) and C (5, 4), then the slope of  $\overleftrightarrow{BD}$  equals .....

(a)  $\frac{4}{3}$  (b)  $-\frac{4}{3}$  (c)  $-\frac{3}{4}$  (d)  $\frac{3}{4}$

- 3 If  $\cos X = \frac{\sqrt{3}}{2}$  where X is the measure of an acute angle, then  $\sin 2X = \dots\dots\dots$

(a) 1 (b)  $\frac{\sqrt{3}}{2}$  (c)  $\frac{1}{2}$  (d)  $\frac{1}{\sqrt{2}}$

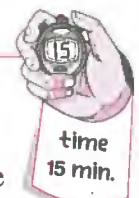
- 2 [a] **Prove that :** The straight line which passes through the two points (3, -2) and (6, 1) is parallel to the straight line which makes an angle of measure  $45^\circ$  with the positive direction of X-axis.

- [b] If A (2, 2), B (1, -1), C (-2, -2) and D (-1, 1) are four points in a perpendicular coordinates plane, **prove that :** the figure ABCD is a rhombus and find its area.

## Quiz

6

Till lesson 4 – unit 5



## 1 Choose the correct answer from those given :

- 1 The straight line whose equation is :  $2X - 3y = 12$  intercepts from the negative part of y-axis a part of length ..... length unit.

(a) 4 (b) -4 (c) 12 (d)  $\frac{2}{3}$

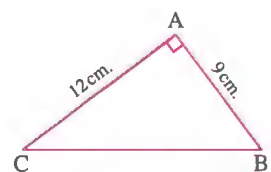
- 2 In the opposite figure :

$\sin B + \cos C = \dots\dots\dots$

(a) 1 (b)  $\frac{8}{5}$   
(c)  $\frac{6}{5}$  (d) zero

- 3 The equation of the straight line whose slope equals  $\frac{1}{2}$  and intercepts 3 length unit from the negative part of y-axis is .....

(a)  $2y = X - 3$  (b)  $2y - X - 6 = 0$   
(c)  $2y - X + 6 = 0$  (d)  $2y + X - 6 = 0$



- 2 If the points A (1, 0), B (-1, 4), C (7, 8) and D (9, 4)

- 1 **Prove that :**  $\overline{BA} \parallel \overline{CD}$  and equal in length.

- 2 **Prove that :**  $m(\angle ABC) = 90^\circ$

- 3 **Find :** The equation of the straight line which passes through the two points A and C

# Final Revision

of Trigonometry and  
Geometry



## Revision for the important rules and laws of trigonometry and geometry

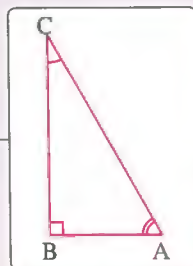
### First Trigonometry

#### Remember

The main trigonometrical ratios of the acute angle and the important relations between them

##### The trigonometrical ratios of the angle A

- $\sin A = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{BC}{AC}$
- $\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{AB}{AC}$
- $\tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{BC}{AB}$



##### The trigonometrical ratios of the angle C

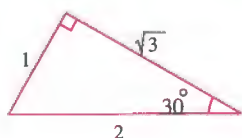
- $\sin C = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{AB}{AC}$
- $\cos C = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{BC}{AC}$
- $\tan C = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{AB}{BC}$

#### Some important relations

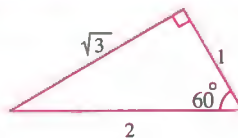
- $\tan A = \frac{\sin A}{\cos A}$
- If  $m(\angle A) + m(\angle C) = 90^\circ$ , then  $\sin A = \cos C$ ,  $\cos A = \sin C$
- If  $\sin A = \cos C$  or  $\cos A = \sin C$ , then  $m(\angle A) + m(\angle C) = 90^\circ$

#### Remember

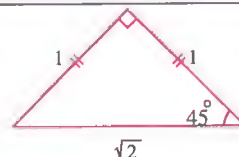
The trigonometrical ratios of some angles



- $\sin 30^\circ = \frac{1}{2}$
- $\cos 30^\circ = \frac{\sqrt{3}}{2}$
- $\tan 30^\circ = \frac{1}{\sqrt{3}}$



- $\sin 60^\circ = \frac{\sqrt{3}}{2}$
- $\cos 60^\circ = \frac{1}{2}$
- $\tan 60^\circ = \sqrt{3}$



- $\sin 45^\circ = \frac{1}{\sqrt{2}}$
- $\cos 45^\circ = \frac{1}{\sqrt{2}}$
- $\tan 45^\circ = 1$

#### Notice that

If  $\cos \theta = 0.7152$ , then we use the calculator to find  $\theta$  by using the keys as the following sequence from left : SHIFT COS . 7 1 5 2 = 0.000

Then  $\theta \approx 44^\circ 20' 25''$

## Second Analytical geometry

### Remember The important laws

If  
 $A(x_1, y_1)$   
 ,  
 $B(x_2, y_2)$

The law of the distance between the two points A , B (the length of  $\overline{AB}$ ) :

$$AB = \sqrt{(\text{difference between } x\text{-coordinates})^2 + (\text{difference between } y\text{-coordinates})^2}$$

The law of finding the coordinates of the midpoint of  $\overline{AB}$  :

$$\text{The midpoint of } \overline{AB} = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

The law of finding the slope of the straight line  $\overleftrightarrow{AB}$  :

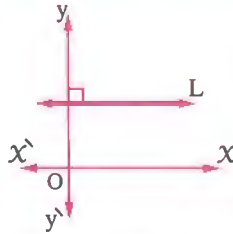
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

### Remember How to find the slope of the straight line

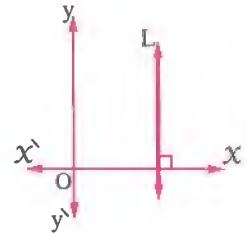
- 1 Given two points on the line as :  
 $A(x_1, y_1), B(x_2, y_2)$  →  $m = \frac{y_2 - y_1}{x_2 - x_1}$
- 2 Given the measure of the positive angle which the straight line makes with the positive direction of  $x$ -axis , say  $\theta$  →  $m = \tan \theta$
- 3 Given the equation of the straight line in the form :  
 $y = b x + c$  →  $m = b$  where  
 $b$  is the coefficient of  $x$
- 4 Given the equation of the straight line in the form :  
 $a x + b y + c = 0$  →  $m = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = \frac{-a}{b}$
- 5 Given the slope of the parallel straight line to it , say  $m_1$  →  $m = m_1$  because the two slopes are equal.
- 6 Given the slope of the perpendicular straight line to it , say  $m_2$  →  $m = \frac{-1}{m_2}$  because :  
 $m \times m_2 = -1$

## Important remarks on the slope of the straight line

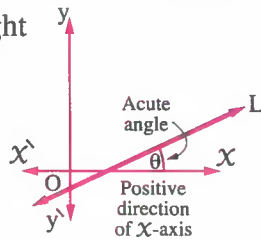
- The slope of X-axis equals 0
- The slope of the straight line parallel to X-axis equals 0



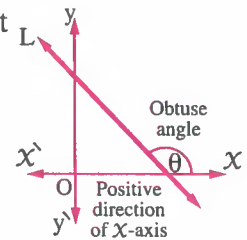
- The slope of y-axis is undefined.
- The slope of the straight line parallel to y-axis is undefined.



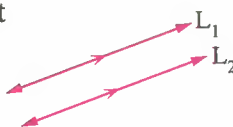
- The slope of the straight line which makes an acute angle with the positive direction of X-axis is positive.



- The slope of the straight line which makes an obtuse angle with the positive direction of X-axis is negative.

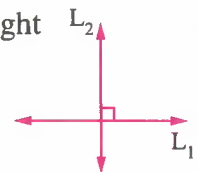


- The two parallel straight lines their slopes are equal.



*i.e.* If  $L_1 \parallel L_2$ , then  $m_1 = m_2$

- The two perpendicular straight lines the product of their slopes equals  $-1$



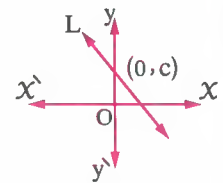
*i.e.* If  $L_1 \perp L_2$ , then  $m_1 \times m_2 = -1$

## Remember The equation of the straight line

- The equation of the straight line whose slope =  $m$  and cuts y-axis at the point  $(0, c)$  is :  
 $y = mX + c$

**For example :**

- The equation of the straight line whose slope is  $-2$  and cuts from the positive part of y-axis 7 units is :  $y = -2X + 7$
- To find the equation of the straight line whose slope is 3 and passes through the point  $(1, -2)$  :  
 $\therefore$  The slope = 3  $\therefore$  The equation of the straight line is :  $y = 3X + c$   
 , then substitute by the point  $(1, -2)$  to find the value of  $c$  as the following :  
 $-2 = 3 \times 1 + c$  , then :  $c = -5$   
 $\therefore$  The equation of the straight line is :  $y = 3X - 5$



**Important remarks on the equation of the straight line**

- 1 The equation of the straight line which passes through the origin point O (0 , 0) is :  
 $y = m X$  where  $m$  is the slope.
- 2 The equation of  $X$ -axis is :  $y = 0$  and the equation of  $y$ -axis is :  $X = 0$
- 3 The equation of the straight line parallel to  $X$ -axis and cuts  $y$ -axis at the point (0 ,  $c$ ) is :  
 $y = c$
- 4 The equation of the straight line parallel to  $y$ -axis and cuts  $X$ -axis at the point ( $a$  , 0) is :  
 $X = a$

**Remember**

Some rules and remarks which help you to solve the exercises

- 1 To prove that the points A , B and C are collinear

We will prove that :

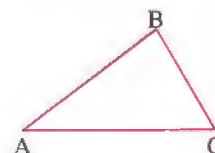
- The slope of  $\overrightarrow{AB}$  = the slope of  $\overrightarrow{BC}$
- or •  $AB + BC = AC$  (where AC is the greatest length)



- 2 To prove that the points A , B and C are vertices of a triangle

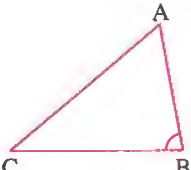
We prove that :

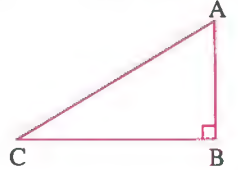
- The slope of  $\overrightarrow{AB} \neq$  the slope of  $\overrightarrow{BC}$
- or •  $AB + BC > AC$  (where AC is the greatest length)

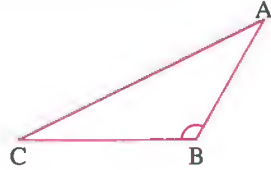


- 3 To determine the type of the triangle ABC according to its angle measures

We compare between :  $(AC)^2$  ,  $(AB)^2 + (BC)^2$  where  $\overline{AC}$  is the longest side , if :

  
 $(AC)^2 < (AB)^2 + (BC)^2$   
 , then :  
 $\Delta ABC$  is acute-angled.

  
 $(AC)^2 = (AB)^2 + (BC)^2$   
 , then :  
 $\Delta ABC$  is right-angled at B

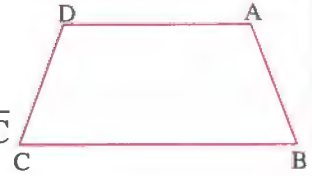
  
 $(AC)^2 > (AB)^2 + (BC)^2$   
 , then :  
 $\Delta ABC$  is obtuse-angled at B

**4 To prove that the quadrilateral ABCD is a trapezium**

**We prove that :**

The slope of  $\overrightarrow{AD}$  = the slope of  $\overrightarrow{BC}$  , then  $\overline{AD} \parallel \overline{BC}$

, the slope of  $\overrightarrow{AB} \neq$  the slope of  $\overrightarrow{DC}$  , then  $\overline{AB}$  is not parallel to  $\overline{DC}$



**5 To prove that the quadrilateral ABCD is a parallelogram**

• **By using the slope , we prove that :**

The slope of  $\overrightarrow{AD}$  = the slope of  $\overrightarrow{BC}$  , then  $\overline{AD} \parallel \overline{BC}$

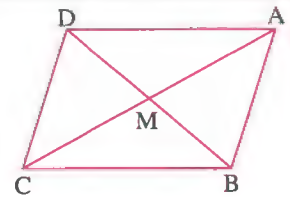
, the slope of  $\overrightarrow{AB}$  = the slope of  $\overrightarrow{DC}$  , then  $\overline{AB} \parallel \overline{DC}$

• **By using the distance between two points , we prove that :**

The length of  $\overline{AD}$  = the length of  $\overline{BC}$  , the length of  $\overline{AB}$  = the length of  $\overline{DC}$

• **By using the midpoint of a line segment , we prove that :**

The midpoint of  $\overline{AC}$  is the midpoint of  $\overline{BD}$  , then :  $\overline{AC}$  ,  $\overline{BD}$  bisect each other.



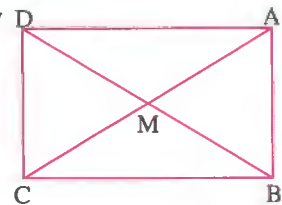
**6 To prove that the quadrilateral ABCD is a rectangle**

\* **First we prove that :** The quadrilateral ABCD is a parallelogram by one of the previous methods

, **then prove that :**

•  $AC = BD$  (By using the distance between two points)

or • The slope of  $\overrightarrow{AB} \times$  the slope of  $\overrightarrow{BC} = -1$  , then :  $\overline{AB} \perp \overline{BC}$



**7 To prove that the quadrilateral ABCD is a rhombus**

\* **First we prove that :** The quadrilateral ABCD is a parallelogram

, **then prove that :**

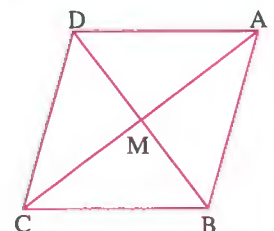
•  $AB = BC$  (By using the distance between two points)

or • The slope of  $\overrightarrow{AC} \times$  the slope of  $\overrightarrow{BD} = -1$  , then  $\overline{AC} \perp \overline{BD}$

\* We can prove that the quadrilateral ABCD is a rhombus directly by using the distance between two points

**We prove that :**

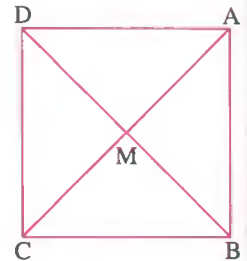
$AB = BC = CD = DA$



**8 To prove that the quadrilateral ABCD is a square**

\* **First we prove that :** The quadrilateral ABCD is a parallelogram  
**, then prove that :**

- $AB = BC$  (By using the distance between two points)  
 and the slope of  $\overrightarrow{AB} \times \text{the slope of } \overrightarrow{BC} = -1$  , then  $\overline{AB} \perp \overline{BC}$
- or •  $AC = BD$  (By using the distance between two points)  
 and the slope of  $\overrightarrow{AC} \times \text{the slope of } \overrightarrow{BD} = -1$  , then  $\overline{AC} \perp \overline{BD}$



\* We can prove that the quadrilateral ABCD is a square by using the distance between two points

**We prove that :**

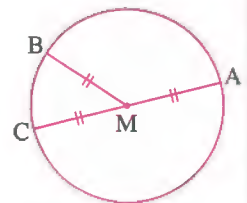
$AB = BC = CD = DA$  , then the quadrilateral is a rhombus

**, then prove that :**  $AC = BD$

**9 To prove that the points A , B , C lie on one circle of centre M**

By using the distance between two points

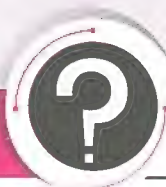
**We prove that :**  $MA = MB = MC$



# Final Examinations 2020

on Trigonometry and  
Geometry





## Model 1

Answer the following questions :

**1** Choose the correct answer from those given :

**1**  $\tan 45^\circ = \dots\dots\dots$

- (a) 1                      (b)  $2\sqrt{2}$                       (c)  $\frac{1}{2}$                       (d)  $\sqrt{2}$

**2** If  $\sin X = \frac{1}{2}$ ,  $X$  is an acute angle, then  $m(\angle X) = \dots\dots\dots$

- (a)  $45^\circ$                       (b)  $60^\circ$                       (c)  $30^\circ$                       (d)  $90^\circ$

**3** The distance between the two points  $(3, 0)$  and  $(0, -4)$  equals  $\dots\dots\dots$  length units.

- (a) 4                      (b) 5                      (c) 6                      (d) 7

**4** If  $X + y = 5$ ,  $kX + 2y = 0$  are perpendicular, then  $k = \dots\dots\dots$

- (a) -2                      (b) -1                      (c) 1                      (d) 2

**5** If  $A(5, 7)$ ,  $B(1, -1)$ , then the midpoint of  $\overline{AB}$  is  $\dots\dots\dots$

- (a)  $(2, 3)$                       (b)  $(3, 3)$                       (c)  $(3, 2)$                       (d)  $(3, 4)$

**6** The equation of the straight line which passes through the point  $(3, -5)$  and parallel to  $y$ -axis is  $\dots\dots\dots$

- (a)  $X = 3$                       (b)  $y = -5$                       (c)  $y = 2$                       (d)  $X = -5$

**2** [a] Without using calculator, prove that :  $\sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ$

[b] Prove that : The points  $A(-3, -1)$ ,  $B(6, 5)$  and  $C(3, 3)$  are collinear.

**3** [a] If  $4 \cos 60^\circ \sin 30^\circ = \tan X$ , find the value of  $X$ , where  $X$  is an acute angle.

[b] If the midpoint of  $\overline{AB}$  is  $C(6, -4)$  where  $A(5, -3)$ , find the point :  $B$

**4** [a] If the straight line  $L_1$  passes through the points  $(3, 1)$ ,  $(2, k)$  and the straight line  $L_2$  makes with the positive direction of the  $X$ -axis an angle of measure  $45^\circ$ , find the value of  $k$  if  $L_1 \parallel L_2$

[b]  $ABC$  is a right-angled triangle at  $C$ ,  $AC = 6$  cm.,  $BC = 8$  cm.

Find : **1**  $\cos A \cos B - \sin A \sin B$

**2**  $m(\angle B)$

- 5 [a] Find the equation of the straight line whose slope is 2 and passes through the point (1, 0)
- [b] **Prove that :** The points A (3, -1), B (-4, 6) and C (2, -2) which belongs to an orthogonal Cartesian coordinates plane lie on the circle whose centre is M (-1, 2). Find the circumference of the circle.

## Model 2

*Answer the following questions :*

- 1 Choose the correct answer from those given :
- 1  $2 \sin 30^\circ \tan 60^\circ = \dots\dots\dots$
- (a)  $\sqrt{3}$  (b) 3 (c)  $\frac{\sqrt{3}}{3}$  (d)  $\frac{1}{2}$
- 2 The equation of the straight line which passes through the point (-2, -3) and parallel to X-axis is  $\dots\dots\dots$
- (a)  $x = -2$  (b)  $x = -3$  (c)  $y = -2$  (d)  $y = -3$
- 3 If  $\cos x = \frac{\sqrt{3}}{2}$ ,  $x$  is an acute angle, then  $\sin 2x = \dots\dots\dots$
- (a) 1 (b)  $\frac{\sqrt{3}}{2}$  (c) -2 (d)  $\frac{1}{\sqrt{3}}$
- 4 A circle of centre at the origin point and its radius length is 2 length units, which of the following points belongs to the circle ?
- (a) (1, -2) (b)  $(-2, \sqrt{5})$  (c)  $(\sqrt{3}, 1)$  (d) (0, 1)
- 5 The perpendicular distance between the two straight lines :  $x - 2 = 0$ ,  $x + 3 = 0$  equals  $\dots\dots\dots$  length units.
- (a) 1 (b) 5 (c) 2 (d) 3
- 6 If  $\frac{-3}{2}$ ,  $\frac{6}{k}$  are the slopes of two parallel straight lines, then  $k = \dots\dots\dots$
- (a) 6 (b) -4 (c)  $\frac{3}{2}$  (d) 2
- 2 [a] If  $\cos E \tan 30^\circ = \cos^2 45^\circ$ , find  $m(\angle E)$ ,  $E$  is an acute angle.
- [b] Show the type of the triangle whose vertices are A (3, 3), B (1, 5) and C (1, 3) due to its side lengths.
- 3 [a] Find the equation of the straight line which passes through the points (1, 3) and (-1, -3) and prove that it is passing through the origin point.
- [b] If the point (3, 1) is the midpoint of (1, y), (x, 3), find the point (x, y)

**4 [a]** Find the equation of the straight line which intercepts the two axes two positive parts of lengths 1 and 4 for  $x$  and  $y$  axes respectively and find its slope.

**[b]** ABC is a right-angled triangle at B,  $AC = 10$  cm. and  $BC = 8$  cm.

**Prove that :**  $\sin^2 A + 1 = 2 \cos^2 C + \cos^2 A$

**5 [a]** **Prove that :** The straight line which passes through the points  $(-1, 3)$ ,  $(2, 4)$  is parallel to the straight line :  $3y - x - 1 = 0$

**[b]** ABCD is a trapezium,  $\overline{AD} \parallel \overline{BC}$ ,  $m(\angle B) = 90^\circ$ ,  $AB = 3$  cm.,  $BC = 6$  cm. and  $AD = 2$  cm.

Find the length of  $\overline{DC}$  and the value of  $\cos(\angle BCD)$

## Model for the merge students

Answer the following questions :

### 1 Put (✓) or (X) :

- 1 The distance between the points  $(9, 0)$  ,  $(4, 0)$  equals 5 length units. ( )
- 2 If  $\tan E = 1$  , then  $m(\angle E) = 45^\circ$  ( )
- 3 The straight line  $y = 2x + 1$  intercepts a part of length  $-1$  from  $y$ -axis ( )
- 4 If  $\overrightarrow{AB} \perp \overrightarrow{CD}$  , then the slope of  $\overrightarrow{AB} \times$  the slope of  $\overrightarrow{CD} = 1$   
(both of  $\overrightarrow{AB}$  and  $\overrightarrow{CD}$  aren't parallel to any axis) ( )
- 5  $\tan 60^\circ = \frac{1}{\sqrt{3}}$  ( )
- 6 If  $A(1, 2)$  ,  $B(3, 4)$  , then the midpoint of  $\overline{AB}$  is  $(2, 3)$  ( )

### 2 Choose the correct answer from those given :

- 1 The distance between the point  $(4, 3)$  and  $x$ -axis is ..... length units.  
(a)  $-3$  (b)  $3$  (c)  $4$  (d)  $-4$
- 2  $4 \cos 30^\circ \tan 60^\circ = \dots\dots\dots$   
(a)  $3$  (b)  $2\sqrt{3}$  (c)  $6$  (d)  $12$
- 3 If  $x + y = 5$  ,  $kx + 2y = 0$  are parallel , then  $k = \dots\dots\dots$   
(a)  $-2$  (b)  $-1$  (c)  $1$  (d)  $2$
- 4 The points  $(0, 1)$  ,  $(3, 0)$  and  $(0, 4)$  .....  
(a) form a right-angled triangle. (b) form an acute-angled triangle.  
(c) form an obtuse-angled triangle. (d) are collinear.
- 5 If  $\overrightarrow{AB} \parallel \overrightarrow{CD}$  and the slope of  $\overrightarrow{AB} = \frac{2}{3}$  , then the slope of  $\overrightarrow{CD} = \dots\dots\dots$   
(a)  $\frac{2}{3}$  (b)  $\frac{3}{2}$  (c)  $-\frac{2}{3}$  (d)  $-\frac{3}{2}$
- 6 If  $\sin x = \frac{1}{2}$  ,  $x$  is an acute angle , then  $\sin 2x = \dots\dots\dots$   
(a)  $1$  (b)  $\frac{1}{4}$  (c)  $\frac{\sqrt{3}}{2}$  (d)  $\frac{1}{\sqrt{3}}$

**3 Join from column (A) to column (B) :**

(A)	(B)
1 The slope of the straight line which is parallel to X-axis is .....	• 10
2 $\sin^2 30^\circ + \cos^2 30^\circ = \dots\dots\dots$	• 0
3 If ABCD is a rectangle where A (-1, -4), C (5, 4), then the length of $\overline{BD} = \dots\dots\dots$ length units.	• 1
4 The equation of the straight line which passes through the origin point and its slope is 2 is $y = \dots\dots\dots X$	• -3
5 The equation of the straight line which passes through the point (2, -3) and parallel to X-axis is $y = \dots\dots\dots$	• 2
6 The value of : $\frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \dots\dots\dots$	• $\frac{\sqrt{3}}{2}$

**4 Complete the following :**

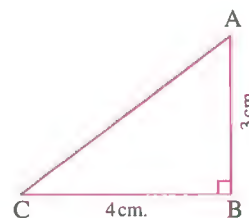
1 If  $\overline{AB} \parallel \overline{CD}$  and the slope of  $\overline{AB} = \frac{1}{2}$ , then the slope of  $\overline{CD} = \dots\dots\dots$

2 In the opposite figure :

ABC is a right-angled triangle at B

, AB = 3 cm. and BC = 4 cm.

, then  $\sin C = \dots\dots\dots$



3 If the point (0, a) belongs to the straight line :  $3X - 4y = -12$ , then  $a = \dots\dots\dots$

4 If  $X \cos 60^\circ = \tan 45^\circ$ , then  $X = \dots\dots\dots$

5 The distance between the point (4, 3) and the origin point in the coordinates plane is .....

6 If the origin point is the midpoint of  $\overline{AB}$  where A (5, -2), then B (....., .....)



1

## Cairo Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 If  $\overrightarrow{AB} \perp \overrightarrow{CD}$  and the slope of  $\overrightarrow{AB} = \frac{1}{2}$ , then the slope of  $\overrightarrow{CD} = \dots\dots\dots$   
 (a) 2 (b)  $\frac{1}{2}$  (c)  $-\frac{1}{2}$  (d) -2

2 The number of symmetry axes of an isosceles triangle equals .....  
 (a) 1 (b) 2 (c) 3 (d) 4

3  $\tan 60^\circ \tan 30^\circ = \dots\dots\dots$   
 (a)  $\sin 30^\circ$  (b)  $\tan 30^\circ$  (c)  $\tan 45^\circ$  (d)  $\cos 60^\circ$

4 The sum of the measures of the interior angles of the quadrilateral equals .....  
 (a)  $540^\circ$  (b)  $360^\circ$  (c)  $180^\circ$  (d)  $90^\circ$

5 The equation of the straight line which passes through the point (2 , 3) and is parallel to X-axis is .....  
 (a)  $x = 2$  (b)  $x = 3$  (c)  $y = 2$  (d)  $y = 3$

6 The perimeter of the square whose surface area is  $100 \text{ cm}^2$  equals ..... cm.  
 (a) 10 (b) 20 (c) 40 (d) 50

2 [a] If  $x \sin 45^\circ \cos 45^\circ = \sin 30^\circ$ , find the value of  $x$  (Showing the steps of the solution).

[b] Find the equation of the straight line which its slope is 2 and passes through the point (1 , 0)

3 [a] XYZ is a right-angled triangle at Y in which  $XY = 6 \text{ cm}$  ,  $YZ = 8 \text{ cm}$ .

Find the value of :  $\cos X \cos Z - \sin X \sin Z$

[b] ABCD is a quadrilateral , where A (2 , 4) , B (-3 , 0) , C (-7 , 5) , D (-2 , 9)

Prove that : The figure ABCD is a square.

4 [a] In the opposite figure :

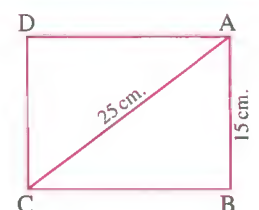
ABCD is a rectangle ,  $AB = 15 \text{ cm}$ .

,  $AC = 25 \text{ cm}$ .

Find : 1 The length of  $\overline{BC}$

2  $m(\angle ACB)$

3 The area of the rectangle ABCD



[b] If C (6 , -4) is the midpoint of  $\overline{AB}$  where A (5 , -3), find the coordinates of the point B

- 5** [a] If the straight line whose equation is  $x + 2y - 7 = 0$  is parallel to the straight line which makes an angle of measure  $45^\circ$  with the positive direction of  $X$ -axis, find the value of  $a$
- [b] Find the equation of the straight line which passes through the two points  $(4, 2)$ ,  $(-2, -1)$ , then prove that it passes through the origin point.

2

Giza Governorate



Answer the following questions :

**1** Choose the correct answer :

- 1** If  $\sin X = \frac{1}{2}$  where  $X$  is an acute angle, then  $\sin 2X = \dots\dots\dots$
- (a)  $\frac{1}{4}$  (b) 1 (c)  $\frac{1}{\sqrt{3}}$  (d)  $\frac{\sqrt{3}}{2}$
- 2** The distance between the point  $(4, 3)$  and  $y$ -axis equals  $\dots\dots\dots$  length unit.
- (a)  $-3$  (b)  $-4$  (c) 3 (d) 4
- 3** The points  $(8, 0)$ ,  $(0, 6)$ ,  $(0, 0)$   $\dots\dots\dots$
- (a) form a right-angled triangle. (b) form an obtuse-angled triangle.  
(c) form an acute-angled triangle. (d) are collinear.
- 4** If  $A(5, 7)$ ,  $B(1, -1)$ , then the midpoint of  $\overline{AB}$  is  $\dots\dots\dots$
- (a)  $(2, 3)$  (b)  $(3, 3)$  (c)  $(3, 2)$  (d)  $(3, 4)$
- 5** The equation of the straight line which passes through the point  $(1, -3)$  and is parallel to  $X$ -axis is  $\dots\dots\dots$
- (a)  $x = 3$  (b)  $y = 1$  (c)  $y = -3$  (d)  $x = -3$
- 6** The opposite figure represents a quarter of a circle with radius 2 cm. long, then its perimeter =  $\dots\dots\dots$  cm.
- (a)  $2\pi$  (b)  $5\pi$   
(c)  $\pi + 4$  (d)  $4\pi + 4$



**2** [a] Find the equation of the straight line which its slope is 2 and passes through the point  $(1, -1)$

[b] ABC is a right-angled triangle at C in which  $AC = 3$  cm.,  $BC = 4$  cm. Find :

- 1**  $\cos A \cos B - \sin A \sin B$  **2**  $m(\angle B)$

**3** [a] Without using calculator, prove that :  $\sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ$

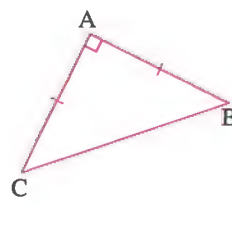
[b] If the straight line  $L_1$  passes through the two points  $(3, 1)$ ,  $(2, k)$  and the straight line  $L_2$  makes with the positive direction of the  $X$ -axis an angle of measure  $45^\circ$ , find the value of  $k$  if  $L_1 \perp L_2$

- 4** [a] If  $\cos E \tan 30^\circ = \cos^2 45^\circ$ , then find  $m(\angle E)$  where  $E$  is an acute angle.
- [b] Show the type of the triangle whose vertices are the points :  
 $A(3, 3)$  ,  $B(1, 5)$  ,  $C(1, 3)$  with respect to its side lengths.
- 5** [a] Find the slope of the straight line  $5x + 4y + 10 = 0$ , then find the length of the y-intercept.
- [b] Prove that the points  $A(3, -1)$  ,  $B(-4, 6)$  ,  $C(2, -2)$  which belong to a perpendicular coordinates plane passing through the circle whose centre is the point  $M(-1, 2)$ , then find the area of the circle.

**3****Alexandria Governorate**

*Answer the following questions : (Calculators are permitted)*

- 1** Choose the correct answer from those given :
- 1** If  $\overrightarrow{AB} \parallel \overrightarrow{CD}$  and the slope of  $\overrightarrow{AB} = \frac{2}{3}$ , then the slope of  $\overrightarrow{CD} = \dots\dots\dots$
- (a)  $\frac{3}{2}$                       (b)  $\frac{2}{3}$                       (c)  $-\frac{3}{2}$                       (d)  $-\frac{2}{3}$
- 2** In the opposite figure :
- ABC is an isosceles triangle and a right-angled triangle at A, then  $\tan C = \dots\dots\dots$
- (a)  $\frac{\sqrt{3}}{2}$                       (b)  $\frac{1}{\sqrt{3}}$                       (c) 1                      (d)  $\frac{1}{2}$
- 3** If A , B are two acute angles and  $m(\angle A) + m(\angle B) = 90^\circ$  ,  $m(\angle A) \neq m(\angle B)$ , then  $\dots\dots\dots$
- (a)  $\sin A = \cos B$                       (b)  $\sin A = \sin B$   
 (c)  $\tan A = \tan B$                       (d)  $\cos A = \cos B$
- 4** A circle of centre at the origin point and its radius length is 2 length unit , then the point  $\dots\dots\dots$  belongs to it.
- (a)  $(1, -2)$                       (b)  $(-2, \sqrt{5})$                       (c)  $(0, 1)$                       (d)  $(\sqrt{3}, 1)$
- 5** If X , Y are two supplementary angles and  $m(\angle X) = m(\angle Y)$ , then  $m(\angle X) = \dots\dots\dots^\circ$
- (a) 30                      (b) 45                      (c) 60                      (d) 90
- 6** The parallelogram whose diagonals are equal in length and perpendicular is the  $\dots\dots\dots$
- (a) square.                      (b) rhombus.                      (c) rectangle.                      (d) trapezium.



**2 [a]** Find the value of  $X$  which satisfies :  $X \sin 30^\circ \cos^2 45^\circ = \sin^2 60^\circ$

**[b]** ABCD is a parallelogram where A (3 , 2) , B (4 , -5) , C (0 , -3) Find the two coordinates of the point at which the two diagonals intersect , then find the coordinates of the point D

**3 [a]** Prove that the points A (3 , -1) , B (-4 , 6) and C (2 , -2) are located on a circle whose centre is the point M (-1 , 2) , then find the circumference of the circle. ( $\pi = 3.14$ )

**[b]** Find the equation of the straight line which is perpendicular to the straight line whose equation is  $X + 2y + 5 = 0$  and intercepts a positive part from y-axis that is equal to 7 units.

**4 [a]** Prove that the straight line passing through the two points (-3 , -2) , (4 , 5) is parallel to the straight line that makes with the positive direction of the X-axis an angle of measure  $45^\circ$

**[b]** ABC is a right-angled triangle at C , AC = 6 cm. , BC = 8 cm.  
Find the value of :  $\cos A \cos B - \sin A \sin B$

**5 [a]** Let A (4 , -6) , B (3 , 7) and C (1 , -3) Find the equation of the straight line which passes through A and the midpoint of  $\overline{BC}$

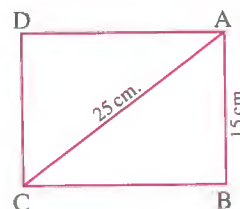
**[b] In the opposite figure :**

ABCD is a rectangle where AB = 15 cm.

, AC = 25 cm.

Find : **1** m ( $\angle ACB$ )

**2** The surface area of the rectangle ABCD



#### 4 El-Kalyoubia Governorate



Answer the following questions :

**1** Choose the correct answer :

**1** If  $\cos \frac{X}{2} = \frac{1}{2}$  where  $\frac{X}{2}$  is the measure of a positive acute angle , then  $X = \dots\dots\dots$

- (a) 30 (b) 90 (c) 60 (d) 120

**2** The triangle whose area is  $24 \text{ cm}^2$  and its height is 8 cm. , then the length of the base corresponding to this height is  $\dots\dots\dots$  cm.

- (a) 16 (b) 6 (c) 3 (d) 2

- 3 If  $\overleftrightarrow{CD}$  is parallel to y-axis where C (k , 4) , D (− 5 , 7) , then k = .....  
 (a) 5 (b) 7 (c) − 5 (d) 4
- 4 The equation of the straight line passing through the origin point and its slope = 1 is .....  
 (a)  $y = x$  (b)  $y = -x$  (c)  $y = 2x$  (d)  $y = 0$
- 5 If the point (0 , a) belongs to the straight line  $3x - 4y + 12 = 0$  , then a = .....  
 (a) 4 (b) − 3 (c) 3 (d) − 4
- 6 In  $\triangle ABC$  , if  $(AB)^2 > (BC)^2 + (AC)^2$  , then  $\angle C$  is ..... angle.  
 (a) an acute (b) a right (c) an obtuse (d) a straight

- 2 [a] If the distance of the point (x , 5) from the point (6 , 1) equals  $2\sqrt{5}$  length unit , then find the value of x

- [b] Without using the calculator , find the numerical value of the expression :  
 $\sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$

- 3 [a] ABCD is a parallelogram where A (3 , 2) , B (4 , − 5) , C (0 , − 3)  
 Find the two coordinates of the point at which the two diagonals intersect , then find the coordinates of the point D

- [b] ABC is a right-angled triangle at B in which AC = 10 cm. , BC = 8 cm.  
**Prove that :**  $\sin^2 A + 1 = 2 \cos^2 C + \cos^2 A$

- 4 [a] If the straight line  $L_1$  passes through the two points (3 , 1) and (2 , k) and the straight line  $L_2$  makes with the positive direction of the x-axis an angle of measure  $45^\circ$  , then find k if  $L_1 \parallel L_2$

- [b] Find the equation of the straight line passing through the point (1 , 2) and perpendicular to the straight line  $x + 3y + 7 = 0$

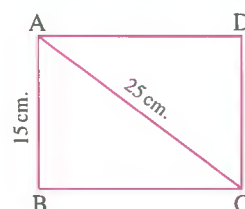
- 5 [a] In the opposite figure :

ABCD is a rectangle in which

AB = 15 cm. and AC = 25 cm.

**Find :** 1  $m(\angle ACB)$

- 2 The surface area of the rectangle ABCD



- [b] Find the equation of the straight line which intersects from the x and y axes two positive parts whose lengths are 4 and 9 length units respectively.

5

El-Sharkia Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

- 1 If  $\cos (X + 25^\circ) = \frac{1}{2}$  ,  $X$  is the measure of an acute angle , then  $X = \dots\dots\dots^\circ$   
 (a) 20 (b) 35 (c) zero (d) 5
- 2 The straight line whose equation is  $3y = 2X - 6$  , its slope =  $\dots\dots\dots$   
 (a) 2 (b)  $\frac{2}{3}$  (c) 6 (d)  $\frac{3}{2}$
- 3 The equation of the straight line which passes through the origin point and makes with the positive direction of  $X$ -axis an angle of measure  $60^\circ$  is  $\dots\dots\dots$   
 (a)  $X = 3y$  (b)  $y = \sqrt{3}X + 2$  (c)  $y = 3X$  (d)  $y = \sqrt{3}X$
- 4 If ABC is a right-angled triangle at B and  $\sin A = \frac{2}{7}$  , then  $\cos C = \dots\dots\dots$   
 (a)  $\frac{2}{7}$  (b)  $\frac{3}{7}$  (c)  $\frac{4}{7}$  (d)  $\frac{5}{7}$
- 5 The distance between the point A ( $\sqrt{2}$  , 4) and the origin point equals  $\dots\dots\dots$  length unit.  
 (a)  $\sqrt{2}$  (b)  $2\sqrt{2}$  (c)  $3\sqrt{2}$  (d)  $4\sqrt{2}$
- 6 If the slope of the straight line  $L_1$  is  $\frac{a}{5}$  and the slope of the straight line  $L_2$  is  $\frac{-b}{3}$  where  $a, b \neq 0$  and  $L_1 \perp L_2$  , then  $a b = \dots\dots\dots$   
 (a)  $\frac{3}{5}$  (b)  $\frac{-3}{5}$  (c) 15 (d) - 15

2 [a] Without using the calculator , prove that :  $\frac{\sin 30^\circ \sin 60^\circ}{\sin 45^\circ \cos 45^\circ} = \cos 30^\circ$

[b] Prove that the points A (3 , - 1) , B (- 4 , 6) , C (2 , - 2) which belong to an orthogonal Cartesian coordinates plane lie on the circle whose centre is M (- 1 , 2) , then find the circumference of the circle.

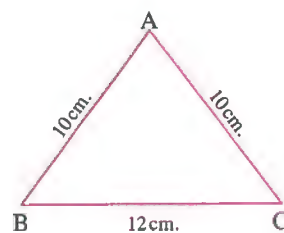
3 [a] If A (5 , 1) , B (3 , - 7) , C (1 , 3) are three noncollinear points , find the equation of the straight line which passes through the point A and is parallel to  $\overrightarrow{BC}$

[b] In the opposite figure :

ABC is an isosceles triangle where  
 $AB = AC = 10$  cm. ,  $BC = 12$  cm.

Find : 1  $\sin B$

2 The area of the triangle ABC



- 4** [a] If ABCD is a parallelogram , A (3 , 3) , B (2 , - 2) , C (5 , - 1)  
 , find : **1** The coordinates of the point of intersection of the two diagonals.  
**2** The coordinates of the point D
- [b] Find the equation of the straight line which passes through the two points (4 , 5) , (0 , 3)  
 , then find the coordinates of the intersection point of the straight line with X-axis.
- 5** [a] If  $\cos X = \sin 30^\circ \cos 60^\circ$   
 , find : **1** The measure of angle X (where X is an acute angle).  
**2**  $\tan X$
- [b] Find the equation of the straight line which cuts 3 units from the positive part of y-axis  
 and is perpendicular to the straight line  $\frac{x}{2} + \frac{y}{3} = 1$

**6****El-Monofia Governorate**

**Answer the following questions : (Using calculator is permitted)**

**1** Choose the correct answer :

- 1** If  $\cos (X + 15)^\circ = \frac{1}{2}$  , then  $\sin (75 - X)^\circ = \dots\dots\dots$   
 (a)  $\frac{1}{2}$  (b)  $\frac{\sqrt{3}}{2}$  (c)  $\frac{1}{\sqrt{2}}$  (d) 1
- 2** A circle is drawn inside a square where the circle touches its four sides. If the perimeter of the square is 56 cm. , then the surface area of the circle is  $\dots\dots\dots \text{cm}^2$   
 (a)  $\frac{77}{2}$  (b) 77 (c) 112 (d) 154
- 3** The number of sides of the regular polygon in which the measure of one of its interior angles is  $144^\circ$  equals  $\dots\dots\dots$  sides.  
 (a) 7 (b) 8 (c) 9 (d) 10
- 4** An isosceles triangle , the lengths of its sides may be 4 cm. , 9 cm. ,  $\dots\dots\dots$  cm.  
 (a) 4 (b) 9 (c) 13 (d) 36
- 5** The distance between the point (- 2 , - 3) and X-axis equals  $\dots\dots\dots$  length units.  
 (a) 2 (b) 3 (c) - 2 (d) - 3
- 6** The equation of the straight line which its slope =  $\frac{1}{2}$  and cuts the y-axis at the point (0 , 3) is  $\dots\dots\dots$   
 (a)  $2y = \frac{1}{2}x + 6$  (b)  $y = \frac{1}{2}x$   
 (c)  $y = \frac{1}{2}x + 3$  (d)  $2y = \frac{1}{2}x + 3$

- 2 [a]** Without using calculator , find the numerical value of the expression :

$$\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ - \tan^2 45^\circ$$

- [b]**  $\overline{AB}$  is a diameter in circle M , if A (7 , - 3) and B (5 , 1) where  $\pi = 3.14$  , find :

- 1** The surface area of the circle.
- 2** The coordinates of the centre of circle M

- 3 [a]** ABC is a right-angled triangle at A , AB = 5 cm. and BC = 13 cm.

**Find the numerical value of the expression :**  $\sin C \cos B + \cos C \sin B$

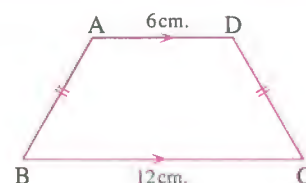
- [b]** Find the equation of the straight line which passes through the point (1 , 3) and is perpendicular to the straight line passing through the two points (5 , 0) and (2 , 1)

- 4 [a]** In the opposite figure :

ABCD is an isosceles trapezium , its area =  $36 \text{ cm}^2$

,  $\overline{AD} \parallel \overline{BC}$  , AD = 6 cm. and BC = 12 cm.

**Find the value of :**  $\sin B + \cos C$



- [b]** Show the type of the triangle ABC according to its angles measures if its vertices are A (- 1 , 3) , B (5 , 1) and C (6 , 4)

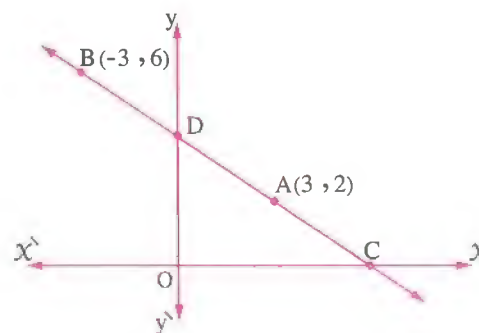
- 5 [a]** Find the slope of the straight line and the length of the intercepted part from y-axis where its equation is  $4x + 5y - 10 = 0$

- [b]** In the opposite figure :

$\overleftrightarrow{CD}$  passes through the two points A (3 , 2) , B (- 3 , 6) and cuts the two axes at C and D respectively.

**Find with the proof :**

- 1** The equation of  $\overleftrightarrow{CD}$
- 2** The area of the triangle DOC where O is the origin point.



**7**

**El-Gharbia Governorate**



**Answer the following questions : (Calculator is allowed)**

- 1** Choose the correct answer :

- 1** The perpendicular distance between the two straight lines  $y - 4 = 0$  and  $y + 5 = 0$  equals ..... length units.

- (a) 1                      (b) 5                      (c) 9                      (d) 4

**2** The equation of the straight line passing through the point  $(3, -2)$  and parallel to  $X$ -axis is .....

- (a)  $X = 3$                       (b)  $y = 2$                       (c)  $y = -2$                       (d)  $X + y = 1$

**3** If the straight line whose equation is  $y = kX + 1$  is parallel to the straight line whose equation is  $2y - X = 0$ , then  $k = \dots\dots\dots$

- (a) 1                                  (b)  $\frac{1}{2}$                                   (c) 2                                  (d) -2

**4** If the lengths 3, 7,  $l$  are lengths of sides of a triangle, then  $l$  can be equal to .....

- (a) 3                                  (b) 7                                  (c) 4                                  (d) 10

**5** The image of the point  $(-3, 5)$  by reflection on the  $y$ -axis is .....

- (a)  $(3, 5)$                       (b)  $(5, 3)$                       (c)  $(-5, 3)$                       (d)  $(-3, -5)$

**6** If ABC is a right-angled triangle at B, then  $\frac{\sin A}{\cos C} = \dots\dots\dots$

- (a)  $\frac{3}{5}$                                   (b)  $\frac{4}{3}$                                   (c)  $\frac{3}{4}$                                   (d) 1

**2** [a] If  $\tan X = 4 \cos 60^\circ \sin 30^\circ$ , then find the value of  $X$  where  $X$  is the measure of an acute angle.

[b] If the triangle XYZ whose vertices are  $X(3, 5)$ ,  $Y(4, 2)$ ,  $Z(-5, a)$  is a right-angled triangle at Y

, find : **1** The value of  $a$

**2** The surface area of the triangle XYZ

**3** [a] If the ratio between the two measures of two supplementary angles is 3 : 5, find the degree measure for each of them by degrees and minutes.

[b] Find the equation of the straight line passing through the point  $(-1, 2)$  and perpendicular to the straight line  $X + y = 5$

**4** [a] Prove that the points  $A(3, -1)$ ,  $B(-4, 6)$ ,  $C(2, -2)$  which belong to an orthogonal Cartesian coordinates plane lie on one circle whose centre is the point  $M(-1, 2)$ , then find the circumference in terms of  $\pi$

[b] ABCD is a trapezium in which  $\overline{AD} \parallel \overline{BC}$ ,  $m(\angle B) = 90^\circ$ ,  $AB = 3$  cm,  $AD = 6$  cm,  $BC = 10$  cm. Find the value of :  $\cos(\angle DCB) - \tan(\angle ACB)$

**5** [a] ABCD is a parallelogram in which  $A(3, 2)$ ,  $B(4, -5)$ ,  $C(0, -3)$

Find : **1** The coordinates of the intersection point of the two diagonals.

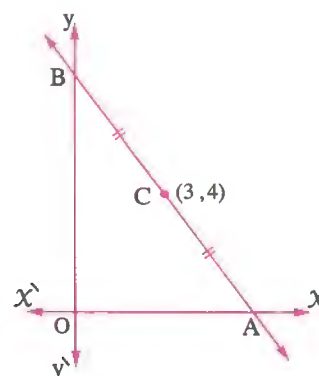
**2** The coordinates of the vertex point D

**[b] In the opposite figure :**

The point C is the midpoint of  $\overline{AB}$   
 where C (3 , 4) , O is the origin point  
 in the perpendicular coordinate system.

**Find :** 1 The coordinates of the two points A and B

2 The equation of  $\overleftrightarrow{AB}$



8

El-Dakahlia Governorate



**Answer the following questions : (Calculator is permitted)**

**1 [a] Choose the correct answer from those given :**

1 ABC is a triangle ,  $m(\angle A) = 85^\circ$  ,  $\sin B = \cos B$  , then  $m(\angle C) = \dots\dots\dots$

- (a)  $30^\circ$                       (b)  $45^\circ$                       (c)  $50^\circ$                       (d)  $60^\circ$

2 The area of the triangle bounded by the straight lines  $x = 0$  ,  $y = 0$   
 ,  $3x + 2y = 12$  equals  $\dots\dots\dots$  square units.

- (a) 6                              (b) 12                              (c) 4                              (d) 5

3 If the straight line passing through the two points (1 , y) , (3 , 4) its slope equals  
 $\tan 45^\circ$  , then  $y = \dots\dots\dots$

- (a) 1                              (b) 2                              (c) - 1                              (d) 4

**[b] ABCD is an isosceles trapezium such that  $\overline{AD} \parallel \overline{BC}$  ,  $AD = 4$  cm.**

**,  $AB = 5$  cm. ,  $BC = 12$  cm. Find the value of :  $\frac{\tan B \times \cos C}{\sin^2 C + \cos^2 B}$**

**2 [a] Choose the correct answer from those given :**

1 The straight line  $a x + (2 - a) y = 5$  is parallel to the straight line passing through  
 the two points (1 , 4) , (3 , 5) , then  $a = \dots\dots\dots$

- (a) 3                              (b) - 2                              (c) 6                              (d) 4

2 ABC is a triangle ,  $2 m(\angle C) = m(\angle A) + m(\angle B)$  , then  $m(\angle C) = \dots\dots\dots^\circ$

- (a) 30                              (b) 60                              (c) 45                              (d) 90

3 The straight line  $\frac{x}{2} - \frac{y}{3} = 6$  cuts the  $x$ -axis at a part with length  $\dots\dots\dots$  units.

- (a) 3                              (b) 2                              (c) 6                              (d) 12

[b]  $\overline{AB}$  is a diameter of circle M , B (8 , 11) , M (5 , 7) **Find :**

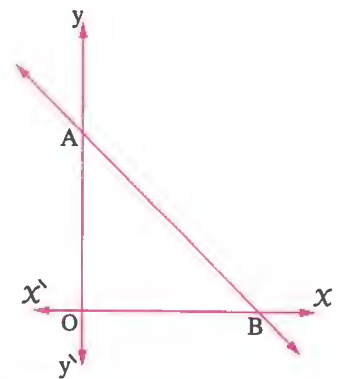
- 1 The circumference of the circle.
- 2 The equation of the straight line perpendicular to  $\overline{AB}$  from point A

3 [a] **Prove that the quadrilateral ABCD whose vertices are :**

A (−1 , 3) , B (5 , 1) , C (7 , 4) , D (1 , 6) is a parallelogram.

[b] The opposite figure represents the straight line  $\overleftrightarrow{AB}$  whose equation is  $y = kx + c$  and cuts the two axes with two equal parts and passes through the point (2 , 3) **Find :**

- 1 The values of k , c
- 2 The area of the triangle ABO



4 [a] **In the opposite figure :**

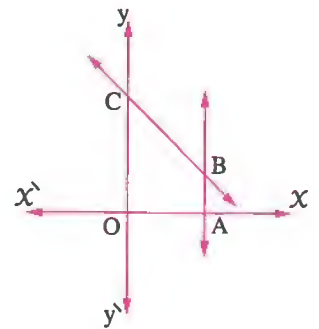
The straight line  $\overleftrightarrow{AB}$  is parallel to y-axis.

The straight line  $\overleftrightarrow{BC}$  its equation is  $y = -x + 3$  , the point B (2 , 1) **Find :**

- 1 The length of  $\overline{BC}$
- 2 The area of the figure OABC
- 3  $m(\angle OCB)$

[b] ABC is a right-angled triangle at B

- 1 **Prove that :**  $\sin^2 A + \cos^2 A = 1$
- 2 If AB = 5 cm. , AC = 13 cm. , **find :**  $m(\angle C)$  to the nearest minute.



5 [a] Find the equation of the straight line passing through the point (3 , 4) and makes with the positive direction of X-axis an angle of measure  $135^\circ$

[b] **Without using calculator , prove that :**

$$\tan^2 60^\circ - \tan^2 45^\circ = \sin^2 60^\circ + \cos^2 60^\circ + 2 \sin 30^\circ$$



Answer the following questions : (Calculator is allowed)

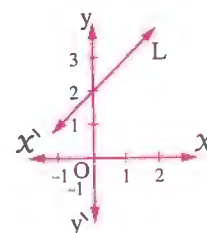
**1 Choose the correct answer from those given :**

- 1** The number of axes of symmetry of the scalene triangle equals .....  
 (a) zero (b) 1 (c) 2 (d) 3
- 2** The midpoint of  $\overline{AB}$  where A (6 , 0) , B (0 , 4) is .....  
 (a) (6 , 4) (b) (4 , 6) (c) (3 , 2) (d) (2 , 3)
- 3** If the lengths of two sides of a triangle are 3 cm. and 4 cm. , then the length of the third side may be ..... cm.  
 (a) 1 (b) 6 (c) 7 (d) 8
- 4** If  $\tan 2X = \frac{1}{\sqrt{3}}$  where  $2X$  is the measure of an acute angle , then  $X = \dots\dots\dots^\circ$   
 (a) 15 (b) 30 (c) 45 (d) 60
- 5** When you stand in front of the mirror and see your image , this is called in mathematics .....  
 (a) rotation. (b) translation. (c) reflection. (d) similarity.

**6 In the opposite figure :**

Which of the following represents the equation of the straight line L ?

- (a)  $y = X$
- (b)  $y = 2$
- (c)  $y + X = 2$
- (d)  $y - X = 2$



**2 [a] Without using the calculator , find the value of X if :**

$$X \cos^2 30^\circ = \tan^2 60^\circ \cos^2 45^\circ$$

- [b]** If A (5 , - 1) , B (3 , 7) , C (1 , - 3) , find the equation of the straight line which passes through the midpoint of  $\overline{BC}$  and the point A

**3 [a] Prove that the points A (1 , - 2) , B (- 4 , 2) , C (1 , 6) are the vertices of an isosceles triangle.**

- [b]** ABC is a right-angled triangle at B , find the value of :  $\frac{\sin A}{\cos C}$  and if  $\tan D = \frac{\sin A}{\cos C}$  where D is an acute angle , find :  $m(\angle D)$

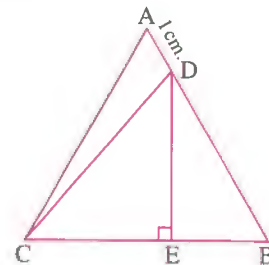
- 4 [a] If the straight line  $L_1$  passes through the two points  $(k, 1)$ ,  $(2, 4)$  and the straight line  $L_2$  makes with the positive direction of  $x$ -axis an angle of measure  $45^\circ$ , find the value of  $k$  if the two straight lines are parallel.

[b] In the opposite figure :

ABC is an equilateral triangle of side length 5 cm.

,  $D \in \overline{AB}$  where  $AD = 1$  cm. ,  $\overline{DE} \perp \overline{BC}$

Find :  $\tan (\angle DCE)$



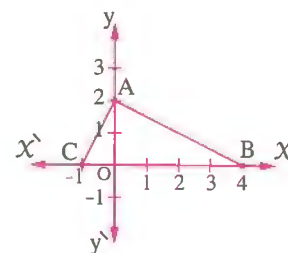
- 5 [a] If ABCD is a rhombus where  $A(3, 3)$ ,  $C(-3, -3)$ , find : 1 The intersection point of the diagonals.

2 The equation of  $\overleftrightarrow{BD}$

[b] In the opposite figure :

A triangle ABC is drawn in the orthogonal Cartesian coordinates plane.

Prove that :  $\triangle ABC$  is a right-angled triangle and find its area.



10

Suez Governorate



Answer the following questions : (Calculator is allowed)

- 1 Choose the correct answer from those given :

1  $\sin^2 60^\circ + \cos^2 60^\circ = \dots\dots\dots$

- (a) 0 (b)  $\frac{1}{4}$  (c)  $\frac{1}{2}$  (d) 1

- 2 ABCD is a parallelogram in which  $m(\angle A) + m(\angle C) = 200^\circ$

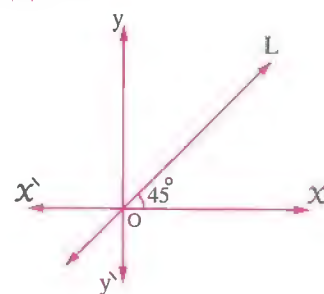
, then  $m(\angle B) = \dots\dots\dots^\circ$

- (a) 80 (b) 50 (c) 100 (d) 160

- 3 In the figure opposite :

The equation of the straight line L is .....

- (a)  $x = 1$   
(b)  $y = -x$   
(c)  $y = x$   
(d)  $y = 1$



- 4 If  $a, b$  are the measures of two complementary angles

where  $a : b = 1 : 2$ , then  $b = \dots\dots\dots^\circ$

- (a) 180                      (b) 90                      (c) 30                      (d) 60

- 5 The perpendicular distance between the straight lines

$x - 2 = 0$ ,  $x + 3 = 0$  equals  $\dots\dots\dots$  length units.

- (a) 1                      (b) 5                      (c) 2                      (d) 3

- 6 If  $A(0, 0)$ ,  $B(5, 7)$ ,  $C(5, h)$  are the vertices of a right-angled triangle at  $C$ , then  $h = \dots\dots\dots$

- (a) 0                      (b) 5                      (c) 7                      (d) -5

- 2 [a] Without using calculator, prove that :

$$2 \sin 30^\circ + 4 \cos 60^\circ = \tan^2 60^\circ$$

- [b] If  $A(-1, -1)$ ,  $B(2, 3)$ ,  $C(6, 0)$ ,  $D(3, -4)$  are four points on an orthogonal Cartesian coordinates plane

, prove that :  $\overline{AC}$  and  $\overline{BD}$  bisect each other.

- 3 [a] If  $\cos 3X = \frac{\sin 60^\circ \sin 30^\circ}{\tan 45^\circ \sin^2 45^\circ}$ , find the value of  $X$  where  $3X$  is an acute angle.

- [b] Find the equation of the straight line passing through the point  $(1, 2)$  and perpendicular to the straight line passing through the two points  $A(2, -3)$ ,  $B(5, -4)$

- 4 [a]  $ABC$  is a right-angled triangle at  $C$  where  $AB = 5$  cm.,  $BC = 4$  cm.

Prove that :  $\sin A \cos B + \cos A \sin B = 1$

- [b] Find the equation of the straight line whose slope is equal to the slope of the straight line

$\frac{y-1}{x} = \frac{1}{3}$  and intersects a part from the negative direction of  $y$ -axis of length 3 units.

- 5 [a]  $ABC$  is a triangle where  $A(0, 0)$ ,  $B(3, 4)$ ,  $C(-4, 3)$

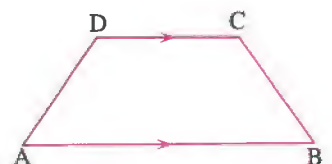
Find the perimeter of  $\triangle ABC$

- [b] In the opposite figure :

$ABCD$  is a trapezoid,  $\overline{AB} \parallel \overline{CD}$

,  $A(9, -2)$ ,  $B(3, 2)$ ,  $C(-x, -x)$ ,  $D(4, -3)$

Find the coordinates of the point  $C$



11

Port Said Governorate



Answer the following questions :

1 Choose the correct answer from those given :

1 If  $-\frac{2}{3}$  ,  $\frac{k}{6}$  are the slopes of two perpendicular straight lines , then  $k = \dots\dots\dots$

- (a) 9 (b) 4 (c) - 9 (d) - 4

2 The distance between the two points (15 , 0) , (6 , 0) equals  $\dots\dots\dots$  unit length.

- (a) - 9 (b) 9 (c) 3 (d) - 3

3 ABC is a right-angled triangle at C ,  $AB = 25$  cm. ,  $AC = 15$  cm.  
 , then the area of the surface of the triangle ABC is  $\dots\dots\dots$   $\text{cm}^2$

- (a) 300 (b) 75 (c) 150 (d) 375

4 If  $\overleftrightarrow{CD}$  is parallel to the y-axis where C (m , 4) , D (- 5 , 7) , then  $m = \dots\dots\dots$

- (a) 5 (b) - 5 (c) - 7 (d) 7

5 If the point of the origin is the midpoint of  $\overline{AB}$  , where A (5 , - 2) , then the point B is  $\dots\dots\dots$

- (a) (2 , 5) (b) (5 , - 2) (c) (- 2 , - 5) (d) (- 5 , 2)

6 If  $\tan (X + 10) = \sqrt{3}$  where  $X$  is the measure of an acute angle , then  $X = \dots\dots\dots$

- (a)  $40^\circ$  (b)  $50^\circ$  (c)  $60^\circ$  (d)  $70^\circ$

2 [a] Prove that the straight line which passes through the points

(- 1 , 3) , (2 , 4) is parallel to the straight line  $3y - x - 1 = 0$

[b] Without using calculator , prove that :

$$\sin 90^\circ = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

3 [a] If  $\cos E = \frac{\cos^2 45^\circ}{\tan 30^\circ}$  , find  $m(\angle E)$  , E is an acute angle.

[b] Prove that the points A (- 3 , 0) , B (3 , 4) , C (1 , - 6) are the vertices of an isosceles triangle.

4 [a] Find the equation of the straight line whose slope is equal to the slope of the straight line  $\frac{y-1}{x} = \frac{1}{3}$  and intercepts a negative part from the y-axis that is equal to 3 units.

[b] ABCD is a quadrilateral , where A (2 , 3) , B (6 , 2) , C (- 2 , - 2) , D (- 2 , 1)  
 Prove that the figure ABCD is a trapezoid.

- 5** [a] If  $A(5, -6)$ ,  $B(3, 7)$  and  $C(1, -3)$ , then find the equation of the straight line passing through the point A and the midpoint of  $\overline{BC}$

- [b]  $XYZ$  is a right-angled triangle at Y, where  $XY = 5$  cm,  $XZ = 13$  cm.  
 , find the value of :  $\sin X \cos Z + \cos X \sin Z$

**12**

**Damietta Governorate**



*Answer the following questions : (Calculator is allowed)*

- 1** Choose the correct answer from the given answers :

- 1** The complement of the angle whose measure is  $40^\circ$  is of measure .....

(a)  $50^\circ$  (b)  $80^\circ$  (c)  $90^\circ$  (d)  $140^\circ$

- 2** If  $D(6, -4)$  is the midpoint of  $\overline{AB}$  where  $A(5, -3)$ , then B is .....

(a)  $(-5, 7)$  (b)  $(5, 7)$  (c)  $(7, 5)$  (d)  $(7, -5)$

- 3** The length of the radius of the circle of centre  $(0, 0)$  and passes through  $(3, 4)$  equals ..... length units.

(a) 7 (b) 1 (c) 12 (d) 5

- 4** The slope of the straight line  $x - 5 = 0$  is .....

(a) 5 (b)  $\frac{1}{5}$  (c) undefined. (d) zero

- 5** If  $\tan(X + 10) = 1$ ,  $X$  is the measure of an acute angle, then  $X =$  .....

(a)  $45^\circ$  (b)  $35^\circ$  (c)  $80^\circ$  (d)  $50^\circ$

- 6** The perpendicular distance between the two straight lines  $x - 3 = 0$ ,  $x + 4 = 0$  equals ..... length units.

(a) 1 (b) 5 (c) 2 (d) 7

- 2** [a] Find the equation of the straight line which passes through the points  $(5, 0)$ ,  $(0, 5)$

- [b]  $ABC$  is a right-angled triangle at B where  $AB = 7$  cm,  $AC = 25$  cm.

**Find the value of the following :  $\sin^2 A + \sin^2 C$**

- 3** [a] If the points  $(0, 1)$ ,  $(a, 3)$ ,  $(2, 5)$  are located on one straight line  
 , then find the value of a

- [b] Find the equation of the straight line passing through the point  $(3, 7)$  and parallel to the straight line  $x + 3y + 5 = 0$

- 4 [a]** Without using the calculator , find the value of  $X$  (Where  $X$  is the measure of an acute angle) which satisfies that :  $2 \sin X = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$

- [b]** Find the equation of the straight line whose slope is 2 and intersects a positive part from the y-axis that equals 7 units.

- 5 [a]** Prove the following equality :  $\tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$

- [b]** State the kind of the triangle whose vertices are the points A ( - 2 , 4 ) , B ( 3 , - 1 ) , C ( 4 , 5 ) with respect to its sides lengths.

### 13 Kafr El-Sheikh Governorate



Answer the following questions : (Calculator is allowed)

- 1** Choose the correct answer :

- 1** The measure of an exterior angle of the equilateral triangle equals .....

(a)  $60^\circ$  (b)  $150^\circ$  (c)  $120^\circ$  (d)  $30^\circ$

- 2** If  $\frac{-2}{3}$  ,  $\frac{6}{k}$  are the slopes of two perpendicular straight lines , then  $k =$  .....

(a) 4 (b) - 9 (c) - 4 (d) 9

- 3** If ABCD is a square , then  $m(\angle CAB) =$  .....

(a)  $90^\circ$  (b)  $45^\circ$  (c)  $60^\circ$  (d)  $630^\circ$

- 4** If  $\sin \frac{X}{3} = \frac{1}{2}$  ,  $\frac{X}{3}$  is the measure of an acute angle , then  $X =$  .....

(a)  $30^\circ$  (b)  $60^\circ$  (c)  $10^\circ$  (d)  $90^\circ$

- 5** The parallelogram whose two diagonals are equal in length and not perpendicular is called a .....

(a) square. (b) rhombus. (c) rectangle. (d) trapezium.

- 6** The equation of the straight line which passes through the point ( 2 , - 3 ) and is parallel to X-axis is .....

(a)  $X = 2$  (b)  $y = 3$  (c)  $X = - 2$  (d)  $y = - 3$

- 2 [a]** Show the type of the triangle whose vertices are A ( 3 , 0 ) , B ( 1 , 4 ) , C ( - 1 , 2 ) due to its side lengths.

- [b]** Without using calculator , find the value of the following :

$$\sin^2 45^\circ \cos 60^\circ + \frac{1}{2} \tan 60^\circ \sin 60^\circ$$

- 3** [a] If the straight line  $L_1 : y = (2 - k)X + 5$  and the straight line  $L_2$  makes with the positive direction of the  $X$ -axis an angle of measure  $45^\circ$ , find the value of  $k$  if  $L_1 \parallel L_2$
- [b] If  $\sqrt{3} \tan X = 4 \sin 60^\circ \cos 30^\circ$ , find :  $X$ , where  $X$  is the measure of an acute angle.
- 4** [a] If the distance between the point  $(X, 3)$  and the point  $(2, 5)$  equals  $2\sqrt{2}$  length units, then find the values of  $X$
- [b] Find the equation of the straight line whose slope is 3 and passes through the point  $(5, -2)$
- 5** [a] If the midpoint of  $\overline{BC}$  is  $A(2, 3)$ , and  $C(-1, 3)$ , find the point  $B$
- [b]  $ABC$  is a right-angled triangle at  $B$ ,  $\sin A + \cos C = 1$ , find :  $m(\angle A)$

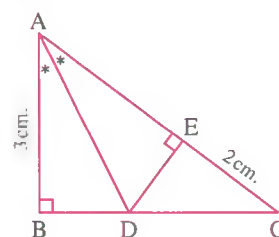
**14**

**El-Beheira Governorate**



Answer the following questions : (Calculator is permitted)

- 1** Choose the correct answer from the given ones :
- 1** If the point of origin is the midpoint of  $\overline{AB}$ , where  $A(5, -2)$ , then the point  $B$  is .....
- (a)  $(-5, -2)$       (b)  $(5, 2)$       (c)  $(-5, 2)$       (d)  $(0, 0)$
- 2** The angle of measure  $50^\circ$  is complementary with an angle of measure .....
- (a)  $50^\circ$       (b)  $40^\circ$       (c)  $30^\circ$       (d)  $130^\circ$
- 3** A circle its centre is  $(3, -4)$  and its radius length is 5 units. Which of the following points belongs to the circle ?
- (a)  $(-3, 4)$       (b)  $(0, 0)$       (c)  $(5, 0)$       (d)  $(0, 4)$
- 4** If  $\cos \frac{X}{2} = \frac{1}{2}$  where  $\frac{X}{2}$  is the measure of an acute angle, then  $X = \dots\dots\dots$
- (a)  $60^\circ$       (b)  $120^\circ$       (c)  $180^\circ$       (d)  $90^\circ$
- 5** If  $ABCD$  is a parallelogram in which  $m(\angle A) + m(\angle C) = 220^\circ$ , then  $m(\angle B) = \dots\dots\dots$
- (a)  $110^\circ$       (b)  $70^\circ$       (c)  $140^\circ$       (d)  $80^\circ$
- 6** In the figure opposite :
- $ABC$  is a right-angled triangle at  $B$   
 $\overline{AD}$  bisects  $\angle A$ ,  $\overline{DE} \perp \overline{AC}$   
 $AB = 3$  cm.,  $CE = 2$  cm.  
 then  $CB = \dots\dots\dots$  cm.
- (a) 2      (b) 3      (c) 4      (d) 5



- 2** [a] Prove that the straight line which passes through the two points  $(-1, 3)$ ,  $(2, 4)$  is parallel to the straight line  $3y - x - 1 = 0$
- [b] ABCD is a trapezium,  $\overline{AD} \parallel \overline{BC}$ ,  $m(\angle B) = 90^\circ$ ,  $AB = 3$  cm.,  $BC = 6$  cm.,  $AD = 2$  cm. Find the length of  $\overline{DC}$  and the value of  $\cos(\angle BCD)$
- 3** [a] Find the equation of the straight line whose slope is 3 and passes through the point  $(1, 2)$
- [b] Without using the calculator, find the value of  $X$  (Where  $X$  is the measure of an acute angle) which satisfies that :  
 $2 \sin X = \tan^2 60^\circ - 2 \tan 45^\circ$
- 4** [a] If the straight line  $L_1$  passes through the two points  $(3, 1)$ ,  $(2, k)$  and the straight line  $L_2$  makes with the positive direction of the  $X$ -axis an angle of measure  $45^\circ$ , then find  $k$  if the two straight lines  $L_1$ ,  $L_2$  are perpendicular.
- [b] ABC is a right-angled triangle at B, if  $\sqrt{2} AB = AC$ , find the main trigonometric ratios of the angle C
- 5** [a] If  $A(X, 3)$ ,  $B(3, 2)$ ,  $C(5, 1)$  and  $AB = BC$ ,  $B \notin \overleftrightarrow{AC}$ , then find the value of  $X$
- [b] Prove that the points  $A(6, 0)$ ,  $B(2, -4)$ ,  $C(-4, 2)$  are the vertices of a right-angled triangle at B, then find the coordinates of the point D that makes the figure ABCD a rectangle.

**15** El-Fayoum Governorate



Answer the following questions : (Using calculators is allowed)

- 1** Choose the correct answer :
- 1** The perpendicular distance between the two straight lines  $x - 2 = 0$  and  $x + 3 = 0$  equals ..... length units.  
 (a) 1 (b) 5 (c) 2 (d) 3
- 2** The sum of the measures of the accumulative angles at a point is .....  
 (a)  $90^\circ$  (b)  $180^\circ$  (c)  $270^\circ$  (d)  $360^\circ$
- 3** If  $\tan(X + 10) = \sqrt{3}$ , where  $X$  is the measure of an acute angle, then  $X =$  .....  
 (a)  $60^\circ$  (b)  $30^\circ$  (c)  $50^\circ$  (d)  $70^\circ$
- 4** The polygon in which the number of its sides is equal to the number of its diagonals is the .....  
 (a) quadrilateral. (b) triangle. (c) pentagon. (d) hexagon.

- 5 A circle of centre at the origin point and its radius length is 2 length units.

Which of the following points belongs to the circle ?

- (a) (1, -2) (b)  $(-2, \sqrt{5})$  (c)  $(\sqrt{3}, 1)$  (d) (0, 1)

- 6 The square which the length of its diagonal is  $8\sqrt{2}$  cm. , its area equals ..... cm<sup>2</sup>

- (a) 4 (b) 32 (c) 64 (d) 16

- 2 [a] Prove that the points A (3, -1) , B (-4, 6) , C (2, -2) which belong to an orthogonal Cartesian coordinates plane lie on the circle whose centre is M (-1, 2) , and find the circumference of the circle where  $\pi = 3.14$

- [b] Without using calculator , prove that :

$$\tan^2 60^\circ - \tan^2 45^\circ = \sin^2 60^\circ + \cos^2 60^\circ + 2 \sin 30^\circ$$

- 3 [a] Find the equation of the straight line perpendicular to  $\overline{AB}$  from its midpoint where A (1, 3) and B (3, 5)

- [b] ABC is a right-angled triangle at B , where AC = 5 cm. , BC = 4 cm. , find the value of :  $2 \cos^2 C + \sin^2 A$

- 4 [a] Prove that the points A (3, -2) , B (-5, 0) , C (0, -7) , D (8, -9) are the vertices of a parallelogram.

- [b] Find the value of X where :  $4X = \cos^2 30^\circ \tan^2 30^\circ \tan^2 45^\circ$

- 5 [a] If the two straight lines  $3X - 4y - 3 = 0$  and  $ky + 4X - 8 = 0$  are both perpendicular , then find the value of k

- [b] Find the equation of the straight line which intercepts from the two axes , two positive parts of length 1 and 4 from X and y axes respectively.

16

Beni Suef Governorate



Answer the following questions : (Calculator is allowed)

- 1 Choose the correct answer from those given :

- 1  $4 \sin 60^\circ \tan 60^\circ = \dots\dots\dots$

- (a) 3 (b) 6 (c) 12 (d)  $2\sqrt{3}$

- 2 The image of the point (4, 5) by the translation (2, 3) is .....

- (a) (6, -8) (b) (-8, 6) (c) (6, 8) (d) (-6, -8)

- 3 The perpendicular distance between the two straight lines  $x - 2 = 0$  ,  $x + 3 = 0$  equals ..... length units.  
 (a) 1 (b) 2 (c) 4 (d) 5
- 4 The equation of the straight line which passes through the point  $(-5, 3)$  and is parallel to y-axis is .....  
 (a)  $x = -5$  (b)  $y = -5$  (c)  $y = 3$  (d)  $x = 3$
- 5 The number of the axes of symmetry of the circle is .....  
 (a) zero (b) 1 (c) 2 (d) an infinite number.
- 6 The points  $(0, 0)$  ,  $(0, 6)$  and  $(8, 0)$  .....  
 (a) form an acute-angled triangle. (b) form a right-angled triangle.  
 (c) form an obtuse-angled triangle. (d) are collinear.

- 2 [a] If the point C  $(6, -4)$  is the midpoint of  $\overline{AB}$  where A  $(5, -3)$  , find the coordinates of the point B

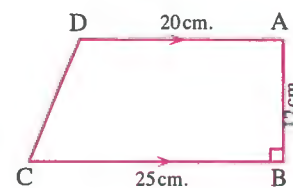
[b] In the opposite figure :

ABCD is a trapezium in which

$\overline{AD} \parallel \overline{BC}$  ,  $m(\angle B) = 90^\circ$

,  $AD = 20$  cm. ,  $AB = 12$  cm. and  $BC = 25$  cm.

Find the length of  $\overline{DC}$  and  $m(\angle C)$



- 3 [a] Prove that :  $\frac{1}{2} \sin 60^\circ = \sin 30^\circ \cos 30^\circ$

- [b] Find the equation of the straight line which passes through the point  $(2, 3)$  and its slope = 2

- 4 [a] If  $\cos E \tan 30^\circ = \sin^2 45^\circ$

, find  $m(\angle E)$  where E is an acute angle.

- [b] Prove that the straight line which passes through the two points  $(2, -1)$  and  $(6, 3)$  is parallel to the straight line which makes a positive angle of measure  $45^\circ$  with the positive direction of x-axis.

- 5 [a] Prove that the points A  $(3, -1)$  , B  $(-4, 6)$  and C  $(2, -2)$  are located on a circle whose centre is M  $(-1, 2)$

- [b] Find the slope of the straight line  $3y - 2x + 5 = 0$  , then find the length of the intersected part from the y-axis.

17

El-Menia Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

- 1 The angle whose measure is  $65^\circ$  complements an angle of measure ..... $^\circ$   
 (a) 35 (b) 25 (c) 115 (d) 45
- 2 ABCD is a parallelogram. If  $m(\angle A) + m(\angle C) = 200^\circ$ , then  $m(\angle B) = \dots\dots\dots^\circ$   
 (a) 50 (b) 80 (c) 100 (d) 160
- 3 The sum of lengths of any two sides in a triangle is ..... the length of the third side.  
 (a) less than (b) equal to (c) greater than (d) twice
- 4 If  $\sin X = \frac{1}{2}$ , then  $m(\angle X) = \dots\dots\dots^\circ$ , X is an acute angle.  
 (a) 45 (b) 60 (c) 90 (d) 30
- 5 The distance between the two points (3 , 0) , (0 , - 4) equals ..... length units.  
 (a) 4 (b) 5 (c) 6 (d) 7
- 6 If  $X + y = 5$  ,  $kX + 2y = 0$  are two parallel straight lines , then  $k = \dots\dots\dots$   
 (a) - 2 (b) - 1 (c) 1 (d) 2

2 [a] Without using calculator , find the value of the expression :

$$\cos 60^\circ \sin 30^\circ - \sin 60^\circ \tan 60^\circ + \cos^2 30^\circ$$

- [b] Find the equation of the straight line which passes through the point (1 , 2) and is perpendicular to the straight line which passes through the two points A (2 , - 3) , B (5 , - 4)

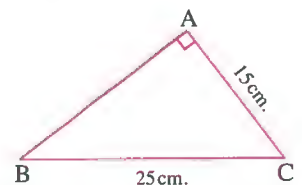
3 [a] Without using calculator , find the value of X which satisfies :

$$2 \sin X = \tan^2 60^\circ - 2 \tan 45^\circ \text{ where } X \text{ is the measure of an acute angle.}$$

- [b] In  $\triangle ABC$  ,  $m(\angle A) = 90^\circ$   
 , AC = 15 cm. , BC = 25 cm.

Prove that :

$$\cos C \cos B - \sin C \sin B = \text{zero}$$



4 [a] Prove that the points A (- 1 , - 4) , B (1 , 0) and C (2 , 2) are collinear.

- [b] If C (6 , - 4) is the midpoint of  $\overline{AB}$  where A (5 , - 3)  
 , find the coordinates of the point B

- 5 [a] Prove that the straight line that makes an angle of measure  $45^\circ$  with the positive direction of the  $X$ -axis is parallel to the straight line whose equation is  $X - y - 1 = 0$
- [b] Find the value of  $a$  if the distance between the two points  $(a, 7)$  and  $(-2, 3)$  equals 5 length units.

18

Assiut Governorate



Answer the following questions : (Calculator is permitted)

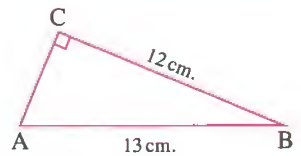
1 Choose the correct answer :

- 1 The measure of the straight angle is ..... $^\circ$   
 (a) 90 (b) 360 (c) 180 (d) 240
- 2 If  $\tan (X + 20)^\circ = \sqrt{3}$  where  $(X + 20)^\circ$  is the measure of an acute angle , then  $X =$  .....  
 (a) 30 (b) 60 (c) 90 (d) 40
- 3 The length of the side opposite to the angle of measure  $30^\circ$  in the right-angled triangle equals ..... the length of the hypotenuse.  
 (a)  $\frac{1}{4}$  (b) twice (c)  $\frac{1}{2}$  (d)  $\frac{1}{3}$
- 4 If  $X + y = 5$  ,  $kX + 2y = 7$  are perpendicular , then  $k =$  .....  
 (a) -2 (b) -1 (c) 1 (d) 2
- 5 The area of the rhombus whose diagonals lengths are 6 cm. and 12 cm. is .....  $\text{cm}^2$   
 (a) 16 (b) 30 (c) 36 (d) 72
- 6 The perpendicular distance between the two straight lines  $X - 3 = 0$  ,  $X + 4 = 0$  equals ..... length units.  
 (a) 2 (b) 7 (c) 12 (d) 6

2 [a] In the opposite figure :

ABC is a right-angled triangle at C ,  $AB = 13$  cm.  
 $BC = 12$  cm.

Prove that :  $\sin A \cos B + \cos A \sin B = 1$



- [b] Show the type of the triangle whose vertices are  $A(1, 1)$  ,  $B(5, 1)$  ,  $C(3, 4)$  due to its side lengths.

3 [a] If  $2 \sin X = \tan^2 60^\circ - 4 \sin 30^\circ$  , find  $X$  , where  $X$  is the measure of an acute angle.

- [b] ABCD is a parallelogram where  $A(3, 2)$  ,  $B(4, -5)$  ,  $C(1, 4)$  , find the two coordinates of the point at which the two diagonals intersect , then find the coordinates of the point D

- 4** [a] Without using the calculator, find the value of :  $\cos 60^\circ + \cos^2 30^\circ + \tan^2 45^\circ$
- [b] Prove that the straight line passing through the two points  $(2\sqrt{3}, 3)$ ,  $(\sqrt{3}, 4)$  is perpendicular to the straight line that makes with the positive direction of the  $X$ -axis an angle of measure  $60^\circ$

- 5** [a] Find the equation of the straight line passing through the point  $(3, -5)$  and parallel to the straight line  $X + 3y = 7$
- [b] Find the slope of the straight line and the length of the  $y$ -intercept by the straight line  $\frac{y-1}{x} = \frac{1}{2}$

**19**

Souhag Governorate



Answer the following questions : (Calculator is permitted)

- 1** Choose the correct answer :
- [1] The point of concurrence of the medians of the triangle divides each median in the ratio of ..... from its base.
- (a) 2 : 3                      (b) 2 : 1                      (c) 1 : 2                      (d) 3 : 2
- [2] If  $\sin X = \cos X$ , then  $X = \dots\dots\dots^\circ$  ( $X$  is the measure of an acute angle)
- (a) 30                      (b) 45                      (c) 60                      (d) 90
- [3] The sum of the measures of the accumulative angles at a point equals ..... $^\circ$
- (a) 30                      (b) 60                      (c) 180                      (d) 360
- [4] The distance between the two points  $(3, 0)$ ,  $(-1, 0)$  equals ..... length units.
- (a) 4                      (b) 5                      (c) 6                      (d) 7
- [5] The side length of a square is  $\sqrt{3}$  cm. , then its area = .....  $\text{cm}^2$
- (a)  $4\sqrt{3}$                       (b) 9                      (c) 3                      (d) 6
- [6] If  $A(5, -3)$ ,  $B(7, -5)$ , then the midpoint of  $\overline{AB}$  is .....
- (a)  $(3, 5)$                       (b)  $(2, 0)$                       (c)  $(5, -5)$                       (d)  $(6, -4)$

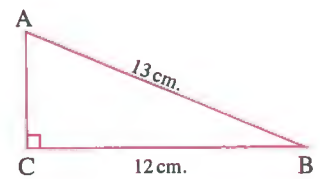
- 2** [a] If  $\cos X = 2 \cos^2 30^\circ - 1$  ( $X$  is the measure of an acute angle), find :  $X$
- [b] Prove that the triangle whose vertices are  $A(1, 4)$ ,  $B(-1, -2)$ ,  $C(2, -3)$  is right-angled at  $B$

**3 [a] In the opposite figure :**

The triangle ABC is right-angled at C  
 , AB = 13 cm. , BC = 12 cm.

**Find :** **1** The length of  $\overline{AC}$

**2** The value of  $\sin A \cos B + \cos A \sin B$



**[b]** Find the equation of the straight line whose slope equals 2 and passes through the point (1 , 0)

**4 [a] Without using the calculator , prove that :  $2 \sin 30^\circ = \tan^2 60^\circ - 2 \tan 45^\circ$**

**[b]** Find the equation of the straight line passing through the points (1 , 3) , (−1 , −3) , then prove that it passes through the origin point.

**5 [a] Prove that the points A (−3 , −1) , B (6 , 5) , C (3 , 3) are collinear.**

**[b]** Prove that the straight line passing through the two points (−3 , −2) , (4 , 5) is parallel to the straight line which makes with the positive direction of the X-axis an angle of measure  $45^\circ$

**20**

**Qena Governorate**



**Answer the following questions :**

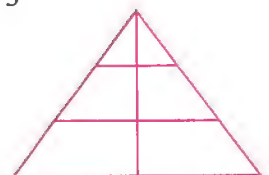
**1 Choose the correct answer :**

**1** If  $\sin X = \frac{1}{2}$  where X is the measure of an acute angle , then  $\sin 2 X = \dots\dots\dots$

- (a)  $\frac{1}{4}$  (b)  $\frac{\sqrt{3}}{2}$  (c) 60 (d)  $\frac{1}{\sqrt{3}}$

**2** The number of quadrilaterals in the opposite figure is  $\dots\dots\dots$

- (a) 3 (b) 6  
(c) 9 (d) 12



**3** If the two straight lines  $X + y = 4$  ,  $a X + 3 y = 0$  are perpendicular , then  $a = \dots\dots\dots$

- (a) −3 (b) −1 (c) 1 (d) 3

**4** The number of axes of symmetry of the rhombus equals  $\dots\dots\dots$

- (a) 1 (b) 2 (c) 3 (d) 4

**5** The straight line whose equation is  $2 y = 3 X - 6$  intercepted a part equal  $\dots\dots\dots$  units from y-axis.

- (a) 6 (b) 2 (c) 3 (d)  $\frac{3}{2}$

6 The image of the point  $(-3, 2)$  by reflection on the origin point is .....

- (a)  $(3, 2)$  (b)  $(3, -2)$  (c)  $(-3, -2)$  (d)  $(-3, 2)$

2 [a]  $\Delta ABC$  is a right-angled triangle at B,  $AC = 10$  cm. ,  $BC = 8$  cm.

**Prove that :**  $\sin^2 A + 1 = 2 \cos^2 C + \cos^2 A$

[b] Prove that the points A  $(1, 1)$  , B  $(0, -1)$  , C  $(2, 3)$  are collinear.

3 [a] If  $\sin X \tan 30^\circ = \sin^2 45^\circ$  , find the value of  $X$  in degrees , where  $X$  is the measure of an acute angle.

[b] Prove that the straight line passing through  $(-1, 3)$  ,  $(2, 4)$  is parallel to the straight line whose equation is  $3y - x - 1 = 0$

4 [a] **Without using calculator , prove that :**  $\sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ$

[b] ABCD is a quadrilateral in which :

A  $(5, 3)$  , B  $(6, -2)$  , C  $(1, -1)$  , D  $(0, 4)$

**Prove that :** ABCD is a rhombus and find its area.

5 [a] Prove that the points A  $(-3, 0)$  , B  $(3, 4)$  , C  $(1, -6)$  are the vertices of an isosceles triangle its vertex A , then find the length of the perpendicular segment from A to  $\overline{BC}$

[b] ABCD is a parallelogram in which A  $(3, 2)$  , B  $(4, -5)$  , C  $(0, -3)$   
Find the coordinates of the point D

21

### Luxor Governorate

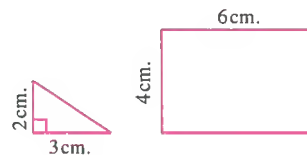


**Answer the following questions :**

1 Choose the correct answer :

1 The number of the right triangles which completely cover the surface of the rectangle equals .....

- (a) 10 (b) 8  
(c) 6 (d) 4



2 If  $m(\angle A) = 85^\circ$  and  $\sin B = \cos B$  in  $\Delta ABC$  , then  $m(\angle C) = \dots^\circ$

- (a) 30 (b) 45 (c) 50 (d) 60

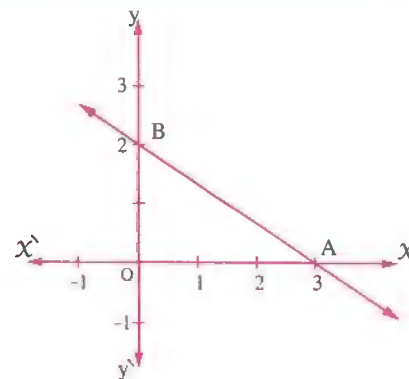
3 The image of the point  $(-5, 6)$  by translation  $(3, -2)$  is .....

- (a)  $(-4, -2)$  (b)  $(4, 2)$  (c)  $(-2, 4)$  (d)  $(-2, -4)$

4 In the opposite figure :

The slope of  $\overleftrightarrow{AB}$  equals .....

- (a)  $\frac{2}{3}$   
 (b)  $-\frac{2}{3}$   
 (c)  $\frac{3}{2}$   
 (d)  $-\frac{3}{2}$



5 The measure of the exterior angle at any vertex of an equilateral triangle equals .....°

- (a) 30                      (b) 60                      (c) 90                      (d) 120

6 If C  $(-3, y)$  is the midpoint of  $\overline{AB}$  where A  $(x, -6)$  and B  $(9, -12)$ , then  $y - x =$  .....

- (a) 7                      (b) 9                      (c) 6                      (d) -18

2 [a] If the distance between the two points  $(a, 5)$  ,  $(3a - 1, 1)$  equals 5 length units, then find a

[b] If  $3 \tan X - 4 \sin^2 30^\circ = 8 \cos^2 60^\circ$ , find X where X is the measure of an acute angle.

3 [a] Find the equation of the straight line passing by  $(1, 2)$  and parallel to the straight line  $2x + 3y - 6 = 0$

[b] Find the measure of the angle made by the straight line passing by the two points  $(-2, \sqrt{3})$  ,  $(1, 4\sqrt{3})$  with the positive direction of the X-axis.

4 [a]  $\overline{AB}$  is a diameter of the circle M where A  $(4, -1)$  , B  $(-2, 7)$ , find the radius length of the circle and find its area.

[b] ABC is a triangle where  $AB = AC = 10$  cm. ,  $BC = 12$  cm.

, draw  $\overline{AD} \perp \overline{BC}$  and intersects it at D **Prove that :**

1  $\sin^2 C + \cos^2 C = 1$

2  $\sin B + \cos C > 1$

5 [a] If  $\overleftrightarrow{AB} \parallel$  the y-axis where A  $(x, 7)$  , B  $(3, 5)$ , find the value of x

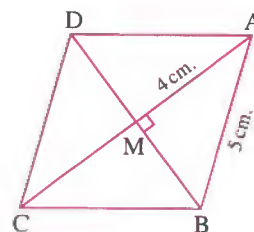
[b] In the opposite figure :

ABCD is a rhombus, its two diagonals intersect at M

, if  $AB = 5$  cm. ,  $AM = 4$  cm. , find :

1  $m(\angle BAD)$

2 The area of the rhombus ABCD



22

Aswan Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 The angle with measure  $65^\circ$  is complement of an angle with measure .....

- (a)  $135^\circ$  (b)  $115^\circ$  (c)  $25^\circ$  (d)  $15^\circ$

2 If  $\overrightarrow{AB} \perp \overrightarrow{CD}$  and the slope of  $\overrightarrow{AB} = \frac{1}{2}$ , then the slope of  $\overrightarrow{CD} = \dots\dots\dots$

- (a) 2 (b) -2 (c)  $\frac{1}{2}$  (d)  $-\frac{1}{2}$

3 If C  $\in$  the axis of symmetry of  $\overline{AB}$ , then CA ..... CB

- (a)  $\perp$  (b)  $<$  (c)  $>$  (d)  $=$

4 If 3 cm. , 7 cm. and y are lengths of sides of a triangle , then y = ..... cm.

- (a) 3 (b) 4 (c) 7 (d) 10

5 The distance between the two points (6 , 0) and (0 , 8) equals ..... length units.

- (a) 6 (b) 8 (c) 10 (d) 14

6 If  $\tan (X + 10) = \sqrt{3}$  where X is the measure of an acute angle , then X = .....

- (a)  $80^\circ$  (b)  $50^\circ$  (c)  $35^\circ$  (d)  $20^\circ$

2 [a] If  $2 \sin X = \tan^2 60^\circ - 2 \tan^2 45^\circ$ , find the value of X where X is the measure of an acute angle.

[b] Find the equation of the straight line which is perpendicular to  $\overline{AB}$  from its midpoint where A (1 , 3) and B (3 , 5)

3 [a] If C (4 , 2) is the midpoint of  $\overline{AB}$  where A (2 , 4) and B (6 , y), find the value of y

[b] If the points A (-1 , -1) , B (2 , 3) , C (6 , 0) are the vertices of a triangle.

, prove that :  $\triangle ABC$  is right-angled at B

4 [a] XYZ is a right-angled triangle at Y , if XY = 5 cm. , XZ = 13 cm.

, find : 1  $\tan X \times \tan Z$

2  $\cos X \cos Z - \sin X \sin Z$

[b] Find the equation of the straight line which intercepts from the positive parts of the coordinates axes two parts of lengths 1 and 4 from X and y axes respectively.

5 [a] Prove that the straight line which passes through the two points (-1 , 3) and (2 , 4) is parallel to the straight line whose equation is  $3y - X - 1 = 0$

[b]  $\triangle ABC$  is a right-angled triangle at B , if  $2 AB = \sqrt{3} AC$

, find the main trigonometric ratios of the angle C

23

New Valley Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

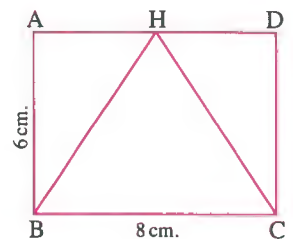
- 1 The quadrilateral ABCD in which  $AB > CD$  ,  $\overline{AB} \parallel \overline{CD}$  is .....  
 (a) a square. (b) a rectangle. (c) a rhombus. (d) a trapezium.

2 In the opposite figure :

ABCD is a rectangle ,  $AB = 6$  cm. ,  $BC = 8$  cm.

,  $H \in \overline{AD}$  , the area of  $\triangle HBC = \dots\dots\dots \text{cm}^2$

- (a) 14 (b) 24  
 (c) 28 (d) 48



3 For any angle A ,  $\frac{\sin A}{\cos A} = \dots\dots\dots$

- (a)  $\sin A$  (b)  $\cos A$  (c)  $\tan A$  (d) 1

4 If ABCD is a rectangle ,  $A(1, 0)$  ,  $C(4, 4)$  , then  $BD = \dots\dots\dots$  length units.

- (a) 5 (b) 8 (c) 9 (d) 10

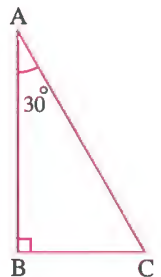
5 If  $x + y = 5$  and  $kx + 2y = 1$  are perpendicular , then  $k = \dots\dots\dots$

- (a) 2 (b) 1 (c) -1 (d) -2

6 In the opposite figure :

$BC : AC : AB = \dots\dots\dots$

- (a)  $1 : \sqrt{3} : 2$   
 (b)  $2 : \sqrt{3} : 1$   
 (c)  $1 : 2 : \sqrt{3}$   
 (d)  $\sqrt{3} : 1 : 2$



2 [a] XYZ is a right-angled triangle at Z ,  $XZ = 3$  cm. ,  $YZ = 4$  cm. Find the value of :

- 1  $\tan X \tan Y$  2  $\sin^2 X + \cos^2 X$

[b] Determine the type of the triangle whose vertices are  $A(3, 3)$  ,  $B(1, 5)$  ,  $C(1, 3)$  according to its side lengths and according to its angles.

3 [a] If  $\tan X = 4 \sin 30^\circ \cos 60^\circ$  ,  $X$  is the measure of an acute angle , then find the value of each of :

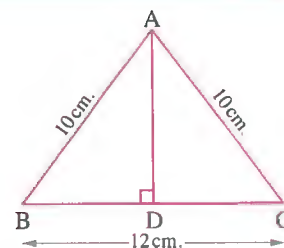
- 1  $X$  2  $\sin X$

[b] Find the equation of the straight line whose slope is 2 and passes through the point  $(1, 0)$

**4 [a] In the opposite figure :**

ABC is a triangle ,  $AB = AC = 10$  cm.  
 ,  $BC = 12$  cm. ,  $\overline{AD} \perp \overline{BC}$  Find the value of :

- 1  $\cos B$       2  $m(\angle B)$       3  $\sin(90^\circ - B)$



**[b]** ABCD is a rhombus ,  $A(-2, 3)$  ,  $B(-1, -2)$  ,  $C(4, -3)$

**Find :** 1 The coordinates of the point of intersection of its diagonals.

2 The coordinates of the point D

**5 [a]** If the straight line  $L_1$  passes through the points  $(2, 1)$  ,  $(3, k)$  and the straight line  $L_2$  makes with the positive direction of the  $X$ -axis an angle of measure  $45^\circ$  , find the value of  $k$  , if  $L_1 \parallel L_2$

**[b]** Find the equation of the straight line which intersects from the two axes two positive parts of lengths 2 and 4 from  $X$  and  $y$  axes respectively.

**24 South Sinai Governorate**


*Answer the following questions :*

**1 Choose the correct answer from those given :**

1 If  $\cos(X + 15^\circ) = \frac{1}{2}$  , then  $\tan X = \dots\dots\dots$  where  $X$  is the measure of an acute angle.

- (a) 1      (b)  $\sqrt{3}$       (c)  $\frac{\sqrt{3}}{3}$       (d)  $\frac{1}{2}$

2 The distance between the two points  $(-3, 0)$  and  $(0, -4)$  equals  $\dots\dots\dots$  length units.

- (a) 4      (b) 5      (c) 3      (d) 2

3 If  $A = (-4, 5)$  and  $B = (-2, -1)$  , then the midpoint of  $\overline{AB}$  is  $\dots\dots\dots$

- (a)  $(0, 1)$       (b)  $(-3, 3)$       (c)  $(-3, 2)$       (d)  $(1, 0)$

4 ABC is a triangle in which  $m(\angle A) = 120^\circ$  ,  $AB = AC$  , then  $m(\angle C) = \dots\dots\dots$

- (a)  $60^\circ$       (b)  $45^\circ$       (c)  $50^\circ$       (d)  $30^\circ$

5 If  $X + y = 5$  and  $kX + 2y = 0$  are two straight lines perpendicular , then  $k = \dots\dots\dots$

- (a) -2      (b) 2      (c) -1      (d) 1

6 ABC is a right-angled triangle at A and  $\overline{AD} \perp \overline{BC}$  , where  $D \in \overline{BC}$  , then  $(AD)^2 = \dots\dots\dots$

- (a)  $BD \times BC$       (b)  $CD \times CB$       (c)  $DB \times DC$       (d)  $(DB)^2 \times (DC)^2$

**2 [a] Without using calculator , prove that :  $\cos 60^\circ = \cos^2 30^\circ - \sin^2 30^\circ$** 

**[b]** If the point  $D = (1, -3)$  is the midpoint of  $\overline{AB}$  ,  $A = (4, -3)$   
 , find the coordinates of the point B

- 3 [a]** Find the equation of the straight line which passes through the points  $(1, 3)$  and  $(-1, -3)$
- [b]** Show the type of the triangle ABC whose vertices are  $A = (3, 3)$  ,  $B = (1, 5)$  and  $C = (1, 3)$  due to its side lengths.

- 4 [a]** Find the equation of the straight line which passes through the point  $(-2, 3)$  and makes with the positive direction of the  $X$ -axis an angle of measure  $45^\circ$

**[b] Find the value of :**  $\frac{2 \tan 45^\circ}{1 + \tan^2 45^\circ}$

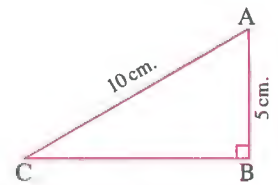
- 5 [a]** Find the equation of the straight line which its slope is 2 , and intersects a positive part from  $y$ -axis that is equal to 5 units.

**[b] In the opposite figure :**

ABC is a triangle right-angled at B  
 , in which  $AC = 10$  cm. ,  $AB = 5$  cm.

**Find :** **1**  $m(\angle C)$

**2**  $\sin^2 C + \cos^2 C$



**25 North Sinai Governorate**



**Answer the following questions :**

- 1 Choose the correct answer from those given :**
- 1** If  $\sin X = \frac{1}{2}$  where  $X$  is the measure of an acute angle , then  $X = \dots\dots\dots$   
 (a)  $90^\circ$  (b)  $60^\circ$  (c)  $45^\circ$  (d)  $30^\circ$
- 2** The measure of the exterior angle of the equilateral triangle equals  $\dots\dots\dots$   
 (a)  $60^\circ$  (b)  $90^\circ$  (c)  $120^\circ$  (d)  $180^\circ$
- 3** The slope of the straight line which makes with the positive direction of  $X$ -axis a positive angle of measure  $45^\circ$  equals  $\dots\dots\dots$   
 (a) 1 (b) -1 (c) zero (d) 1.4
- 4** The angle whose measure is  $40^\circ$  complements an angle of measure  $\dots\dots\dots$   
 (a)  $30^\circ$  (b)  $140^\circ$  (c)  $50^\circ$  (d)  $40^\circ$
- 5** If  $A(2, -2)$  ,  $B(-2, 2)$  , then the midpoint of  $\overline{AB}$  is  $\dots\dots\dots$   
 (a)  $(-1, 1)$  (b)  $(1, -1)$  (c)  $(4, -4)$  (d)  $(0, 0)$
- 6** If 3 , 7 ,  $l$  are the lengths of the sides of a triangle , then  $l$  can be equal to  $\dots\dots\dots$   
 (a) 3 (b) 4 (c) 7 (d) 10

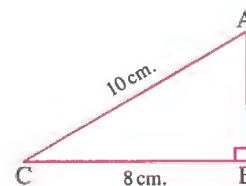
**2 [a] Prove that :**  $\cos 60^\circ = 2 \cos^2 30^\circ - 1$  (Without using the calculator)

**[b]** Prove that the triangle whose vertices are A (1 , - 2) , B (- 4 , 2) and C (1 , 6) is an isosceles triangle.

**3 [a]** Find the equation of the straight line whose slope = 2 and cuts 7 units from the positive part of y-axis.

**[b] In the opposite figure :**

ABC is a right-angled triangle at B in which AC = 10 cm.  
BC = 8 cm.



**1 Find the length of :  $\overline{AB}$**

**2 Prove that :**  $\sin^2 A + \cos^2 A = 1$

**4 [a]** If  $\cos X = \frac{\sin 60^\circ \sin 30^\circ}{\sin^2 45^\circ}$

, find the value of X where X is the measure of an acute angle. (Without using the calculator)

**[b]** Find the equation of the straight line passing through the point (1 , 2) and perpendicular to the straight line passing through the two points (2 , - 3) , (5 , - 4)

**5** If A (3 , - 1) , B (- 4 , 6) , C (2 , - 2) and M (- 1 , 2) :

**1** Prove that the points A , B , C lie on the circle whose centre is M

**2** Find the circumference of the circle M ( $\pi = 3.14$ )

**26**

Red Sea Governorate



*Answer the following questions :*

**1 Choose the correct answer from those given :**

**1** If A (5 , 7) , B (1 , - 1) , then the midpoint of  $\overline{AB}$  is .....

- (a) (2 , 3)                      (b) (3 , 3)                      (c) (3 , 2)                      (d) (3 , 4)

**2** A rhombus whose diagonals lengths are 6 cm. , 8 cm. , then its area is .....  $\text{cm}^2$

- (a) 48                      (b) 28                      (c) 24                      (d) 14

**3** If  $\cos X = \frac{\sqrt{3}}{2}$  where X is the measure of an acute angle , then  $\sin 2 X = \dots\dots\dots$

- (a)  $\frac{\sqrt{3}}{2}$                       (b) 1                      (c) - 2                      (d)  $\frac{1}{\sqrt{3}}$

**4** If the lengths of two sides of an isosceles triangle are 5 cm. and 13 cm. , then the length of the third side is ..... cm.

- (a) 5                      (b) 8                      (c) 13                      (d) 16

5 If the two straight lines  $3x - 4y = 3$  and  $4x + ky = 8$  are perpendicular, then  $k = \dots\dots\dots$

- (a) 4 (b) 3 (c) -4 (d) -3

6 The number of axes of symmetry of the equilateral triangle equals  $\dots\dots\dots$

- (a) zero (b) 1 (c) 2 (d) 3

2 [a] Without using calculator, prove that :  $\sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ \tan 45^\circ$

[b] Find the equation of the straight line which passes through the two points  $(4, 2)$ ,  $(-2, -1)$

3 [a] Find the value of  $x$  if  $\tan x = 4 \cos 60^\circ \sin 30^\circ$  where  $x$  is the measure of an acute angle.

[b] Prove that the points A  $(2, 4)$ , B  $(-3, 0)$  and C  $(-7, 5)$  are the vertices of a right-angled triangle, then find its area.

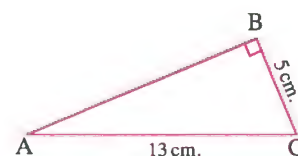
4 [a] Find the equation of the straight line which its slope is 2 and intercepts from the positive part of y-axis 7 length units.

[b] In the opposite figure :

ABC is a right-angled triangle at B

, AC = 13 cm. , BC = 5 cm.

Find the value of :  $\sin A \cos C + \cos A \sin C$



5 [a] If the distance between the two points  $(x, 7)$ ,  $(-2, 3)$  equals 5 length units, find the value of  $x$

[b] If the straight line  $L_1$  passes through the two points  $(3, 1)$ ,  $(2, k)$  and the straight line  $L_2$  makes with the positive direction of the  $x$ -axis a positive angle its measure is  $45^\circ$ , find the value of  $k$  if  $L_1 \parallel L_2$

27

Matrouh Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 If  $\cos 2x = \frac{1}{2}$ , then  $m(\angle x) = \dots\dots\dots$

- (a)  $15^\circ$  (b)  $30^\circ$  (c)  $45^\circ$  (d)  $60^\circ$

2 The angle measured  $37^\circ$  is complemented by an angle of measurement  $\dots\dots\dots$

- (a)  $53^\circ$  (b)  $143^\circ$  (c)  $37^\circ$  (d)  $90^\circ$

3 If  $\frac{-2}{3}$  ,  $\frac{k}{2}$  are the slopes of two parallel straight lines , then k = .....

- (a)  $\frac{-4}{3}$  (b)  $\frac{-3}{4}$  (c) 3 (d)  $\frac{1}{3}$

4 The area of the circle equals .....

- (a)  $\pi r$  (b)  $2 \pi r$  (c)  $\pi r^2$  (d)  $2 \pi r^2$

5 In  $\Delta ABC$  ,  $AB + BC$  .....  $AC$

- (a)  $>$  (b)  $\geq$  (c)  $<$  (d)  $\leq$

6 If  $\overline{AB}$  is a diameter of a circle , where A (3 , -5) , B (5 , 1) , then the centre of the circle is .....

- (a) (8 , -2) (b) (4 , 2) (c) (2 , 2) (d) (4 , -2)

2 [a] Without using calculator , prove that :

$$\tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

[b] Prove that the points A (6 , 0) , B (2 , -4) , C (-4 , 2) are the vertices of a right-angled triangle at B

3 [a] If the distance between the two points (a , 7) and (-2 , 3) equals 5 length units , find the value of a

[b] ABC is a right-angled triangle at B ,  $AB = 3$  cm. ,  $BC = 4$  cm.

Find the value of :  $\sin A \cos C + \cos A \sin C$

4 [a] If A , B are the measures of two complementary angles

, where  $A : B = 1 : 2$

, find :  $\sin A + \cos B$

[b] Find the slope and the intercepted part of y-axis of the straight line

whose equation is  $\frac{x}{2} + \frac{y}{2} = 1$

5 [a] If C is the midpoint of  $\overline{AB}$  , where A = (x , -6) , B = (9 , -12) and C = (-3 , y) , find the values of x , y

[b] Find the equation of the straight line passing through the point (3 , -5) and parallel to the straight line  $x + 2y = 7$

# Final Examinations 2021

on Trigonometry and Geometry



1

## Cairo Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given answers :

- 1 If  $\sin X = \frac{1}{2}$ , where  $X$  is the measure of an acute angle, then  $X = \dots\dots\dots^\circ$   
(a) 30 (b) 45 (c) 60 (d) 90
- 2 The straight line whose equation is  $y = 3X + 4$  intercepts from the positive part of y-axis a part of length  $\dots\dots\dots$  length units.  
(a) 3 (b) 4 (c) 5 (d) 7
- 3 The measure of the exterior angle of an equilateral triangle equals  $\dots\dots\dots^\circ$   
(a) 120 (b) 90 (c) 60 (d) 30
- 4 If  $\triangle ABC \equiv \triangle XYZ$ , then  $AB = \dots\dots\dots$   
(a) BC (b) YZ (c) XZ (d) XY
- 5 The equation of the straight line whose slope equals 1 and passes through the origin point is  $\dots\dots\dots$   
(a)  $y = X + 1$  (b)  $X = 1$  (c)  $y = 1$  (d)  $y = X$
- 6 The angle whose measure is  $30^\circ$  supplements an angle of measure  $\dots\dots\dots^\circ$   
(a) 60 (b) 120 (c) 150 (d) 180

2 [a] Without using calculator, prove that :

$$4 \sin 45^\circ \cos 45^\circ = 2 \text{ (showing the steps of the solution).}$$

[b] Find the equation of the straight line which passes through the point (1, 2) and is parallel to the straight line whose equation is  $y = 3X + 5$

3 [a] Find the value of  $X$  which satisfies that :

$$X \sin 30^\circ = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$$

[b] Prove that the straight line which passes through the points (0, 5), (3, 2) is perpendicular to the straight line which makes an angle of measure  $45^\circ$  with the positive direction of X-axis.

- 4 [a] ABCD is a parallelogram , M is the point of intersection of its diagonals where , A (3 , - 1) , C (1 , 7) Find the coordinates of the point M
- [b] If A (2 , 8) , B (- 1 , 4) and C (3 , 1) are the vertices of the triangle ABC , prove that :
- 1 The triangle ABC is a right-angled triangle at B
  - 2 The triangle ABC is an isosceles triangle.
- 5 [a] The triangle ABC is a right-angled triangle at B where AB = 7 cm. and BC = 24 cm. Find the value of :
- 1  $3 \tan A \times \tan C$
  - 2  $\sin^2 A + \sin^2 C$
- [b] If the points (0 , 1) , (a , 3) and (2 , 5) are collinear , find the value of a

2

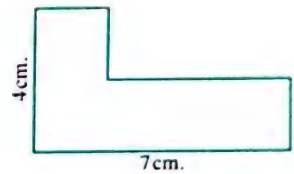
Giza Governorate



Answer the following questions :

1 Choose the correct answer :

- 1 The perimeter of the opposite figure equals ..... cm.
- (a) 44 (b) 22  
(c) 18 (d) 11
- 2 If  $\angle X$  ,  $\angle Y$  are two complementary angles and  $\sin X = \frac{3}{5}$  , then  $\cos Y =$  .....
- (a)  $\frac{4}{5}$  (b)  $\frac{3}{5}$  (c)  $\frac{3}{4}$  (d)  $\frac{5}{3}$
- 3 ABCD is a parallelogram and  $m(\angle A) : m(\angle B) = 1 : 2$  , then  $m(\angle B) =$  .....°
- (a) 45 (b) 135 (c) 120 (d) 115
- 4 The straight line whose equation is :  $y - 2x - 5 = 0$  cuts from the positive part of y-axis a part of length ..... length units.
- (a) 2 (b) 5 (c) 7 (d) 10
- 5 In  $\triangle ABC$  , if the angles  $\angle A$  ,  $\angle B$  are complementary , then  $m(\angle C) =$  .....°
- (a) 45 (b) 30 (c) 90 (d) 60
- 6 The slope of the straight line which makes with the positive direction of X-axis an angle whose positive measure is  $X^\circ$  equals .....
- (a)  $\sin X$  (b)  $\cos X$  (c)  $\frac{\sin X}{\cos X}$  (d)  $\sin X + \cos X$



- 2 [a] ABCD is a trapezoid in which  $\overline{AD} \parallel \overline{BC}$  ,  $m(\angle B) = 90^\circ$  If AB = 3 cm. , AD = 6 cm. , BC = 10 cm. , then prove that :  $\cos(\angle DCB) - \tan(\angle ACB) = \frac{1}{2}$
- [b] If the straight line  $L_1$  , passes through the points (3 , 1) , (2 , k) and the straight line  $L_2$  makes with the positive direction of X-axis an angle of measure  $45^\circ$  , then find the value of k which makes the two straight lines  $L_1$  ,  $L_2$  parallel.

1

Cairo Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given answers :

1 If  $\sin X = \frac{1}{2}$ , where  $X$  is the measure of an acute angle, then  $X = \dots\dots\dots^\circ$

- (a) 30 (b) 45 (c) 60 (d) 90

2 The straight line whose equation is  $y = 3X + 4$  intercepts from the positive part of y-axis a part of length  $\dots\dots\dots$  length units.

- (a) 3 (b) 4 (c) 5 (d) 7

3 The measure of the exterior angle of an equilateral triangle equals  $\dots\dots\dots^\circ$

- (a) 120 (b) 90 (c) 60 (d) 30

4 If  $\triangle ABC \equiv \triangle XYZ$ , then  $AB = \dots\dots\dots$

- (a) BC (b) YZ (c) XZ (d) XY

5 The equation of the straight line whose slope equals 1 and passes through the origin point is  $\dots\dots\dots$

- (a)  $y = X + 1$  (b)  $X = 1$  (c)  $y = 1$  (d)  $y = X$

6 The angle whose measure is  $30^\circ$  supplements an angle of measure  $\dots\dots\dots^\circ$

- (a) 60 (b) 120 (c) 150 (d) 180

2 [a] Without using calculator, prove that :

$$4 \sin 45^\circ \cos 45^\circ = 2 \text{ (showing the steps of the solution).}$$

[b] Find the equation of the straight line which passes through the point (1, 2) and is parallel to the straight line whose equation is  $y = 3X + 5$

3 [a] Find the value of  $X$  which satisfies that :

$$X \sin 30^\circ = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$$

[b] Prove that the straight line which passes through the points (0, 5), (3, 2) is perpendicular to the straight line which makes an angle of measure  $45^\circ$  with the positive direction of X-axis.

- 4 [a] ABCD is a parallelogram, M is the point of intersection of its diagonals where  $A(3, -1)$ ,  $C(1, 7)$  Find the coordinates of the point M
- [b] If  $A(2, 8)$ ,  $B(-1, 4)$  and  $C(3, 1)$  are the vertices of the triangle ABC
- prove that : 1 The triangle ABC is a right-angled triangle at B
- 2 The triangle ABC is an isosceles triangle.
- 5 [a] The triangle ABC is a right-angled triangle at B where  $AB = 7$  cm. and  $BC = 24$  cm.
- Find the value of : 1  $3 \tan A \times \tan C$  2  $\sin^2 A + \sin^2 C$
- [b] If the points  $(0, 1)$ ,  $(a, 3)$  and  $(2, 5)$  are collinear, find the value of a

2

Giza Governorate

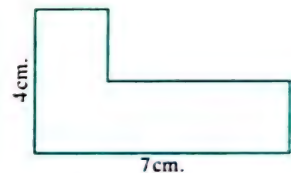


Answer the following questions :

1 Choose the correct answer :

- 1 The perimeter of the opposite figure equals ..... cm.

(a) 44 (b) 22  
(c) 18 (d) 11



- 2 If  $\angle X$ ,  $\angle Y$  are two complementary angles and  $\sin X = \frac{3}{5}$ , then  $\cos Y =$  .....

(a)  $\frac{4}{5}$  (b)  $\frac{3}{5}$  (c)  $\frac{3}{4}$  (d)  $\frac{5}{3}$

- 3 ABCD is a parallelogram and  $m(\angle A) : m(\angle B) = 1 : 2$ , then  $m(\angle B) =$  .....°

(a) 45 (b) 135 (c) 120 (d) 115

- 4 The straight line whose equation is :  $y - 2x - 5 = 0$  cuts from the positive part of y-axis a part of length ..... length units.

(a) 2 (b) 5 (c) 7 (d) 10

- 5 In  $\triangle ABC$ , if the angles  $\angle A$ ,  $\angle B$  are complementary, then  $m(\angle C) =$  .....°

(a) 45 (b) 30 (c) 90 (d) 60

- 6 The slope of the straight line which makes with the positive direction of X-axis an angle whose positive measure is  $X^\circ$  equals .....

(a)  $\sin X$  (b)  $\cos X$  (c)  $\frac{\sin X}{\cos X}$  (d)  $\sin X + \cos X$

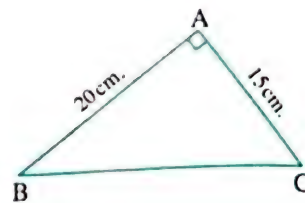
- 2 [a] ABCD is a trapezoid in which  $\overline{AD} \parallel \overline{BC}$ ,  $m(\angle B) = 90^\circ$  If  $AB = 3$  cm.,  $AD = 6$  cm.,  $BC = 10$  cm., then prove that :  $\cos(\angle DCB) - \tan(\angle ACB) = \frac{1}{2}$
- [b] If the straight line  $L_1$ , passes through the points  $(3, 1)$ ,  $(2, k)$  and the straight line  $L_2$  makes with the positive direction of X-axis an angle of measure  $45^\circ$ , then find the value of k which makes the two straight lines  $L_1$ ,  $L_2$  parallel.

**3 [a] In the opposite figure :**

ABC is a triangle ,  $m(\angle A) = 90^\circ$  ,  $AC = 15$  cm.

,  $AB = 20$  cm.

**Prove that :**  $\cos C \cos B - \sin C \sin B = 0$


**[b] ABCD is a parallelogram its diagonals intersect at M where :**

$A(3, -1)$  ,  $B(6, 2)$  ,  $C(1, 7)$

Find the coordinates of the two points M and D

**4 [a] Without using calculator , find  $m(\angle X)$  which satisfies the equation :**

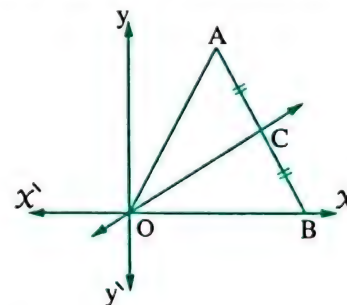
$\tan X = 4 \sin 30^\circ \cos 60^\circ$  where  $X$  is a positive acute angle.

**[b] Find the equation of the straight line passing through the point  $(3, 4)$  and perpendicular to the straight line  $5x - 2y + 7 = 0$** 
**5 [a] If the distance between the point  $(a, 7)$  and the point  $(0, 3)$  is equal to 5 length units. , then find the value of  $a$** 
**[b] In the opposite figure :**

AOB is an equilateral triangle

,  $C$  is the midpoint of  $\overline{AB}$

Find the equation of  $\overline{OC}$  where  $O$  is the origin point.


**3**

Alexandria Governorate



**Answer the following questions : (Calculators are permitted)**

**1 Choose the correct answer from those given :**

**1** If  $C(6, -4)$  is the midpoint of  $\overline{AB}$  where  $A(5, -3)$  , then  $B$  is .....

- (a)  $(7, -5)$       (b)  $(-5, -7)$       (c)  $(-5, 7)$       (d)  $(11, -7)$

**2** The measure of the angle that complements an angle of measure  $60^\circ$  is .....

- (a) 120      (b) zero      (c) 30      (d) 90

**3** If  $\sin \theta = 0.6$  , then  $m(\angle \theta) \approx$  .....

- (a)  $51^\circ 33' 35''$       (b)  $36^\circ 52' 12''$       (c)  $47^\circ 15' 48''$       (d)  $45^\circ 15' 6''$

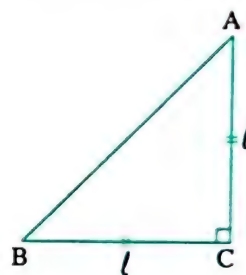
- 4 The square whose area is  $100 \text{ cm}^2$ , then its diagonal length is ..... cm.  
 (a) 10 (b) 50 (c)  $2\sqrt{10}$  (d)  $10\sqrt{2}$
- 5 ABC is a right-angled triangle at B where A (1, 4), B (-1, -2), then the slope of  $\overrightarrow{BC}$  equals .....  
 (a)  $-\frac{1}{3}$  (b) 3 (c)  $\frac{1}{3}$  (d) -3
- 6 The sum of the lengths of any two sides of a triangle is ..... the length of the third side.  
 (a) smaller than (b) equal to (c) greater than (d) twice

2 [a] In the opposite figure :

ABC is an isosceles triangle and right-angled at C  
 and the length of each of its legs is  $l$

Find : 1 The ratio between the lengths of the triangle  
 sides AC : BC : AB

2  $\tan B$ ,  $\sin A$



- [b] If the distance between the two points  $(X, 5)$ ,  $(6, 1)$  equals  $2\sqrt{5}$  length units, find the values of  $X$

3 [a] If the points A (3, 2), B (4, -3), C (-1, -2), D (-2, 3) are the vertices of a rhombus

, find : 1 The coordinates of the intersection point of its diagonals.

2 The area of the rhombus ABCD

- [b] Without using calculator, find the value of  $X$  (where  $X$  is the measure of an acute angle) which satisfies :  $2 \sin X = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$

4 [a] Find the equation of the straight line passing through the point (1, 2) and perpendicular to the straight line passing through the two points A (2, -3), B (5, -4)

[b] Prove the following equality with indicating the steps :  $\tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$

5 [a] If the straight line  $L_1$  passes through the two points (3, 1), (2, k) and the straight line  $L_2$  makes with the positive direction of the  $X$ -axis an angle of measure  $45^\circ$ , find the value of  $k$ , if  $L_1 \parallel L_2$

[b] Prove that the points A (-2, 5), B (3, 3), C (-4, 2) are not collinear.



Answer the following questions :

1 Choose the correct answer :

1 If  $\cos X = \frac{\sqrt{2}}{2}$  where  $X$  is the measure of an acute angle , then  $\sin 2X = \dots\dots\dots$

- (a)  $\frac{1}{\sqrt{2}}$  (b)  $-\frac{\sqrt{2}}{2}$  (c) 1 (d)  $\frac{2}{\sqrt{2}}$

2 The number of the axes of symmetry of the circle equals .....

- (a) zero (b) 1 (c) 2 (d) an infinite number.

3 If ABCD is a rectangle A ( - 4 , - 1 ) , C ( 4 , 5 ) , then the length of  $\overline{BD} = \dots\dots\dots$  length units.

- (a) 10 (b) 6 (c) 5 (d) 4

4 The perpendicular length between  $X = 5$  ,  $X + 3 = 0$  equals .....

- (a) 2 (b) 8 (c) - 8 (d) 5

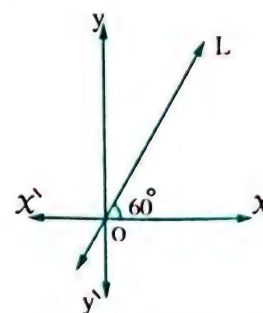
5  $\Delta ABC$  is an isosceles triangle and right-angled at C and the length of each leg is  $l$  , then  $AB : BC : CA = \dots\dots\dots$

- (a)  $1 : 1 : \sqrt{2}$  (b)  $1 : \sqrt{2} : 1$  (c)  $\sqrt{2} : 1 : 2$  (d)  $\sqrt{2} : 1 : 1$

6 In the opposite figure :

The equation of the straight line L is .....

- (a)  $X = \sqrt{3} y$  (b)  $y = \sqrt{3} X$   
(c)  $X = y$  (d)  $y = \sqrt{3}$



2 [a] Find the slope and the length of the y-intercept for the straight line :  $\frac{X}{2} + \frac{y}{3} = 1$

[b] If  $\sin X = \tan 30^\circ \sin 60^\circ$  where  $X$  is the measure of an acute angle , find :  $4 \cos X \sin X$

3 [a] Find the equation of the straight line which passes through the point ( 2 , - 5 ) and is parallel to the straight line which passes through the two points ( - 2 , 1 ) , ( 2 , 7 )

[b] ABC is a right-angled triangle at B , if  $2 AB = \sqrt{3} AC$

Find : 1  $m(\angle C)$

2  $\sin^2 A - \cos^2 C$

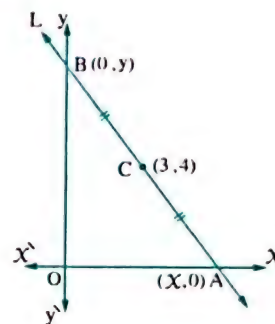
- 4 [a] If the two straight lines  $L_1 : 3x - 4y - 3 = 0$  ,  $L_2 : ay + 4x - 8 = 0$  are perpendicular , find the value of  $a$
- [b] If the points  $A(3, 2)$  ,  $B(4, -3)$  ,  $C(-1, -2)$  ,  $D(-2, 3)$  are the vertices of a rhombus , find the area of the rhombus ABCD

- 5 [a] Prove that :  $\cos^2 60^\circ = \cos^2 30^\circ \tan^2 30^\circ \tan^2 45^\circ$

- [b] In the opposite figure :

The point C is the midpoint of  $\overline{AB}$  where  $C(3, 4)$

Find the perimeter of the triangle AOB



5

El-Sharkia Governorate



Answer the following questions : (Calculator is allowed)

- 1 Choose the correct answer from those given :

- 1 In  $\triangle ABC$  , if  $m(\angle B) = 90^\circ$  , then  $\sin A + \cos C = \dots\dots\dots$

(a)  $2 \sin C$  (b)  $2 \cos A$  (c)  $2 \cos C$  (d)  $\tan A$

- 2 If  $\sin 2x = \frac{1}{2}$  where  $2x$  is the measure of an acute angle , then  $x = \dots\dots\dots^\circ$

(a) 15 (b) 60 (c) 70 (d) 30

- 3 In the opposite figure :

If  $AO = 8$  length units

,  $OB = 6$  length units

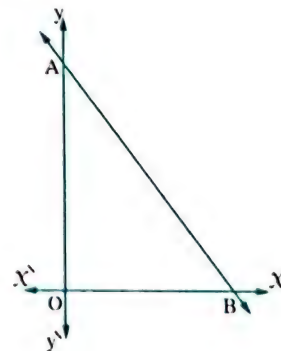
, then the equation of  $\overleftrightarrow{AB}$  is  $\dots\dots\dots$

(a)  $y = \frac{4}{3}x + 8$

(b)  $y = -\frac{4}{3}x - 8$

(c)  $y = \frac{3}{4}x - 8$

(d)  $y = -\frac{4}{3}x + 8$



- 4 The perpendicular distance between the point  $(3, -4)$  and  $x$ -axis equals  $\dots\dots\dots$  length units.

(a) 3 (b) -4 (c) 5 (d) 4

5 In the square XYZL, if the slope of  $\overrightarrow{XZ} = 1$ , then the slope of  $\overrightarrow{YL} = \dots\dots\dots$

- (a) 1 (b) -1 (c)  $\pm 1$  (d)  $45^\circ$

6 ABC is a right-angled triangle at B, where  $3 AC = 5 BC$ , then  $\tan A = \dots\dots\dots$

- (a)  $\frac{3}{5}$  (b)  $\frac{5}{3}$  (c)  $\frac{3}{4}$  (d)  $\frac{4}{3}$

2 [a] If the point C (4, y) is the midpoint of  $\overline{AB}$  where A (x, 3) and B (6, 5)

, find the value of :  $x + y$

[b] Prove that the points A (5, 3), B (3, -2), C (-2, -4) are the vertices of a triangle

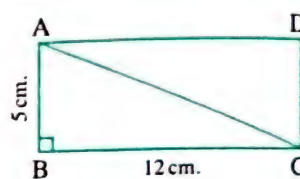
, then prove that the triangle is an obtuse-angled triangle at B

3 [a] In the opposite figure :

If ABCD is a rectangle in which  $AB = 5 \text{ cm.}$ ,  $BC = 12 \text{ cm.}$

, find : 1 The length of  $\overline{AC}$

2 The value of :  $5 \tan (\angle ACD) - 13 \sin (\angle DAC)$



[b] If the two points A (3, -1), B (5, 3)

, find the equation of the axis of symmetry of  $\overline{AB}$

4 [a] Without using the calculator, find the value of :  $\frac{\cos^2 60^\circ + \cos^2 30^\circ}{\sin 60^\circ \tan 60^\circ}$

[b] If the two equations of the two straight lines  $L_1$  and  $L_2$  are :

$L_1 : 6x + ky - 3 = 0$  and  $L_2 : 3y = 2x + 6$  respectively.

, find the value of k which makes :

1 The two straight lines parallel.

2 The two straight lines perpendicular.

5 [a] Find the equation of the straight line which passes through the point (1, 4) and is parallel to the straight line :  $x + 2y - 4 = 0$

[b] If ABCD is a square where : A (2, 4), B (-3, zero), C (-7, 5)

, find : 1 The coordinates of the point D 2 the area of the square ABCD

6

El-Monofia Governorate



Answer the following questions : (Using calculator is permitted)

1 Choose the correct answer :

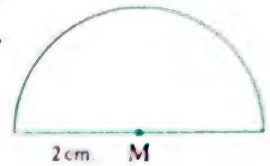
1 The surface area of a square is  $25 \text{ cm}^2$ , then the length of its diagonal is ..... cm.

- (a) 5 (b) 10 (c)  $5\sqrt{2}$  (d)  $10\sqrt{2}$

- 2 ABC is a triangle. If  $(AC)^2 > (AB)^2 + (BC)^2$ , then  $\angle C$  is .....
- (a) acute. (b) obtuse. (c) right. (d) straight.

- 3 The opposite figure represents a semicircle with the radius length of its circle is 2 cm. , then the perimeter of this figure = ..... cm.

- (a)  $2\pi$  (b)  $4\pi$   
(c)  $2\pi + 4$  (d)  $4\pi + 2$



- 4 If  $\cos \frac{x}{2} = \frac{\sqrt{3}}{2}$  where  $\frac{x}{2}$  is the measure of an acute angle , then  $\tan (x - 15^\circ) = \dots\dots\dots$

- (a)  $\sqrt{3}$  (b)  $\frac{1}{\sqrt{3}}$  (c) 1 (d)  $\frac{\sqrt{3}}{2}$

- 5 The equation of a straight line is :  $\frac{x}{2} - \frac{y}{3} = 6$  , then it intercepts from X-axis a part of length ..... length units.

- (a) 3 (b) 12 (c) 6 (d) 18

- 6 If  $\frac{-2}{3}$  ,  $\frac{6}{k}$  are the slopes of two perpendicular straight lines , then  $k = \dots\dots\dots$

- (a) 4 (b) -9 (c) -4 (d) 9

- 2 [a] Determine the type of the triangle ABC where : A (3 , 0) , B (1 , 4) and C (-1 , 2) with respect to the lengths of its sides.

- [b] Without using calculator , prove that :  $\frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ} = 2 + \sqrt{3}$

- 3 [a] ABCD is a quadrilateral where A (2 , 4) , B (-3 , 0) , C (-7 , 5) and D (-2 , 9) Prove that : ABCD is a square.

- [b] ABC is a right-angled triangle at C , AC = 6 cm. and BC = 8 cm.  
Find the value of :  $\cos A \cos B - \sin A \sin B$

- 4 [a] Prove that the straight line which passes through the two points (-3 , -2) and B (4 , 5) is parallel to the straight line which makes with the positive direction of X-axis an angle its measure is  $45^\circ$

- [b] If  $\sqrt{3} \sin x \tan 30^\circ = \tan 45^\circ \cos 2x$  , find the value of  $x$  (where  $x$  is the measure of an acute angle).

- 5 [a] Find the equation of the straight line which is perpendicular to the straight line :  $3x - 4y + 7 = 0$  and intercepts from the positive part of y-axis a part of length 4 units.

- [b] ABCD is a rectangle in which AB = 3 cm. , AC = 5 cm.

Find : 1  $m(\angle ACB)$

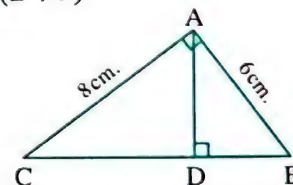
2 The area of the rectangle ABCD



Answer the following questions : (Calculator is allowed)

**1 Choose the correct answer :**

- 1 The number of the axes of symmetry of the scalene triangle equals .....  
 (a) zero (b) 1 (c) 2 (d) 3
- 2 In the triangle XYZ , if  $(YZ)^2 + (XZ)^2 < (XY)^2$  , then  $\angle Z$  is .....  
 (a) acute. (b) right. (c) obtuse. (d) straight.
- 3 If the distance between the two points (a , 0) and (0 , 1) is one length unit , then a = .....  
 (a) 1 (b) - 1 (c) 0 (d) 2
- 4 If the origin point is the midpoint of  $\overline{AB}$  where A (2 , - 3) , then the point B is .....  
 (a) (- 3 , 2) (b) (- 2 , 3) (c) (- 2 , - 3) (d) (2 , 3)
- 5 In the opposite figure : ABC is a right-angled triangle at A in which  $\overline{AD} \perp \overline{BC}$  cutting it at D , AB = 6 cm. and AC = 8 cm. , then AD = ..... cm.  
 (a) 3.6 (b) 8.4 (c) 4.8 (d) 6.4
- 6 ABC is a right-angled triangle at B , then  $\sin A + 2 \cos C =$  .....  
 (a)  $2 \sin C$  (b)  $3 \sin A$  (c)  $2 \sin A$  (d)  $3 \cos A$



- 2 [a]** XYZ is a right-angled triangle at Y in which : XY = 5 cm. and XZ = 13 cm.

Find the value of :  $\cos X \cos Z - \sin X \sin Z$

- [b]** Find the measure of the positive angle that  $\overline{AB}$  makes where :  
 A (3 , - 2) , B (6 , 1) with the negative direction of the X-axis.

- 3 [a]** Find the value of X if :  $\cos (3X + 6^\circ) = \frac{1}{2}$  where  $(3X + 6^\circ)$  is the measure of an acute angle.

- [b]** Find the equation of the straight line which is parallel to the straight line  $\frac{y-1}{X} = \frac{1}{3}$  and intersects from the negative part of y-axis a part equals 3 length units.

- 4 [a]** Find the value of X which satisfies :  $X - \sin 30^\circ \cos^2 45^\circ = \sin^2 60^\circ$

- [b]** If the points A (- 3 , 0) , B (3 , 4) and C (1 , - 6) are the vertices of an isosceles triangle of vertex A , find the length of the drawn line segment from A perpendicular to  $\overline{BC}$

- 5 [a] If the point M  $(-1, 2)$  is the centre of the circle passing through the point A  $(3, -1)$ , find the circumference of the circle (where  $\pi = \frac{22}{7}$ )
- [b] Find the equation of the straight line passing through the point  $(1, 2)$  and perpendicular to the straight line passing through the two points A  $(2, -3)$  and B  $(5, -4)$

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El-Dakahlia Governorate



Answer the following questions : (Calculator is permitted)

- 1 [a] Choose the correct answer :

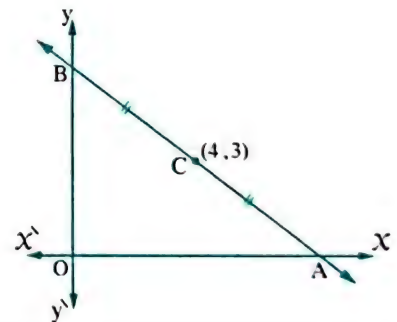
- 1 If  $m(\angle A) = 75^\circ$ ,  $\sin A = \cos B$ ,  $\angle B$  is acute, then  $m(\angle B) = \dots\dots\dots$   
 (a)  $45^\circ$  (b)  $75^\circ$  (c)  $15^\circ$  (d)  $105^\circ$
- 2 If ABC is a right-angled triangle at B,  $AB = BC$ , then  $\tan A = \dots\dots\dots$   
 (a)  $\frac{1}{3}$  (b)  $\sqrt{3}$  (c) 1 (d)  $\frac{1}{\sqrt{2}}$
- 3 If  $\overrightarrow{AB} \perp \overrightarrow{CD}$  and the slope of  $\overrightarrow{AB} = 0$ , then the slope of  $\overrightarrow{CD} = \dots\dots\dots$   
 (a) 1 (b) -1 (c) zero (d) not defined.

- [b] In the opposite figure :

The point C is the midpoint of  $\overline{AB}$   
 where C  $(4, 3)$ , O is the origin  
 point in the perpendicular coordinates system.

Find : 1 The coordinates of the two points A, B

- 2 The area of the triangle AOB



- 2 [a] Choose the correct answer :

- 1 If  $\cos 3X = \frac{1}{2}$ ,  $3X$  is the measure of an acute angle, then  $X = \dots\dots\dots$   
 (a)  $20^\circ$  (b)  $30^\circ$  (c)  $45^\circ$  (d)  $60^\circ$
- 2 The radius length of the circle whose centre is  $(0, 0)$  and passes through  $(3, 4)$  equals  $\dots\dots\dots$  length units.  
 (a) 7 (b) 1 (c) 12 (d) 5
- 3 The measure of the exterior angle of the equilateral triangle equals  $\dots\dots\dots$   
 (a)  $60^\circ$  (b)  $90^\circ$  (c)  $120^\circ$  (d)  $80^\circ$

- [b] Without using calculator, find the value of  $X$  which satisfies :

$$2 \sin X = \tan^2 60^\circ - 2 \tan 45^\circ \text{ where } X \text{ is the measure of an acute angle.}$$

- 3 [a]** Find the equation of the straight line which intercepts from the positive parts of the two axes two parts of lengths 2 units , 3 units from  $X$  and  $y$ -axes respectively.

- [b]** ABC is a right-angled triangle at C , AC = 5 cm. , BC = 12 cm. Find the value of :  
 $\cos A \cos B - \sin A \sin B$

- 4 [a]** ABCD is a parallelogram where A (3 , 2) , B (4 , -5) , C (0 , -3)

Find the coordinates of the point at which the two diagonals intersect , then find the coordinates of the point D

- [b]** Without using calculator , prove that :  $2 \sin 30^\circ + 4 \cos 60^\circ = \tan^2 60^\circ$

- 5 [a]** Prove that A (5 , 1) , B (3 , -7) , C (1 , 3) are not collinear points.

- [b]** Find the equation of the straight line perpendicular to  $\overline{AB}$  from its midpoint where A (2 , 1) , B (4 , 5)

9

Ismailia Governorate



Answer the following questions : (Calculator is allowed)

- 1** Choose the correct answer from those given :

- 1** The parallelogram whose two diagonals are equal in length and perpendicular is the .....

(a) rectangle. (b) rhombus. (c) square. (d) trapezium.

- 2** If C is the midpoint of  $\overline{AB}$  where A (-3 , 6) , B (3 , -6) , then C = .....

(a) (6 , -6) (b) (0 , 0) (c) (3 , 3) (d) (-3 , 0)

- 3** The number of diagonals of the triangle equals .....

(a) 3 (b) 2 (c) 1 (d) 0

- 4** ABC is a triangle in which  $m(\angle A) = 75^\circ$  ,  $\sin B = \cos B$  , then  $m(\angle C) = \dots^\circ$

(a) 90 (b) 60 (c) 45 (d) 30

- 5** If the ratio between the measures of two adjacent supplementary angles is 1 : 2 , then the measure of the greater angle equals .....

(a) 120 (b) 90 (c) 180 (d) 60

- 6** The equation of the straight line which passes through the origin point and its slope = 3 is .....

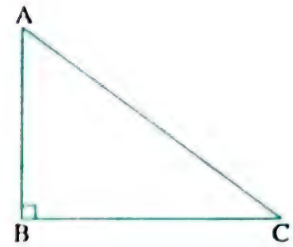
(a)  $y = x$  (b)  $y = 3$  (c)  $x = 3$  (d)  $y = 3x$

- 2 [a] In the opposite figure :**

ABC is a right-angled triangle at B

Prove that :  $\sin^2 A + \sin^2 C = 1$

- [b]** Prove that the straight line which passes through the two points  $(-1, 3)$  ,  $(2, 4)$  is parallel to the straight line whose equation is  $3y - x - 1 = 0$



- 3 [a] In the opposite figure :**

ABCD is a rectangle ,  $AB = 15$  cm. ,  $AC = 25$  cm.

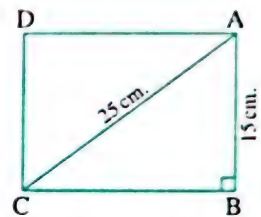
Find :  $m(\angle ACB)$  in degree measure

, then find the area of the rectangle ABCD

- [b]** The opposite table shows a linear relation.

Find : **1** The equation of the straight line.

**2** The length of the intercepted part from y-axis.



$x$	1	2	3
$y$	1	3	5

- 4 [a]** prove that the quadrilateral ABCD whose vertices are

$A(-1, 3)$  ,  $B(5, 1)$  ,  $C(7, 4)$  and  $D(1, 6)$  is a parallelogram.

- [b]** Find the slope of the straight line which intersects from the positive parts of two coordinates  $x$ -axis and  $y$ -axis two parts of lengths 3 units , 4 units respectively , then find the equation of this straight line.

- 5 [a] Without using calculator , find the value of :  $\sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$**

- [b] In the opposite figure :**

A represents the location of Ahmed's house

, B represents the location of Saeid's house

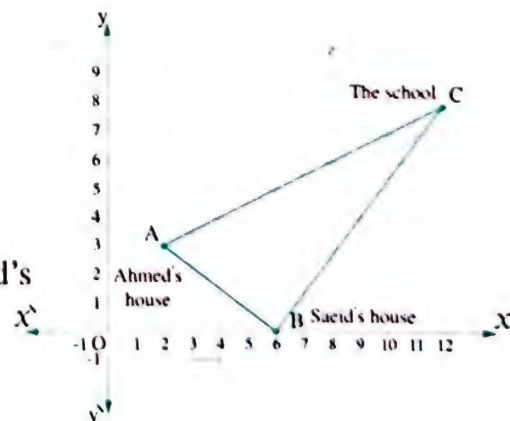
, C represents the location of the School.

- 1** Which is nearer (closer) to the school : Ahmed's house or Saeid's house ? Why ?

Without measuring.

- 2** Are the two roads  $\overline{AB}$  and  $\overline{BC}$  perpendicular ?

giving reason , without measuring.



10

Suez Governorate



Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

- 1 If  $\sin 30^\circ = \cos \theta$  where  $\theta$  is an acute angle , then  $\theta = \dots\dots\dots^\circ$   
 (a) 15 (b) 30 (c) 60 (d) 90
- 2 ABC is a triangle in which :  $(AB)^2 > (BC)^2 + (AC)^2$  , then  $\angle C$  is .....  
 (a) acute. (b) obtuse. (c) right. (d) reflex.
- 3 If A ( - 2 , 5 ) , B ( 2 , - 5 ) , then the midpoint of  $\overline{AB}$  is .....  
 (a) ( 0 , 0 ) (b) ( 2 , 5 ) (c) ( 5 , 2 ) (d) ( - 5 , - 2 )
- 4 If  $\overline{XY}$  is the axis of symmetry of  $\overline{AB}$  , then  $XA \dots\dots\dots XB$   
 (a) > (b) < (c) = (d)  $\leq$
- 5 If  $m_1$  ,  $m_2$  are the slopes of two perpendicular straight lines , then  $m_1 \times m_2 = \dots\dots\dots$   
 (a) - 1 (b) zero (c) 1 (d) 2
- 6 The surface area of the rhombus ABCD = .....  
 (a)  $\frac{1}{2} AB \times DC$  (b)  $\frac{1}{2} AC \times BD$  (c)  $\frac{1}{2} AB \times AD$  (d)  $\frac{1}{2} AD \times BC$

2 [a] Find the equation of the straight line whose slope is 2 and intersects from the positive part of the y-axis a part equals 7 units.

[b] Find the value of  $X$  if :  $4X = \cos^2 30^\circ \tan^2 30^\circ \tan^2 45^\circ$

3 [a] ABCD is a parallelogram whose diagonals intersect at E  
 if A ( 4 , 3 ) , B ( 0 , 2 ) , C ( - 2 , - 3 ) , then find the coordinates of E , D

[b] Without using calculator , prove that :

$$\tan^2 60^\circ - \tan^2 45^\circ = \sin^2 60^\circ + \cos^2 60^\circ + 2 \sin 30^\circ$$

4 [a] Prove that the straight line passing through the two points ( 2 , - 1 ) , ( 6 , 3 ) is parallel to the straight line that makes with the positive direction of the X-axis an angle of measure  $45^\circ$

[b] ABC is a right-angled triangle at B , if  $2AB = \sqrt{3}AC$   
 , find :  $\sin C$  ,  $\tan A$

5 [a] Prove that the points A ( - 3 , 0 ) , B ( 3 , 4 ) , C ( 1 , - 6 ) are the vertices of an isosceles triangle of vertex A

[b] Find the equation of the straight line which passes through the point ( 3 , 5 ) and is perpendicular to the straight line whose slope equals  $-\frac{1}{2}$

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Port Said Governorate



Answer the following questions :

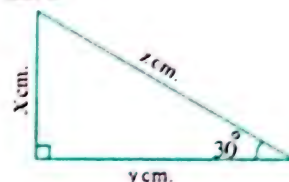
1 Choose the correct answer from those given :

1 The product of multiplying the slopes of two perpendicular straight lines equals .....

- (a) 1 (b) -1 (c)  $\pm 1$  (d) zero

2 In the opposite figure :

- (a)  $x + y = \frac{1}{2} z$  (b)  $z = x^2 + y^2$   
 (c)  $x = \frac{1}{2} z$  (d)  $2y = z$



3  $\sin 30^\circ = \cos \dots$

- (a)  $10^\circ$  (b)  $45^\circ$  (c)  $30^\circ$  (d)  $60^\circ$

4  $\tan 45^\circ = \dots$

- (a) 1 (b)  $2\sqrt{2}$  (c)  $\frac{1}{2}$  (d)  $\sqrt{2}$

5 If A (5, 7) , B (1, -1) , then the midpoint of  $\overline{AB}$  is .....

- (a) (2, 3) (b) (3, 3) (c) (3, 2) (d) (3, 4)

6 If  $\overline{AB} \parallel \overline{CD}$  and the slope of  $\overline{AB} = \frac{2}{3}$  , then the slope of  $\overline{CD} = \dots$

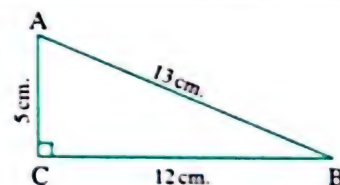
- (a)  $\frac{3}{2}$  (b)  $-\frac{3}{2}$  (c)  $-\frac{2}{3}$  (d)  $\frac{2}{3}$

2 [a] In the opposite figure :

ABC is a right-angled triangle at C  
 , AB = 13 cm. , BC = 12 cm. , AC = 5 cm.

1 Prove that :  $\sin A \cos B + \cos A \sin B = 1$

2 Find :  $1 + \tan^2 A$



[b] Find the value of the following :  $\sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$

3 [a] Find  $m(\angle E)$  , where  $\angle E$  is an acute angle :  $\sin E = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

[b] Prove that the straight line passing through the two points (-3, -2) , (4, 5) is parallel to the straight line that makes with the positive direction of the X-axis an angle of measure  $45^\circ$ .

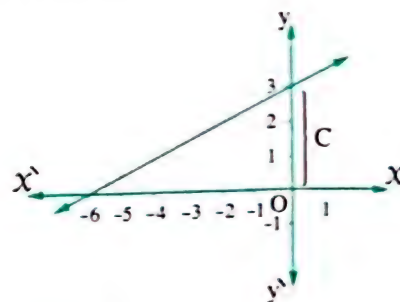
4 [a] Find the equation of the straight line passing through the point (1, 2) and perpendicular to the straight line passing through the two points A (2, -3) , B (5, -4)

[b] Prove that the points A (3, -1) , B (-4, 6) and C (2, -2) are located on the circle whose centre is the point M (-1, 2)

- 5 [a]** ABCD is a parallelogram where A (3, 2), B (4, -5), C (0, -3), find the coordinates of the point at which the two diagonals intersect, then find the coordinates of the point D

**[b]** Using the opposite figure, find the following :

- 1 The length of the y-intercept (c)
- 2 The length of the x-intercept.
- 3 The slope of the straight line (m)



12

Damietta Governorate



Answer the following questions : (Calculator is allowed)

**1** Choose the correct answer from the given answers :

- 1 If the lengths of two sides of an isosceles triangle are 2 cm. and 5 cm. , then the length of the third side is ..... cm.  
 (a) 2 (b) 3 (c) 5 (d) 7
- 2 If  $\sin X = \frac{1}{2}$ , X is the measure of an acute angle, then  $\sin 2X = \dots\dots\dots$   
 (a)  $\frac{\sqrt{3}}{3}$  (b)  $\frac{\sqrt{3}}{2}$  (c)  $\frac{\sqrt{2}}{2}$  (d) 1
- 3 The surface area of the square is equal to the square of the length of the diagonal divided by .....  
 (a) 1 (b) 2 (c) 3 (d) 4
- 4 The equation of the straight line which passes through the point (-2, 5) and is parallel to X-axis is .....  
 (a)  $X = -2$  (b)  $X = 5$  (c)  $y = -2$  (d)  $y = 5$

**5** In the opposite figure :

$A \in \overline{AB}$ ,  $B \in \overline{AB}$ ,  $m(\angle C) = 90^\circ$

, then  $X + y = \dots\dots\dots$

- (a)  $90^\circ$  (b)  $180^\circ$  (c)  $270^\circ$  (d)  $360^\circ$



**6** If  $\overline{AB}$ ,  $\overline{DC}$  are parallel, their slopes are  $m_1$ ,  $m_2$ , then .....

- (a)  $m_1 = -m_2$  (b)  $m_1 - m_2 = 0$  (c)  $m_1 m_2 = -1$  (d)  $m_1 m_2 = 1$

**2 [a]** ABC is a right-angled triangle at C, AC = 6 cm., BC = 8 cm.

Find :  $\cos A \cos B - \sin A \sin B$

[b] Find the equation of the straight line which intercepts from the positive parts of the two parts of lengths 3 units and 2 units for  $x$  and  $y$  axes respectively and find its slope.

3 [a] If the distance of the point  $(x, 5)$  from the point  $(6, 1)$  equals  $2\sqrt{5}$  length units, then find the value of  $x$

[b] Find the equation of the straight line which passes through the points  $(2, -1)$ ,  $(1, 1)$  and if the point  $(0, k) \in$  the straight line, find the value of  $k$

4 [a] Find the value of  $x$  if :  $4x = \cos^2 30^\circ \tan^2 30^\circ \tan^2 45^\circ$  (Indicating the steps of the solution)

[b] If the straight line passing through the two points  $(a, 0)$ ,  $(0, 3)$  is perpendicular to the straight line that makes an angle of measure  $30^\circ$  with the positive direction of the  $x$ -axis find  $a$ .

5 [a] Prove that :  $\sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ = 0$  (Indicating the steps of the solution)

[b] Find the equation of the straight line perpendicular to  $\overline{AB}$  from its midpoint  $C$  where  $A(1, 3)$  and  $B(3, 5)$

13

Kafr El-Sheikh Governorate



Answer the following questions : (Calculators are permitted)

1 Choose the correct answer from those given :

1 In  $\triangle ABC$ , if  $m(\angle A) = 60^\circ$ ,  $\sin B = \cos B$ , then  $m(\angle C) = \dots\dots\dots$

(a)  $30^\circ$  (b)  $75^\circ$  (c)  $90^\circ$  (d)  $105^\circ$

2 The area of the triangle bounded by the straight lines :  $x = 0$ ,  $y = 0$ ,  $5x + 2y = 10$  is  $\dots\dots\dots$  square units.

(a) 10 (b) 8 (c) 7 (d) 5

3 If the straight line passing through the two points  $(\sqrt{3}, 1)$ ,  $(2\sqrt{3}, y)$  its slope equals  $\tan 60^\circ$ , then  $y = \dots\dots\dots$

(a) 2 (b) 3 (c) 4 (d) 5

4 If the straight line  $ax + (2 - a)y = 5$  is parallel to the straight line passing through the two points  $(1, 4)$ ,  $(3, 5)$ , then  $a = \dots\dots\dots$

(a) 3 (b) -2 (c) 1 (d) zero

5 If the point  $(l - 3, 2)$  is in the first quadrant, then  $l$  can be equal to  $\dots\dots\dots$

(a) -3 (b) 2 (c) 7 (d) zero

6 The complement of the angle whose measure is  $65^\circ$  is of measure  $\dots\dots\dots$

(a)  $35^\circ$  (b)  $25^\circ$  (c)  $115^\circ$  (d)  $45^\circ$

- 2 [a]** ABC is a right-angled triangle at B , AC = 13 cm. , BC = 12 cm.

**Prove that :**  $\sin^2 C + \sin^2 A = 1$

- [b]** If the point A (5 , 2) lies on the circle of centre M (1 , - 1) , then find :

- 1** The surface area of the circle in terms of  $\pi$
- 2** The equation of the straight line which passes through A and M

- 3 [a]** If A (- 3 , 5) , B (- 1 , 7) , find the equation of the axis of symmetry of  $\overline{AB}$

- [b]** Without using the calculator , prove that :

$$\tan^2 60^\circ - \tan^2 45^\circ = \sin^2 60^\circ + \cos^2 60^\circ + 2 \sin 30^\circ$$

- 4 [a]** Prove that the points A (- 1 , 3) , B (5 , 1) , C (7 , 4) , D (1 , 6) are the vertices of the parallelogram ABCD

- [b]** ABCD is an isosceles trapezoid in which  $\overline{AD} \parallel \overline{BC}$  , AD = 4 cm. , AB = 5 cm.

, BC = 12 cm. , then calculate :  $\frac{\tan B \cos C}{\cos^2 C + \sin^2 C}$

- 5 [a]** If the straight line  $L_1$  passes through the two points (3 , 1) , (2 , k) and the straight line  $L_2$  makes with the positive direction of X-axis an angle of measure  $45^\circ$  , find the value of k if : **1**  $L_1 \parallel L_2$  **2**  $L_1 \perp L_2$

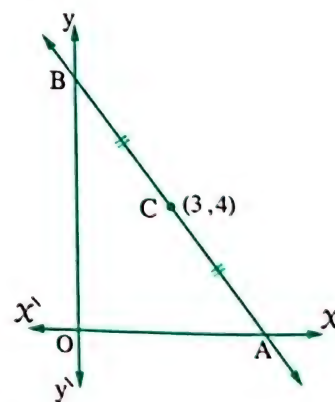
- [b]** In the opposite figure :

The point C is the midpoint of  $\overline{AB}$

where C (3 , 4) , O is the origin point of the perpendicular coordinates system.

**Find :** **1** The coordinates of the two points A and B

- 2** The equation of  $\overline{AB}$



**14**

**El-Beheira Governorate**

**Answer the following questions : (Calculator is permitted)**

- 1** Choose the correct answer from the given ones :

- 1** If A (5 , 7) , B (1 , - 1) , then the midpoint of  $\overline{AB}$  is .....

- (a) (2 , 3) (b) (3 , 3) (c) (3 , 2) (d) (3 , 4)

- 2** If  $m(\angle B) = 80^\circ$  , then  $m(\text{reflex } \angle B) = \dots\dots\dots$

- (a)  $10^\circ$  (b)  $100^\circ$  (c)  $80^\circ$  (d)  $280^\circ$

- 3 The slope of the straight line which is parallel to the straight line passing through the two points  $(2, 3)$ ,  $(-2, 4)$  equals .....

(a)  $-1$  (b)  $-\frac{1}{4}$  (c)  $\frac{1}{4}$  (d)  $1$

- 4 If  $\tan(X + 10^\circ) = \sqrt{3}$  where  $X$  is the measure of an acute angle, then  $X = \dots\dots\dots$

(a)  $30^\circ$  (b)  $45^\circ$  (c)  $50^\circ$  (d)  $60^\circ$

- 5 In a parallelogram, the two diagonals are .....

(a) perpendicular. (b) equal in length.  
(c) equal in length and perpendicular. (d) bisecting each other.

- 6 The triangle whose sides lengths are  $2$  cm.,  $(X + 2)$  cm. and  $5$  cm. becomes an isosceles triangle when  $X = \dots\dots\dots$

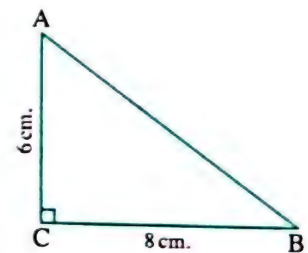
(a) zero (b)  $2$  (c)  $3$  (d)  $5$

- 2 [a] In the opposite figure :

ABC is a right-angled triangle  
at C,  $AC = 6$  cm.,  $BC = 8$  cm.

Find : 1  $\cos A \cos B - \sin A \sin B$

2  $m(\angle B)$



- [b] State the kind of the triangle whose vertices are the points  $A(-2, 4)$ ,  $B(3, -1)$ ,  $C(4, 5)$  with respect to its sides.

- 3 [a] Without using the calculator, prove that :

$$\tan^2 60^\circ - \tan^2 45^\circ = \cos^2 30^\circ + \cos^2 60^\circ + 2 \sin 30^\circ$$

- [b] Find the equation of the straight line whose slope equals  $2$  and intersects from the negative part of the  $y$ -axis a part equals  $3$  units and draw it.

- 4 [a] Find the value of  $X$  which satisfies :  $X \sin 30^\circ \cos^2 45^\circ = \sin^2 60^\circ$

- [b] If the straight line  $L_1$  passes through the two points  $(3, 1)$ ,  $(2, k)$  and the straight line  $L_2$  makes with the positive direction of the  $X$ -axis an angle of measure  $45^\circ$ , find the value of  $k$ , if  $L_1 \parallel L_2$

- 5 [a] If the point  $(3, 1)$  is the midpoint of  $\overline{AB}$  where  $A(1, y)$  and  $B(X, 3)$ , find the point  $(X, y)$

- [b] Find the equation of the straight line passing through the point  $(3, -5)$  and perpendicular to the straight line :  $X + 2y - 7 = 0$



Answer the following questions : (Using calculators is allowed)

1 Choose the correct answer :

1 If  $\tan 3X = \sqrt{3}$  where  $X$  is the measure of an acute angle , then  $X = \dots\dots\dots^\circ$

- (a) 10 (b) 15 (c) 20 (d) 30

2 If the perimeter of a square is 16 cm. , then its area is  $\dots\dots\dots \text{cm}^2$

- (a) 4 (b) 16 (c) 60 (d) 90

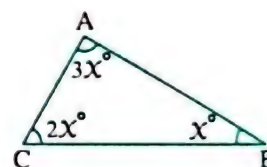
3 The perpendicular distance between the two straight lines :  $X - 2 = 0$  ,  $X + 3 = 0$  equals  $\dots\dots\dots$  length units.

- (a) 1 (b) 5 (c) 2 (d) 3

4 In the opposite figure :

$\Delta ABC$  is  $\dots\dots\dots$  triangle.

- (a) an isosceles. (b) an equilateral.  
(c) an obtuse-angled. (d) a right-angled.



5 The area of the triangle identified by the straight lines :

$3X - 4y = 12$  ,  $X = 0$  ,  $y = 0$  equals  $\dots\dots\dots$  square units.

- (a) 6 (b) 7 (c) 12 (d) 5

6 The measure of the angle of the regular hexagon is  $\dots\dots\dots$

- (a)  $108^\circ$  (b)  $90^\circ$  (c)  $120^\circ$  (d)  $60^\circ$

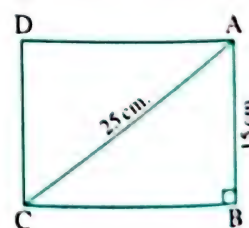
2 [a] In the opposite figure :

ABCD is a rectangle in which  $AB = 15 \text{ cm}$ .

,  $AC = 25 \text{ cm}$ .

Find : 1  $m(\angle ACB)$

2 The surface area of the rectangle ABCD



[b] If the distance between the two points  $(a, 7)$  ,  $(-2, 3)$  equals 5 length units , find the values of a

3 [a] Without using the calculator , find the value of  $X$  (where  $X$  is the measure of an acute angle) which satisfies :

$$2 \sin X = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$$

[b] Prove that the straight line passing through the two points  $(-1, 3)$  ,  $(2, 4)$  is parallel to the straight line  $3y - x - 1 = 0$

- 4 [a] ABCD is a quadrilateral, where A (5, 3), B (6, -2), C (1, -1), D (0, 4).  
Prove that : ABCD is a rhombus.

- [b] If A (5, -6), B (3, 7) and C (1, -3), find the equation of the straight line passing through the point A and the midpoint of  $\overline{BC}$

- 5 [a] Without using the calculator, prove that :

$$\frac{\cos^2 60^\circ + \cos^2 30^\circ + \tan^2 45^\circ}{\sin 60^\circ \tan 60^\circ - \sin 30^\circ} = 2$$

- [b] If the straight line  $L_1$  passes through the two points A (3, 1), B (2, y) and the straight line  $L_2$  makes an angle whose measure is  $45^\circ$  with the positive direction of X-axis, then find the value of y if  $L_1 \perp L_2$

16

Beni Suef Governorate



Answer the following questions : (Calculator is allowed)

- 1 Choose the correct answer from those given :

- 1 The product of multiplying the slopes of two perpendicular straight lines equals .....

(a) zero (b) 1 (c) -1 (d)  $\frac{1}{2}$

- 2 If  $\overline{AB}$  is a diameter in a circle of centre M, where A (2, 4) and B (-2, 0), then M = .....

(a) (0, 2) (b) (2, 0) (c) (0, 0) (d) (2, 2)

- 3 The quadrilateral whose diagonals are equal in length and perpendicular is the .....

(a) parallelogram. (b) rhombus. (c) rectangle. (d) square.

- 4 If the lengths of two sides of a triangle are 2 cm. and 5 cm., then the length of the third side  $\in$  .....

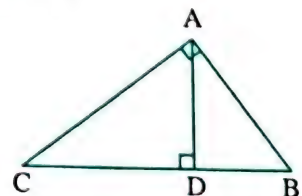
(a)  $]2, 5[$  (b)  $]3, 7[$  (c)  $]2, 7[$  (d)  $]3, 5[$

- 5 In the opposite figure :

If  $m(\angle BAC) = 90^\circ$ ,  $\overline{AD} \perp \overline{BC}$

, then  $(AD)^2 =$  .....

(a)  $AB \times AC$  (b)  $DB \times DC$  (c)  $BD \times BC$  (d)  $(AB)^2 + (BD)^2$



- 6 If  $\tan(X + 15^\circ) = 1$ , where X is the measure of an acute angle, then X = .....

(a)  $60^\circ$  (b)  $45^\circ$  (c)  $30^\circ$  (d)  $15^\circ$

- 2** [a] Find the area of the rectangle ABCD where A (−1, 3) , B (5, 1) , C (6, 4) and D (0, 6)

[b] Find the value of  $X$  if :  $X \cos 60^\circ = \sin 30^\circ + \tan 45^\circ$

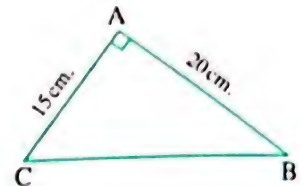
- 3** [a] Prove that the straight line passing through the two points (−1, 0) and (3, 4) is parallel to the straight line that makes a positive angle of measure  $45^\circ$  with the positive direction of the  $X$ -axis.

[b] In the opposite figure :

ABC is a right-angled triangle at A

, AB = 20 cm. and AC = 15 cm.

Prove that :  $\cos C \cos B - \sin C \sin B = \text{zero}$



- 4** [a] If C ( $X, -3$ ) is the midpoint of  $\overline{AB}$  where A (−3,  $y$ ) , B (9, 11) , find the value of :  $X + y$

[b] Without using the calculator , find the value of the expression :  
 $\sin 45^\circ \cos 45^\circ + 3 \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$

- 5** [a] Find the equation of the straight line passing through the point (2, −5) and perpendicular to the straight line whose equation is  $y - 2X + 7 = \text{zero}$
- [b] Prove that the points A (2, 3) , B (6, 2) , C (0, −1) and D (−2, 1) are the vertices of a trapezoid.

**17**

El-Menia Governorate



Answer the following questions : (Calculator is allowed)

**1** Choose the correct answer :

- 1** The measure of the exterior angle of the equilateral triangle equals .....  
 (a)  $60^\circ$  (b)  $90^\circ$  (c)  $120^\circ$  (d)  $180^\circ$
- 2** If  $L_1, L_2$  are two lines parallel and their slopes are  $-\frac{2}{3}, \frac{k}{6}$  , then  $k =$  .....  
 (a) −12 (b) −9 (c) 4 (d) −4
- 3** The lengths of two sides of an isosceles triangle equal 2 cm. , 5 cm. , then the length of the third side equals ..... cm.  
 (a) 5 (b) 2 (c) 3 (d) 7
- 4** The distance between the point (5, 12) and the point of origin equals ..... units.  
 (a) 5 (b) 13 (c) 12 (d)  $\sqrt{17}$

5 The area of the square whose perimeter is 16 cm, equals .....  $\text{cm}^2$

- (a) 4 (b) 8 (c) 16 (d) 256

6 XYZ is an isosceles triangle right-angled at Z, then  $\tan X = \dots\dots\dots$

- (a)  $\frac{1}{\sqrt{3}}$  (b)  $\sqrt{3}$  (c) 1 (d)  $\frac{1}{3}$

2 [a] Prove that the triangle whose vertices are A (6, 0), B (2, -4), C (-4, 2) is right-angled at B

[b] XYZ is a right-angled triangle at Z where  $XZ = 7 \text{ cm}$ . Find the value of :  $\tan X \times \tan Y$

3 [a] Find X where :  $4X = \cos^2 30^\circ \tan^2 30^\circ \tan^2 45^\circ$

[b] Find the equation of the straight line passing through the point (3, -5) and parallel to the straight line  $X + 2y - 7 = 0$

4 [a] ABCD is a parallelogram, A (-2, 5), B (3, 3), C (-4, 2) Find the two coordinates of the point at which the two diagonals intersect, then find the coordinates of the point D

[b] Without using the calculator, prove that :  $\sin^2 30^\circ = 5 \cos^2 60^\circ - \tan^2 45^\circ$

5 [a] If the straight line  $L_1$  passes through the two points (3, 1), (2, k) and the straight line  $L_2$  makes with the positive direction of the X-axis an angle whose measure is  $45^\circ$ , then find k, if the two straight lines  $L_1, L_2$  are perpendicular.

[b] Find the equation of the straight line which intersects from the positive parts of X and y axes two parts of lengths 2 units, 3 units respectively.

18

Assiut Governorate



Answer the following questions : (Calculator is permitted)

1 Choose the correct answer :

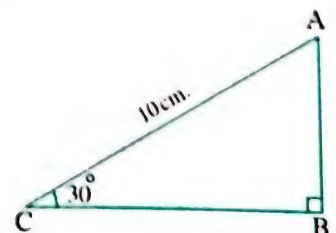
1 The sum of the measures of the interior angles of a triangle equals .....

- (a)  $90^\circ$  (b)  $180^\circ$  (c)  $360^\circ$  (d)  $540^\circ$

2 In the opposite figure :

AB = ..... cm.

- (a) 5 (b) 15  
(c) 20 (d) 40



3 The measure of the interior angle of a regular hexagon equals .....

- (a)  $108^\circ$  (b)  $120^\circ$  (c)  $90^\circ$  (d)  $180^\circ$

4 If  $2 \sin X = 1$  (where  $X$  is the measure of an acute angle) , then  $X =$  .....

- (a)  $45^\circ$  (b)  $90^\circ$  (c)  $30^\circ$  (d)  $60^\circ$

5 The equation of the straight line which passes through the point  $(2, -3)$  and is parallel to  $X$ -axis is .....

- (a)  $X = 2$  (b)  $y = -3$  (c)  $X = -2$  (d)  $y = 3$

6 If the origin point is the midpoint of  $\overline{AB}$  ,  $A(5, -2)$  , then  $B =$  .....

- (a)  $(5, 2)$  (b)  $(-5, -2)$  (c)  $(-5, 2)$  (d)  $(0, 0)$

2 [a] Prove that the points  $A(-3, -1)$  ,  $B(6, 5)$  and  $C(3, 3)$  are collinear.

[b] Find the value of  $X$  that satisfies :  $X \sin 30^\circ \cos^2 45^\circ = \sin^2 60^\circ$

3 [a] If the triangle whose vertices are  $Y(4, 2)$  ,  $X(3, 5)$  and  $Z(-5, a)$

is right-angled at  $Y$  , then find the value of  $a$

[b] Find the equation of the straight line whose slope is 2 and intersects from the positive part of the  $y$ -axis a part that equals 7 units.

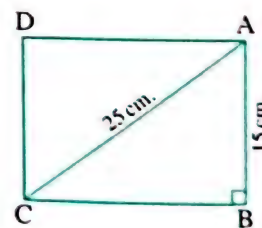
4 [a] In the opposite figure :

$ABCD$  is a rectangle in which  $AB = 15$  cm.

and  $AC = 25$  cm.

Find : 1  $m(\angle ACB)$

2 The surface area of the rectangle  $ABCD$



[b] Prove that the straight line which passes through the points  $(2, 3)$  ,  $(0, 0)$  is parallel to the straight line which passes through  $(-1, 4)$  ,  $(1, 7)$

5 [a]  $ABCD$  is a quadrilateral , where  $A(5, 3)$  ,  $B(6, -2)$  ,  $C(1, -1)$  , and  $D(0, 4)$

Prove that :  $ABCD$  is a rhombus.

[b] Find the slope and the intercepted part of  $y$ -axis by the straight line :

$$2X - 3y - 6 = 0$$

19

Souhag Governorate

Answer the following questions : (Calculator is permitted)

1 Choose the correct answer :

1 If  $\sin \frac{X}{2} = \frac{1}{2}$ ,  $X$  is the measure of an acute angle, then  $X = \dots\dots\dots^\circ$

- (a) 30 (b) 60 (c) 10 (d) 90

2 The perimeter of the square whose surface area is  $100 \text{ cm}^2$  equals  $\dots\dots\dots \text{ cm}$ .

- (a) 10 (b) 20 (c) 40 (d) 50

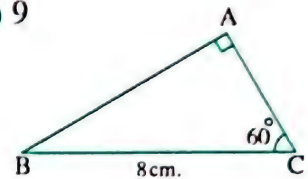
3 If  $-\frac{2}{3}$ ,  $\frac{6}{k}$  are the slopes of two perpendicular straight lines, then  $k = \dots\dots\dots$

- (a) 4 (b) -9 (c) -4 (d) 9

4 In the opposite figure :

The length of  $\overline{AC} = \dots\dots\dots \text{ cm}$ .

- (a) 2 (b) 6  
(c) 4 (d) 8



5 The equation of the straight line passing through the origin point and its slope = 1 is  $\dots\dots\dots$

- (a)  $y = X$  (b)  $y = -X$  (c)  $y = 2X$  (d)  $y = 0$

6 If the numbers 3, 7,  $l$  are lengths of sides of a triangle, then  $l$  can be equal to  $\dots\dots\dots$

- (a) 3 (b) 7 (c) 4 (d) 10

2 [a] If the midpoint of  $\overline{BC}$  is A (2, 3) and C (-1, 3), find the point B

[b] If  $\cos X = \sin 30^\circ \cos 60^\circ$ , find :

1 The measure of  $\angle X$  (where  $X$  is an acute angle)

2  $\tan X$

3 [a] If the straight line whose equation is :  $aX + 2y - 7 = 0$  is parallel to the straight line which makes an angle of measure  $45^\circ$  with the positive direction of  $X$ -axis, find the value of  $a$

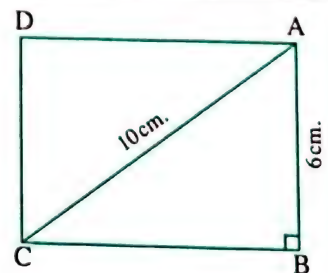
[b] Without using calculator, prove that :  $\tan^2 60^\circ - \tan^2 45^\circ = 4 \sin 30^\circ$

4 [a] In the opposite figure :

ABCD is a rectangle where  $AB = 6 \text{ cm}$ ,  $AC = 10 \text{ cm}$ .

Find : 1  $m(\angle ACB)$

2 The surface area of the rectangle ABCD



[b] Find the equation of the straight line passing through the point (1, 2) and perpendicular to the straight line  $X + 3y + 7 = 0$

5 [a] Prove that the points A (3, -1), B (-4, 6), C (2, -2) which belong to a perpendicular coordinates plane lie on the circle whose centre is the point M (-1, 2), then find the area of the circle.

[b] Find the slope and the intercepted part of y-axis by the straight line where its equation is  $4X + 5y - 10 = 0$

20

Qena Governorate



Answer the following questions :

1 Choose the correct answer from those given :

1  $\sin 30^\circ = \dots\dots\dots$

- (a) 1 (b)  $\frac{\sqrt{3}}{2}$  (c)  $\cos 60^\circ$  (d)  $\frac{1}{\sqrt{2}}$

2 The number of diagonals of the hexagon equals .....

- (a) 5 (b) 6 (c) 2 (d) 9

3 If O the origin point is the midpoint of  $\overline{AB}$  as  $A = (-2, 5)$ , then  $B = \dots\dots\dots$

- (a) (2, 5) (b) (2, -5) (c) (-2, 5) (d) (-2, -5)

4 If the measure of two angles of a triangle are  $70^\circ, 40^\circ$ , then the number of its axes equal .....

- (a) 1 (b) 2 (c) 3 (d) zero

5 If  $L_1, L_2$  are two parallel straight lines of slopes  $m_1, m_2$  respectively, then .....

- (a)  $m_1 - m_2 = \text{zero}$  (b)  $m_1 = -m_2$  (c)  $m_1 \times m_2 = 1$  (d)  $m_1 \times m_2 = -1$

6 If the lengths of two sides of a triangle are 2 cm., 5 cm., then the length of the third side can be .....

- (a) 2 cm. (b) 3 cm. (c) 4 cm. (d) 1 cm.

2 [a] Without using calculator, find the value of :  $\cos 60^\circ \sin 30^\circ - \sin 60^\circ \cos 30^\circ$

[b] Find the equation of the straight line which makes with the positive direction of X-axis a positive angle of measure  $135^\circ$  and intercepts from the positive part of y-axis a part of length 5 length units.

3 [a] Prove that the points A (1, 4), B (-1, -2), C (2, -3) are the vertices of a right-angled triangle, find its area.

[b] In the opposite figure :

$\Delta ABC$  is a right-angled triangle at C

,  $AB = 6 \text{ cm}$  ,  $m(\angle B) = 60^\circ$

Find : The length of  $\overline{AC}$



4 [a] Find the slope of the straight line whose equation is :

$2x - 6y = 12$  , then find the points of intersection with the coordinates axes.

[b] Without using calculator , find the value of  $x$  (where  $x$  is the measure of an acute angle) that satisfies :

$$\tan x = 4 \cos 60^\circ \sin 30^\circ$$

5 [a] Prove that the straight line which passes through the two points  $(1, 3)$  ,  $(2, 4)$  is parallel to the straight line whose equation is :  $y - x = 5$

[b] Prove that the figure ABCD is a rectangle where  $A(1, 0)$  ,  $B(-1, 4)$  ,  $C(7, 8)$  ,  $D(9, 4)$

21

Luxor Governorate



Answer the following questions :

1 Choose the correct answer :

1 The length of the side opposite to the angle of measure  $30^\circ$  in the right-angled triangle equals ..... the length of the hypotenuse.

- (a) quarter.      (b) twice.      (c) half.      (d) third.

2 If  $\tan(2x - 5) = 1$  where  $x$  is the measure of an acute angle , then  $x =$  .....

- (a)  $15^\circ$       (b)  $75^\circ$       (c)  $50^\circ$       (d)  $25^\circ$

3 If the diagonal length of a square is 10 cm. , then its area = .....  $\text{cm}^2$

- (a) 100      (b) 75      (c) 50      (d) 25

4 The straight line passing by the two points  $(0, 0)$  ,  $(2, 3)$  is parallel to the straight line whose slope is .....

- (a)  $\frac{3}{2}$       (b)  $\frac{2}{3}$       (c)  $-\frac{3}{2}$       (d)  $-\frac{2}{3}$

5 The image of the point  $(3, -2)$  by reflection in the  $x$ -axis is .....

- (a)  $(-2, 3)$       (b)  $(3, 2)$       (c)  $(2, -3)$       (d)  $(-3, -2)$

6 The slope of the straight line  $x - 5 = 0$  is .....

- (a) 5      (b)  $\frac{1}{5}$       (c) zero      (d) undefined.

- 2** [a] Find in degrees the value of  $X$  if :  $\tan 2X = 4 \sin 30^\circ \cos 30^\circ$  where  $0^\circ < X < 90^\circ$
- [b] Find the equation of the straight line passing by the point  $(3, 5)$  and is parallel to the straight line  $2X - 3y + 6 = 0$

- 3** [a] Prove that the straight line passing by the two points  $(7, -3)$  ,  $(5, -1)$  is perpendicular to the straight line which makes an angle of measure  $45^\circ$  with the positive direction of  $X$ -axis.

[b] Without using the calculator , prove that :  $2 \sin 30^\circ + 4 \cos 60^\circ = \tan^2 60^\circ$

- 4** [a] If the distance between the points  $(a, 0)$  ,  $(0, 1)$  equals  $\sqrt{2}$  length unit find  $a$
- [b] If  $\overline{AB}$  is a diameter in the circle  $M$  where  $A(4, -1)$  ,  $B(-2, 7)$  , find the coordinates of the point  $M$  and the radius length of the circle.

- 5** [a] Prove that the points  $A(-1, -4)$  ,  $B(1, 0)$  ,  $C(2, 2)$  are collinear.

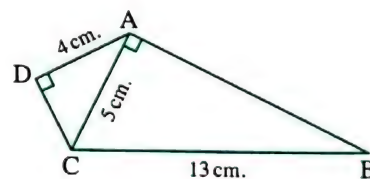
[b] In the opposite figure :

$$m(\angle ADC) = m(\angle BAC) = 90^\circ$$

$$, AD = 4 \text{ cm.} , AC = 5 \text{ cm.} , BC = 13 \text{ cm.}$$

Find the value of :

$$\tan(\angle DAC) \sin(\angle ACB) - \sin(\angle B) \cos(\angle CAD)$$



22

Aswan Governorate



Answer the following questions : (Calculator is allowed)

- 1** Choose the correct answer from those given :

**1** The measure of the exterior angle of the equilateral triangle is .....°

- (a) 60                      (b) 90                      (c) 120                      (d) 180

**2**  $4 \sin 30^\circ \cos 60^\circ = \dots\dots\dots$

- (a) 1                      (b) 2                      (c) 3                      (d) 4

**3** The length of the opposite side of the angle with measure  $30^\circ$  in the right-angled triangle equals ..... the length of the hypotenuse.

- (a)  $\frac{1}{4}$                       (b)  $\frac{1}{3}$                       (c)  $\frac{1}{2}$                       (d)  $\frac{3}{4}$

- 4 The equation of the straight line passing through the point  $(-2, -3)$  and parallel to  $X$ -axis is .....  
 (a)  $y = -2$  (b)  $y = -3$  (c)  $X = -2$  (d)  $X = -3$
- 5  $\Delta ABC$  is an isosceles triangle in which  $AB = 3$  cm. ,  $BC = 7$  cm. , then  $AC =$  ..... cm.  
 (a) 3 (b) 4 (c) 7 (d) 10
- 6 The distance between the two straight lines  $X - 2 = 0$  ,  $X + 3 = 0$  equals ..... length units  
 (a) 1 (b) 2 (c) 3 (d) 5

- 2 [a] Find the equation of the straight line which passes through the two points  $(1, 3)$  ,  $(-1, -3)$   
 [b] Prove that the points  $A(3, -1)$  ,  $B(-4, 6)$  ,  $C(2, -2)$  lie on the circle whose centre is  $M(-1, 2)$  , then find the circumference of the circle.

- 3 [a] Without using calculator , find the measure of  $\angle E$  (Such that  $E$  is an acute angle)  
 If :  $2 \sin E = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$   
 [b] If  $C$  is the midpoint of  $\overline{AB}$  , then find  $x, y$  where  $A(x, 3)$  ,  $B(6, y)$  ,  $C(4, 6)$

- 4 [a]  $\Delta ABC$  is right-angled at  $C$  in which  $AC = 6$  cm. ,  $BC = 8$  cm.  
 Find : 1  $\cos A \cos B - \sin A \sin B$  2  $m(\angle B)$   
 [b] If the straight line  $L_1$  passes through the two points  $(3, 1)$  ,  $(2, k)$  and the straight line  $L_2$  makes with the positive direction of the  $X$ -axis an angle of measure  $45^\circ$  , find the value of  $k$  if the two straight lines are : 1 Parallel. 2 Perpendicular.

- 5 [a] Find the equation of the straight line which passes through the point  $(3, -5)$  and is parallel to the straight line  $X + 2y - 7 = 0$   
 [b] Find the value of  $X$  if :  $X \sin 30^\circ \cos^2 45^\circ = \sin^2 60^\circ$



Answer the following questions : (Calculator is allowed)

- 1 Choose the correct answer from those given :
- 1 The area of the square whose perimeter is 16 cm. equals .....  $\text{cm}^2$   
 (a) 4 (b) 8 (c) 16 (d) 256
- 2 If the lengths of two sides of an isosceles triangle are 3 cm. , 7 cm. , then the length of the third side is ..... cm.  
 (a) 4 (b) 7 (c) 10 (d) 3

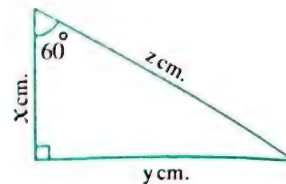
**3 In the opposite figure :**

(a)  $x + y = z$

(b)  $z = x^2 + y^2$

(c)  $2x = z$

(d)  $y = \frac{1}{2}z$



**4**  $2 \sin 30^\circ \tan 60^\circ = \dots\dots\dots$

(a)  $\sqrt{3}$

(b) 3

(c)  $\frac{3}{2}$

(d)  $\frac{1}{2}$

**5** If  $x + y = 5$  ,  $kx + 2y = 0$  are perpendicular , then  $k = \dots\dots\dots$

(a) 1

(b) -1

(c) 2

(d) -2

**6** If A (5 , 7) , B (1 , -1) , then the midpoint of  $\overline{AB}$  is  $\dots\dots\dots$

(a) (2 , 3)

(b) (3 , 3)

(c) (3 , 2)

(d) (3 , 4)

**2 [a]** ABC is a triangle ,  $m(\angle B) = 90^\circ$  ,  $AB = 15$  cm. ,  $BC = 20$  cm.

**Prove that :**  $\cos A \cos C - \sin A \sin C = 0$

**[b]** If the point C (3 , 1) is the midpoint of  $\overline{AB}$  where A (1 , y) , B (x , 3) , find the point : (x , y)

**3 [a]** If the points (0 , 1) , (a , 3) , (2 , 5) are located on one straight line , then find the value of a

**[b]** Prove that the points A (3 , -1) , B (-4 , 6) , C (2 , -2) which belong to an orthogonal Cartesian coordinates plane lie on the circle whose centre is M (-1 , 2) , then find the circumference of the circle in terms of  $\pi$

**4 [a]** Find the equation of the straight line passing through the point (3 , 5) and parallel to the straight line  $x + 3y = 7$

**[b]** Find the value of  $x$  (where  $x$  is the measure of an acute angle) :

$$2 \sin x = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$$

**5 [a]** Find the equation of the straight line whose slope is 2 and intersects from the negative part of y-axis a part of length 3 units.

**[b]** Without using the calculator , prove that :  $\sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ$

**24**

South Sinai Governorate



**Answer the following questions :**

**1 Choose the correct answer from those given :**

**1** The distance between the two points (4 , 0) and (0 , -3) equals  $\dots\dots\dots$  length units.

(a) 5

(b) 4

(c) 3

(d) 12

- 2 If  $\cos (X + 30^\circ) = \frac{1}{2}$  where  $X$  is the measure of an acute angle , then  $X = \dots\dots\dots^\circ$   
 (a) 60 (b) 30 (c) 45 (d) 20
- 3 ABC is a triangle in which  $AB = AC$  and  $m (\angle B) = 30^\circ$  , then  $m (\angle A) = \dots\dots\dots^\circ$   
 (a) 60 (b) 30 (c) 120 (d) 40
- 4 If  $A = (5 , 7)$  and  $B = (-1 , -3)$  , then the midpoint of  $\overline{AB}$  is  $\dots\dots\dots$   
 (a)  $(2 , -2)$  (b)  $(-2 , 2)$  (c)  $(-2 , -2)$  (d)  $(2 , 2)$
- 5 The number of symmetry axes of an isosceles triangle equals  $\dots\dots\dots$   
 (a) 1 (b) zero (c) 2 (d) 3
- 6 ABC is a right-angled triangle at B , D is the midpoint of  $\overline{AC}$  and  $BD = 5$  cm. , then  $AC = \dots\dots\dots$  cm.  
 (a) 5 (b) 10 (c) 15 (d) 20

- 2 [a] ABC is a right-angled triangle at B ,  $AC = 13$  cm. ,  $BC = 12$  cm.

**Prove that :**  $\sin^2 C + \cos^2 C = 1$

- [b] Find the equation of the straight line passing through the point  $(2 , -3)$  and parallel to the straight line  $y = X + 4$

- 3 [a] Without using calculator , prove that :  $\cos 60^\circ = 2 \cos^2 30^\circ - 1$

- [b] Find the equation of the straight line which passes through the point  $(3 , -4)$  and makes with the positive direction of the  $X$ -axis an angle of measure  $45^\circ$

- 4 [a] If the distance between the two points  $(a , 7)$  and  $(-2 , 3)$  equals 5 length units , find the values of  $a$

- [b] Find the value of  $X$  where  $X$  is the measure of an acute angle satisfying the equation :

$$\sin X = 2 \sin 30^\circ \cos 30^\circ$$

- 5 [a] Find the equation of the straight line passing through the point  $(3 , 4)$  and perpendicular to the straight line whose slope  $= \frac{-1}{2}$

- [b] Prove that the triangle of the vertices  $A = (0 , 0)$  ,  $B = (4 , 0)$  and  $C = (0 , 3)$  is right-angled , and find its surface area.

25

North Sinai Governorate



Answer the following questions :

1 Choose the correct answer from those given :

1 If  $a = b$  ,  $a$  ,  $b$  are the measures of two complementary angles , then  $a = \dots\dots\dots^\circ$

- (a) 30 (b) 45 (c) 60 (d) 90

2 If  $\tan 3X = \sqrt{3}$  , where  $X$  is the measure of an acute angle , then  $X = \dots\dots\dots^\circ$

- (a) 10 (b) 20 (c) 30 (d) 60

3 The sum of measures of the interior angles of the quadrilateral equals  $\dots\dots\dots^\circ$

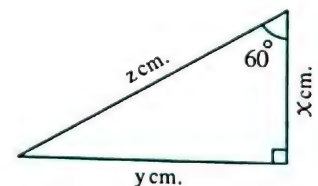
- (a) 360 (b) 180 (c) 90 (d) 540

4 If  $A(1, -6)$  ,  $B(9, 2)$  , then the midpoint of  $\overline{AB}$  is  $\dots\dots\dots$

- (a)  $(-2, 5)$  (b)  $(2, -5)$  (c)  $(5, -2)$  (d)  $(-5, 2)$

5 In the opposite figure :

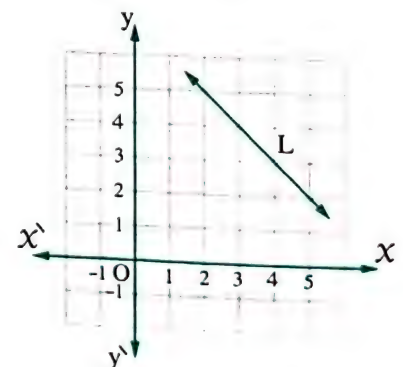
- (a)  $X + y = z$  (b)  $z = X^2 + y^2$   
(c)  $2X = z$  (d)  $y = \frac{1}{2}z$



6 In the opposite figure :

$L$  is a straight line passing through the two points  $(2, 5)$  ,  $(5, 2)$  , then the point  $\dots\dots\dots \in L$

- (a)  $(1, 6)$  (b)  $(2, 3)$   
(c)  $(0, 0)$  (d)  $(3, -4)$



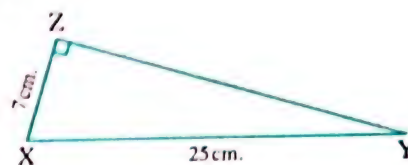
2 [a] Without using the calculator , prove that :  $\sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ$

[b] ABCD is a quadrilateral , where  $A(2, 4)$  ,  $B(-3, 0)$  ,  $C(-7, 5)$  ,  $D(-2, 9)$  , prove that : ABCD is a square.

3 [a] Find the equation of the straight line whose slope is 3 and passes through the point  $(5, 0)$

- [b] In the opposite figure : XYZ is a right-angled triangle at Z  
 , XZ = 7 cm. , XY = 25 cm.

- 1 Find the value of :  $\tan X \times \tan Y$   
 2 Prove that :  $\sin^2 X + \sin^2 Y = 1$



- 4 [a] Without using the calculator , find the value of  $X$  if :  $2 \sin X = \tan^2 60^\circ - 2 \tan 45^\circ$   
 where  $X$  is the measure of an acute angle.  
 [b] Prove that the points A ( - 1 , - 4 ) , B ( 1 , 0 ) , C ( 2 , 2 ) are collinear.
- 5 [a] Prove that the straight line passing through the two points ( - 3 , - 2 ) , ( 4 , 5 ) is  
 parallel to the straight line which makes with the positive direction of the  $X$ -axis an  
 angle of measure  $45^\circ$   
 [b] If the straight line passing through the two points ( - 2 , 3 ) , ( 1 , k ) is perpendicular to  
 the straight line whose slope equals - 3 , then find the value of  $k$

26

Red Sea Governorate



Answer the following questions :

- 1 Choose the correct answer from those given :

- 1  $2 \sin 30^\circ = \dots\dots\dots$   
 (a)  $\frac{1}{2}$  (b)  $\frac{\sqrt{3}}{2}$  (c) 1 (d) 2
- 2 The measure of the exterior angle of the equilateral triangle equals .....  
 (a)  $30^\circ$  (b)  $60^\circ$  (c)  $90^\circ$  (d)  $120^\circ$
- 3 The distance between the point ( 3 , 4 ) and the point of origin equals ..... length units.  
 (a) 3 (b) 4 (c) 5 (d) 7
- 4 If 3 cm. , 7 cm. ,  $l$  are the lengths of the sides of a triangle  
 , then  $l$  can be equal to ..... cm.  
 (a) 3 (b) 7 (c) 4 (d) 10
- 5 If  $\overline{AB} \perp \overline{CD}$  and the slope of  $\overline{AB} = \frac{2}{3}$  , then the slope of  $\overline{CD} = \dots\dots\dots$   
 (a)  $\frac{2}{3}$  (b)  $-\frac{2}{3}$  (c)  $\frac{3}{2}$  (d)  $-\frac{3}{2}$
- 6 The image of the point ( 3 , - 2 ) by reflection in the origin point is .....  
 (a) ( - 3 , 2 ) (b) ( - 3 , - 2 ) (c) ( 3 , 2 ) (d) ( - 2 , 3 )

- 2 [a] Find the value of :  $\cos 60^\circ \sin 30^\circ - \sin 60^\circ \tan 60^\circ + \cos^2 30^\circ$

- [b] Prove that the straight line which passes through the two points ( - 3 , - 2 ) , ( 4 , 5 )  
 is parallel to the straight line which makes an angle of measure  $45^\circ$  with the positive  
 direction of the  $X$ -axis.

- 3 [a]** Find the slope of the straight line  $3x + 4y - 5 = 0$ , then find the length of the intercepted part from y-axis.

- [b]** Find the value of  $x$  where :  $x \sin 30^\circ \cos^2 45^\circ = \sin^2 60^\circ$

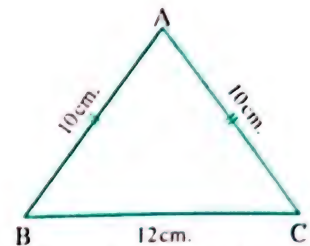
- 4 [a]** In the opposite figure :

ABC is a triangle in which  $AB = AC = 10$  cm.

,  $BC = 12$  cm.

- 1 Find :**  $m(\angle B)$

- 2 Prove that :**  $\sin^2 B + \cos^2 B = 1$



- [b]** Prove that the triangle whose vertices are  $A(1, 4)$ ,  $B(-1, -2)$ ,  $C(2, -3)$  is right-angled, then find its area.

- 5 [a]** Find the equation of the straight line which passes through the point

$A(4, 6)$  and the midpoint of  $\overline{BC}$  where  $B(3, 7)$ ,  $C(1, -3)$

- [b]** ABCD is a parallelogram where  $A(3, 3)$ ,  $B(2, -2)$ ,  $C(5, -1)$ ,  $M$  is the intersection point of its diagonals. **Find :**

- 1** The coordinates of  $M$

- 2** The coordinates of  $D$

**27**

Matrouh Governorate



**Answer the following questions : (Calculator is allowed)**

- 1** Choose the correct answer from those given :

- 1** The area of the square whose perimeter is 16 cm. equals .....  $\text{cm}^2$

(a) 4 (b) 8 (c) 16 (d) 256

- 2** The equation of the straight line whose slope is 1 and passes through the origin point is .....

(a)  $x = 1$  (b)  $y = 1$  (c)  $y = x$  (d)  $y = -x$

- 3** If  $\cos 2x = \frac{1}{2}$ , then  $x =$  .....

(a)  $15^\circ$  (b)  $30^\circ$  (c)  $45^\circ$  (d)  $60^\circ$

- 4** A right circular cylinder, if its height equals the length of its base radius =  $r$  cm., then its volume = .....  $\text{cm}^3$

(a)  $\pi r^3$  (b)  $2\pi r^2$  (c)  $2\pi r^3$  (d)  $\frac{4}{3}\pi r^3$

- 5** The slope of the straight line which is parallel to the  $x$ -axis is .....

(a) -1 (b) zero (c) 1 (d) undefined.

6 In the opposite figure :

$$m(\angle C) = 120^\circ$$

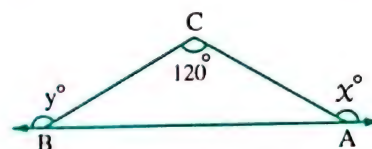
, then  $x^\circ + y^\circ = \dots\dots\dots$

(a)  $90^\circ$

(b)  $180^\circ$

(c)  $300^\circ$

(d)  $360^\circ$



2 [a] Without using calculator , find the value of  $x$  if :  $4x = \cos^2 30^\circ \tan^2 30^\circ \tan^2 45^\circ$

[b]  $\overline{AB}$  is a diameter of the circle  $M$  , if  $B(8, 11)$  ,  $M(5, 7)$

Find : 1 The coordinates of  $A$

2 The length of the radius of the circle.

3 [a] Prove that the points  $A(-2, 5)$  ,  $B(3, 3)$  ,  $C(-4, 2)$  are not collinear and if  $D(-9, 4)$  , prove that the figure  $ABCD$  is a parallelogram.

[b] Explaining the steps and without using calculator , find :

$$\frac{\cos^2 60^\circ + \cos^2 30^\circ - \tan^2 45^\circ}{\sin 60^\circ \tan 60^\circ - \sin 30^\circ}$$

4 [a] Find the equation of the straight line which passes through the point  $(3, 4)$  and is perpendicular to the straight line  $5x - 2y + 7 = 0$

[b]  $ABCD$  is an isosceles trapezoid ,  $\overline{AD} \parallel \overline{BC}$  ,  $AD = 4$  cm. ,  $AB = 5$  cm.  
where  $BC = 12$  cm.

Prove that :  $\frac{5 \tan B \cos C}{\sin^2 C + \cos^2 C} = 3$

5 [a] If the straight line  $L_1$  passes through the two points  $(3, 1)$  ,  $(2, k)$  and the straight line  $L_2$  makes with the positive direction of the  $x$ -axis an angle whose measure is  $45^\circ$  , then find  $k$  if the two straight lines  $L_1$  ,  $L_2$  are :

1 Parallel.

2 Perpendicular.

[b] Find the slope and the intercepted part of  $y$ -axis by the straight line :  $2x = 3y + 6$

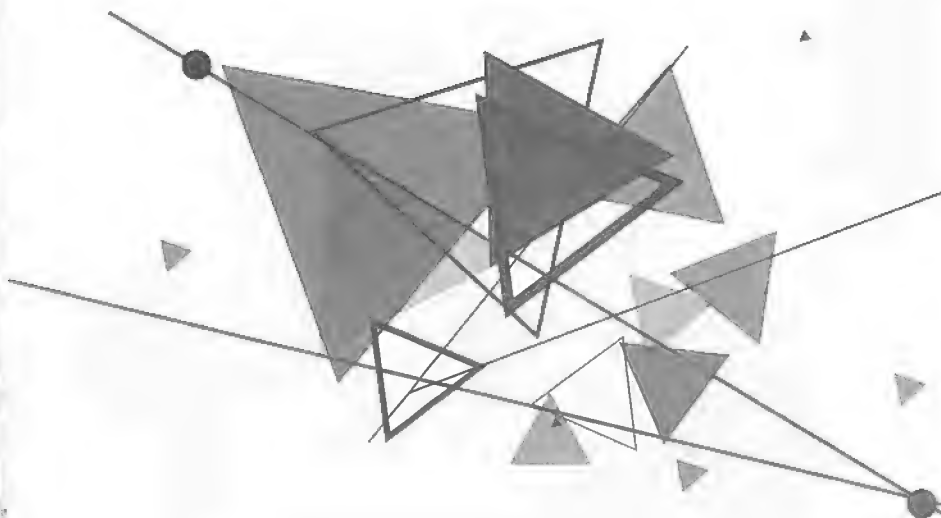


**EL-MOASSER**

# In **Mathematics**

Guide Answers

For **3<sup>rd</sup>** Prep.  
First Term



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**Guide  
Answers**

# **of Algebra and Statistics Exercises**

---



## Answers of unit one

## Answers of Exercise 1

1

$$[1] a = -5, b = 9 \quad [2] a = \sqrt{25} = 5, b = \sqrt[3]{27} = 3$$

$$[3] a - 2 = 2 \quad \therefore a = 4, b + 1 = -3 \quad \therefore b = -4$$

$$[4] 2 - a = 6 \quad \therefore a = -4, b - 3 = -1 \quad \therefore b = 2$$

$$[5] a - 7 = -2 \quad \therefore a = 5, b^3 - 1 = 26$$

$$\therefore b^3 = 27 \quad \therefore b = \sqrt[3]{27} = 3$$

$$[6] \therefore a = 2 - a \quad \therefore 2a = 2 \quad \therefore a = 1$$

$$, b = 2b - 3 \quad \therefore b = 3$$

$$[7] a^5 = 32 \quad \therefore a^5 = 2^5 \quad \therefore a = 2$$

$$, b^2 - 1 = \sqrt[3]{27} \quad \therefore b^2 - 1 = 3$$

$$\therefore b^2 = 4 \quad \therefore b = \pm 2$$

$$[8] b = 7, a = b^2 = 49$$

$$[9] a = 7, 2a = 2b + 1 \quad \therefore 2b + 1 = 14$$

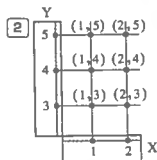
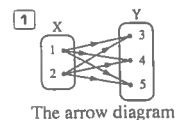
$$\therefore 2b = 13 \quad \therefore b = 6.5$$

$$[10] 5a - 1 = 3 \quad \therefore 5a = 4 \quad \therefore a = \frac{4}{5}$$

$$, b = 4a \quad \therefore b = 4 \times \frac{4}{5} \quad \therefore b = \frac{16}{5}$$

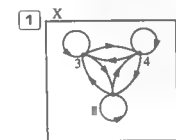
2

$$X \times Y = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$$

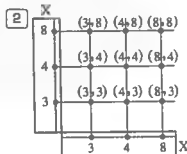


3

$$X^2 = \{(3, 3), (3, 4), (3, 8), (4, 3), (4, 4), (4, 8), (8, 3), (8, 4), (8, 8)\}$$



The arrow diagram



The Cartesian diagram

4

$$[1] \{(1, 4), (2, 4), (3, 4)\}$$

$$[2] \{(a, 5), (a, 6)\} \quad [3] \emptyset$$

$$[4] \{(2, 4), (2, 5), (3, 4), (3, 5)\}$$

$$[5] \{5\} \times \{6, 7\}$$

$$[6] X = \{2, 3, 5\}, Y = \{6, 9\}$$

$$[7] \{1, 2\} \quad [8] 5$$

5

$$[1] c \quad [2] b \quad [3] d \quad [4] d \quad [5] a$$

$$[6] d \quad [7] a \quad [8] a \quad [9] b \quad [10] d$$

$$[11] a \quad [12] c \quad [13] c \quad [14] d \quad [15] a$$

6

$$[1] X \times Y = \{(2, 4), (2, 0), (-1, 4), (-1, 0)\}$$

$$[2] Y \times Z = \{(4, 4), (4, 5), (4, -2), (0, 4), (0, 5), (0, -2)\}$$

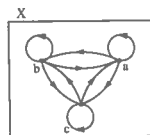
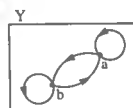
$$[3] X^2 = \{(2, 2), (2, -1), (-1, 2), (-1, -1)\}$$

$$[4] n(X \times Z) = 2 \times 3 = 6$$

$$[5] n(Y^2) = 2 \times 2 = 4$$

$$[6] n(Z^2) = 3 \times 3 = 9$$

7



$$\therefore Y \times Y \subset X \times X \text{ because } Y \subset X$$

8

$$[1] X = \{1\}, Y = \{1, 3, 5\}$$

$$[2] Y \times X = \{(1, 1), (3, 1), (5, 1)\}$$

$$[3] Y^2 = \{(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)\}$$

9

$$X = \{3\}$$

$$\therefore X^2 = \{3\} \times \{3\} = \{(3, 3)\}$$

10

1  $(X \cap Y) \times Y =$

$$\{3, 4\} \times \{3, 4, 5\} \\ = \{(3, 3), (3, 4), (3, 5), (4, 3), (4, 4), (4, 5)\}$$

2  $(X - Y) \times Y = \{1, 2\} \times \{3, 4, 5\}$

$$= \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$$

3  $(Y - X) \times X = \{5\} \times \{1, 2, 3, 4\}$

$$= \{(5, 1), (5, 2), (5, 3), (5, 4)\}$$

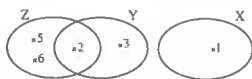
11

1  $X \times (Y \cap Z) = \{3, 4\} \times \{5\} = \{(3, 5), (4, 5)\}$

2  $(X - Y) \times Z = \{3\} \times \{6, 5\} = \{(3, 6), (3, 5)\}$

3  $(X - Y) \times (Y - Z) = \{3\} \times \{4\} = \{(3, 4)\}$

12



First :

1  $X \times Y = \{(1, 2), (1, 3)\}$

2  $Y \times Z = \{(2, 2), (2, 5), (2, 6), (3, 2), (3, 5), (3, 6)\}$

3  $X \times Z = \{(1, 2), (1, 5), (1, 6)\}$

4  $Y^2 = \{(2, 2), (2, 3), (3, 2), (3, 3)\}$

Second :

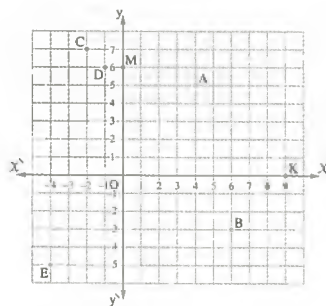
$$(X \times Y) \cup (Y \times Z) = \{(1, 2), (1, 3), (2, 2), (2, 5), (2, 6), (3, 2), (3, 5), (3, 6)\}$$

Third :  $X \times (Y \cap Z) = \{1\} \times \{2\} = \{(1, 2)\}$

Fourth :  $(X \times Y) \cap (X \times Z) = \{(1, 2)\}$

Fifth :  $(Z - Y) \times (X \cup Y) = \{5, 6\} \times \{1, 2, 3\} \\ = \{(5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\}$

13



A Lies on the first quadrant

B Lies on the fourth quadrant

C Lies on the second quadrant

D Lies on the second quadrant

E Lies on the third quadrant

M Lies on y-axis

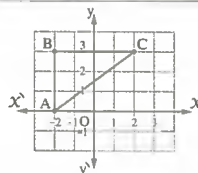
K Lies on x-axis

14

1 b   2 a   3 c   4 b   5 d   6 d

7 c   8 a   9 c   10 a   11 c   12 b   13 c

15

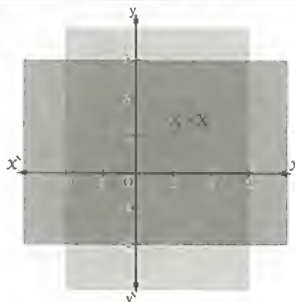


$$\therefore AB = 3 \text{ length unit}, BC = 4 \text{ length unit}$$

$$\therefore \text{The area of } \triangle ABC = \frac{1}{2} \times AB \times BC$$

$$= \frac{1}{2} \times 3 \times 4 = 6 \text{ square unit}$$

16



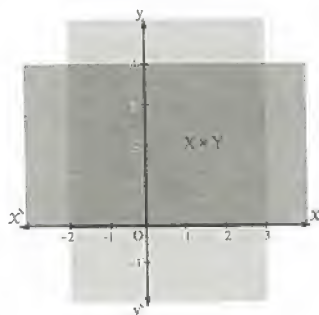
$$A \in X \times X$$

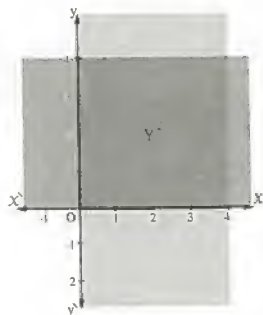
$$C \notin X \times X$$

$$B \in X \times X$$

$$D \in X \times X$$

17

 1  $X \times Y$ 

 2  $Y \times X$ 

 3  $Y^2$ 


18

$$\because X = \{a, 2\}, Y = \{1, 2, 3\}$$

$$\because X \subset Y \quad \therefore a = 1 \text{ or } a = 3$$

19

$$X = \{4, 1\}, Y = \{4, 1, 7\}$$

$$X \times Y = \{(4, 4), (4, 1), (4, 7), (1, 4), (1, 1), (1, 7)\}$$

20 (a)

## Answers of Exercise 2

1

1 b

2 b

3 c

4 a

5 c

6 a

2

 $R_1$  is a function and its range =  $\{1, 9\}$ 
 $R_2$  is a function and its range =  $\{1, 4, 9\}$ 
 $R_3$  is not a function.

 $R_4$  is not a function.

3

The arrow diagram in fig.(2) represents a function.

4

 $t_1$  is a function

$$t_1 = \{(a, b), (b, a), (c, d), (d, c), (e, e)\}$$

$$\text{and its range} = \{a, b, c, d, e\} = X$$

 $t_2$  is not a function because  $a \in X$  has two images and  $b \in X$  has no image.

$$t_3 \text{ is a function, } t_3 = \{(a, 3), (b, 3), (c, 3), (d, 3), (e, 3)\}, \text{ its range} = \{3\}$$

5

 1  $R_1$  is not a function because  $c \in X$  has no image.

 2  $R_2$  is not a function because  $b \in X$  has two images.

 3  $R_3$  is a function because each element of  $X$  appears only once as a first projection in an ordered pair of the relation.

$$\text{and the range} = \{2, 8, 10\}$$

6

1  $R = \{(-3, 1), (-3, 8), (1, 2), (4, 4)\}$

2  $R$  is not a function because  $-3 \in X$  has two images.

3  $\because (X, 2) \in R \quad \therefore X = 1$

7

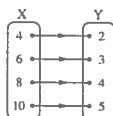
$R = \{(1, 3), (2, 6), (3, 9)\}$

$R$  is a function from  $X$  to  $Y$  because each element of  $X$  has one image in  $Y$

and the range =  $\{3, 6, 9\}$

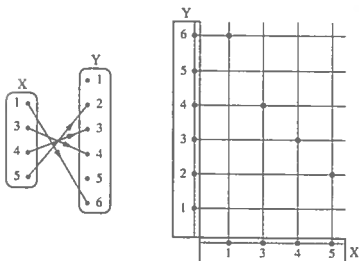
8

$R = \{(4, 2), (6, 3), (8, 4), (10, 5)\}$



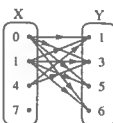
9

$R = \{(1, 6), (3, 4), (4, 3), (5, 2)\}$



10

$R = \{(0, 1), (0, 3), (0, 5), (0, 6), (1, 1), (1, 3), (1, 5), (1, 6), (4, 1), (4, 3)\}$



$R$  is not a function because  $0 \in X, 1 \in X, 4 \in X$  each of them has more than one image in  $Y$   
also  $7 \in X$  has no image in  $Y$

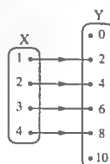
11

$R = \{(2, 4), (2, 5), (2, 6), (2, 7), (2, 9), (4, 4), (4, 5), (4, 6), (4, 7), (4, 9), (5, 5), (5, 6), (5, 7), (5, 9), (7, 7), (7, 9)\}$

Represent by yourself.

12

1  $R = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$



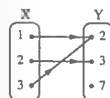
2  $R$  is a function from  $X$  to  $Y$  because each element of  $X$  has one image in  $Y$   
its range =  $\{2, 4, 6, 8\}$

13

1  $R = \{(1, 2), (2, 3), (3, 2)\}$

Yes,  $R$  is a function.

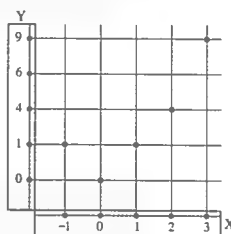
2  $\because (2, 3) \in R \quad \therefore 2a + 3$   
 $\therefore 2a = 2 \quad \therefore a = 1$



14

1  $R = \{(-1, 1), (0, 0), (1, 1), (2, 4), (3, 9)\}$

2  $R$  is a function from  $X$  to  $Y$  because each element of  $X$  has one image in  $Y$



15

$R = \{(1, 1), (2, 8)\}$  Represent by yourself.

16

$R = \{(-2, \frac{1}{4}), (-1, \frac{1}{2}), (0, 1), (1, 2), (2, 4)\}$   
Represent by yourself.

$R$  is a function from  $X$  to  $Y$  because each element of  $X$  has one image in  $Y$

The range =  $\{\frac{1}{4}, \frac{1}{2}, 1, 2, 4\}$

17

$$R = \{(2, 10), (2, 16), (2, 24), (2, 30), (5, 10), (5, 30), (8, 16), (8, 24)\}$$

$R$  is not a function because  $2 \in X$

has more than one image in  $Y$

also  $5 \in X, 8 \in X$  each of them

has two images in  $Y$

Represent by yourself.

18

$$R = \{(2, 6), (2, 8), (2, 10), (3, 6), (3, 15), (4, 8)\}$$

19

$$R = \{(6, -6), (4, -4), (2, -2), (0, 0), (-2, 2), (-4, 4), (-6, 6)\}$$

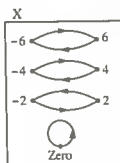
$\therefore R$  is a function

on  $X$  because each

element of  $X$  has

a unique image in  $X$

its range =  $X$



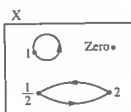
20

$$R = \{(1, 1), (2, \frac{1}{2}), (\frac{1}{2}, 2)\}$$

$\therefore R$  is not a function

because  $0 \in X$

has no image in  $X$



21

$$R = \{(1, 1), (1, 2), (1, 3), (1, 6), (1, 11), (3, 1), (3, 2), (3, 3), (3, 6), (3, 11), (11, 1), (11, 2), (11, 3), (11, 6), (11, 11)\}$$

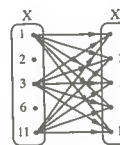
$R$  is not a function because

each of  $1 \in X, 3 \in X, 11 \in X$

has more than one image in  $X$

also each of  $2 \in X, 6 \in X$

has no image in  $X$



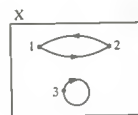
22

$$\therefore X = \{1, 2, 3\}$$

$$R = \{(1, 2), (2, 1), (3, 3)\}$$

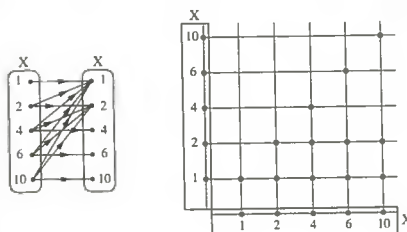
$\therefore R$  is a function.

its range =  $\{1, 2, 3\}$



23

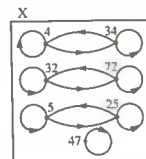
$$R = \{(1, 1), (2, 1), (2, 2), (4, 1), (4, 2), (4, 4), (6, 1), (6, 2), (6, 6), (10, 1), (10, 2), (10, 10)\}$$



$R$  is not a function because each of  $2 \in X, 4 \in X, 6 \in X$  and  $10 \in X$  has more than one image in  $X$

24

$$R = \{(34, 34), (34, 4), (72, 72), (72, 32), (25, 25), (25, 5), (4, 4), (4, 34), (5, 5), (5, 25), (32, 32), (32, 72), (47, 47)\}$$



25

1  $\therefore R$  is a function from  $X$  to  $Y$

$\therefore$  each element in  $X$  has only one image in  $Y$

$\therefore$  the image of  $-2 = (-2)^2 - 1 = 3 \in Y$

the image of  $2 = (2)^2 - 1 = 3 \in Y$

the image of  $5 = (5)^2 - 1 = 24 \in Y$

$\therefore l = 24$

2 Represent by yourself.

26

$$1 \ R_1 = \{(0, 0), (4, 2), (16, 4)\}$$

 $\therefore R_1$  is a function from  $X$  to  $Y$ 

$$2 \ R_2 = \{(0, 0)\}$$

 $\therefore R_2$  is not a function because each of  $4 \in X$ ,  $16 \in X$  has no image in  $Y$ 

$$3 \ R_3 = \{(0, 0), (4, 2)\}$$

 $\therefore R_3$  is not a function because  $16 \in X$  has no image in  $Y$ 

27

$$\therefore aRb \quad , \quad a \times b = 12$$

$$1 \quad \therefore X R 4 \quad \therefore X \times 4 = 12 \quad \therefore X = 3$$

$$2 \quad \therefore y R 3 y \quad \therefore y \times 3 y = 12$$

$$\therefore 3 y^2 = 12 \quad \therefore y^2 = 4$$

$$\therefore y = \pm 2$$

28

$$\therefore (a, 2) \in R \quad , \quad y^2 = 2x$$

$$\therefore 2^2 = 2a \quad \therefore a = 2$$

$$\therefore \left(\frac{2}{9}, b\right) \in R \quad \therefore b^2 = 2 \times \frac{2}{9} = \frac{4}{9}$$

$$\therefore b = -\frac{2}{3} \text{ (refused) or } b = \frac{2}{3}$$

$$\therefore (c, 3) \in R \quad \therefore 3^2 = 2c \quad \therefore c = 4\frac{1}{2}$$

$$\therefore \left(\frac{9}{32}, d\right) \in R \quad \therefore d^2 = 2 \times \frac{9}{32} = \frac{9}{16}$$

$$\therefore d = -\frac{3}{4} \text{ (refused) or } d = \frac{3}{4}$$

29

$$\therefore R_1 = \{(1, -1), (0, 0), (-1, 1)\}$$

$$\therefore R_2 = \{(1, 1), (-1, -1)\}, \quad R = R_1 \cap R_2$$

$$\therefore R = \emptyset, \quad R \text{ is not a function}$$

30

$$1 \ R = \{(1, 13), (1, 31), (2, 23), (3, 13), (3, 31), (3, 23)\} \text{ (Represent by yourself)}$$

$$2 \ 2 R 65 \text{ false (say why by yourself)}$$

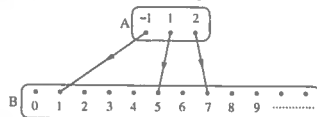
$$1 R 31 \text{ true (say why by yourself)}$$

$$3 R 13 \text{ true (say why by yourself)}$$

$$3 \ M = \{(2, 23), (3, 23)\}$$

31

$$R = \{(-1, 1), (1, 5), (2, 7)\}$$



32

$$1 \ L \text{ does not represent a relation because : } L \not\subseteq X \times Y$$

$$2 \ M \text{ represents a relation because : } M \subseteq X \times Y$$

33

$$\therefore f \text{ is a function on } X$$

 $\therefore$  each element in  $X$  has to appear only once as a first projection in  $R$ 

$$\therefore a = 3, b = 5 \text{ or } a = 5, b = 3$$

$$\therefore a + b = 3 + 5 = 8$$

34

$$R = \{(-1, 1), (0, 0), (1, 1)\}$$

$$R \text{ is not a function because } -2 \in X, 2 \in X$$

 $\therefore$  did not appear as a first projection in the ordered pairs of  $R$ 

35

$$\therefore a \text{ divides } b$$

$$\therefore X \cup Y = \{2, 3, 5, 11, 14, 9, 35\}$$

$$\therefore f \text{ is function from } X \text{ to } Y$$

$$\therefore 2 \text{ divides } 14 \quad \therefore 2 \in X, 14 \in Y$$

$$\therefore 3 \text{ divides } 9 \quad \therefore 3 \in X, 9 \in Y$$

$$\therefore 5 \text{ divides } 35 \quad \therefore 5 \in X, 35 \in Y$$

$$\therefore n(X) = 3 \quad \therefore X = \{2, 3, 5\}$$

$$\therefore n(X \times Y) = 12 \quad \therefore n(Y) = 4$$

$$\therefore Y = \{14, 9, 35, 11\}$$

$$\therefore R = \{(2, 14), (3, 9), (5, 35)\}$$

$$\therefore \text{its range} = \{14, 9, 35\}$$

36

$$\therefore X \cup Y = \{4, 8, 9, 27\}, \quad n(X) = 4$$

$$\therefore X = \{4, 8, 9, 27\}$$

$$\therefore a \text{ is a multiple of } b, \quad f \text{ is a function from } X \text{ to } Y$$

$$\therefore n(Y) = 2 \quad \therefore Y = \{4, 9\}$$

$$\therefore R = \{(4, 4), (8, 4), (9, 9), (27, 9)\}$$

$$\therefore \text{the range of the function} = \{4, 9\}$$

## Answers of Exercise 3

- 1 c    2 d    3 d    4 c    5 a  
 6 d    7 c    8 d    9 d    10 c  
 11 d    12 c    13 c    14 a    15 c  
 16 b    17 d    18 d

The polynomial functions are numbers ①, ②, ④

	Degree	$f(-2)$	$f(0)$	$f(\frac{1}{2})$
1	First	7	3	2
2	Second	zero	-4	$-3\frac{3}{4}$

4  
 $\therefore f(2) = 2 \times (2)^2 - 5 \times 2 + 2 = \text{zero}$   
 $f(\frac{1}{2}) = 2(\frac{1}{2})^2 - 5 \times \frac{1}{2} + 2 = \text{zero}$   
 $\therefore f(2) = f(\frac{1}{2})$

5  
 $\therefore f(2) = 2 \times 2 - 1 = 3, f(1) = 2 \times 1 - 1 = 1$   
 $\therefore f(2) - 3f(1) = 3 - 3 \times 1 = \text{zero}$

6  
 1  $f(\sqrt{2}) + 3g(\sqrt{2}) = (\sqrt{2})^2 - 3(\sqrt{2}) + 3(\sqrt{2} - 3)$   
 $= 2 - 3\sqrt{2} + 3\sqrt{2} - 9 = -7$

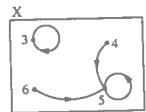
2  $\therefore f(3) = (3)^2 - 3 \times 3 = 9 - 9 = \text{zero}$   
 $g(3) = 3 - 3 = \text{zero}$   
 $\therefore f(3) = g(3) = \text{zero}$

7  
 $\therefore f(1 + \sqrt{6}) = (1 + \sqrt{6})^2 - 2(1 + \sqrt{6}) - 5$   
 $= 1 + 2\sqrt{6} + 6 - 2 - 2\sqrt{6} - 5 = \text{zero}$   
 $f(1 - \sqrt{6}) = (1 - \sqrt{6})^2 - 2(1 - \sqrt{6}) - 5$   
 $= 1 - 2\sqrt{6} + 6 - 2 + 2\sqrt{6} - 5 = \text{zero}$   
 $\therefore f(1 + \sqrt{6}) = f(1 - \sqrt{6}) = \text{zero}$

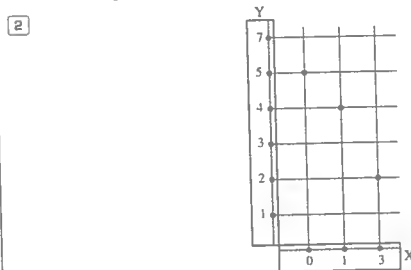
8  
 1  $\therefore a = \text{zero} \quad \therefore f(X) = bX + 5$   
 $\therefore f$  is of first degree.  
 2  $\therefore f(3) = 11 \quad \therefore b \times 3 + 5 = 11$   
 $\therefore 3b = 6 \quad \therefore b = \frac{6}{3} = 2$

9  
 $f(1) = 5 \times 1 - b = 5 - b, h(3) = 3 - 2b$   
 $\therefore f(1) + h(3) = -7 \quad \therefore 5 - b + 3 - 2b = -7$   
 $\therefore 8 - 3b = -7 \quad \therefore 8 + 7 = 3b$   
 $\therefore 15 = 3b \quad \therefore b = \frac{15}{3} = 5$   
 $\therefore f(X) = 5X - 5$   
 $\therefore f(3) = 5 \times 3 - 5 = 15 - 5 = 10$   
 $\therefore h(X) = X - 10 \quad \therefore h(1) = 1 - 10 = -9$   
 $\therefore f(3) + h(1) = 10 - 9 = 1$

10  
 $\therefore f(X) = X(X-3) \quad \therefore (X-3)^2 = X-3$   
 $\therefore X^2 - 6X + 9 - X + 3 = 0$   
 $\therefore X^2 - 7X + 12 = 0 \quad \therefore (X-3)(X-4) = 0$   
 $\therefore X = 3 \text{ or } X = 4$

11  
 1   
 2  $f = \{(3, 3), (4, 5), (5, 5), (6, 5)\}$   
 $\therefore$  its range =  $\{3, 5\}$

12  
 1  $\therefore f(0) = 5 - 0 = 5$   
 $\text{also, } f(1) = 4, f(3) = 2$   
 $\therefore$  The range of  $f = \{5, 4, 2\}$



13

1  $t(0) = 2 \times 0 + 3 = 3$

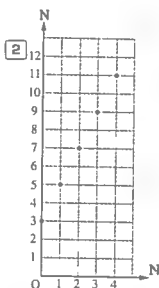
also  $t(1) = 5$

$t(2) = 7, t(3) = 9$

$t(4) = 11$

$t(5) = 13$

- 3 The range of  $t$   
 $= \{3, 5, 7, 9, 11, 13, \dots\}$   
 The set of odd natural  
 numbers except  $\{1\}$



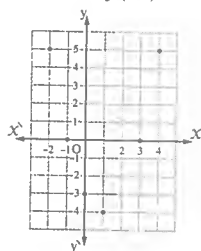
14

1  $f(4) = (4)^2 - 2 \times 4 - 3 = 16 - 8 - 3 = 5$

$f(3) = 0, f(2) = -3, f(1) = -4,$

$f(0) = -3, f(-1) = 0, f(-2) = 5$

2



- 3 From (1) :  $\therefore f(4) = 5, f(-2) = 5$   
 $\therefore X = 4$  or  $-2$

15

$\therefore f(a) = b \quad \therefore b = a^2 + b \quad \therefore a^2 = 0$

$\therefore a = 0 \quad \therefore a b^2 + 5 = 0 \times b^2 + 5 = 5$

16

1 The domain  $= \{1, 2, 3, 4, 5\}$

2 The range  $= \{3, 5, 7, 9, 11\}$

3 The rule of the function  $f$  is :  $f(X) = 2X + 1$

17

$\therefore f(0) = 0 \quad \therefore 2 \times (0)^2 + b \times 0 + c = 0$

$\therefore c = 0 \quad \therefore f(X) = 2X^2 + bX$

$\therefore f(3) = \text{zero} \quad \therefore 0 = 2(3)^2 + 3b$

$\therefore 0 = 18 + 3b \quad \therefore b = -6$

### Answers of Exercise 4

1

- 1 a    2 b    3 c    4 c    5 b

- 6 a    7 c    8 c    9 c    10 a

- 11 a    12 d    13 c    14 c

2

1 X-axis,  $(0, 5)$     2 6    3  $(0, -1)$

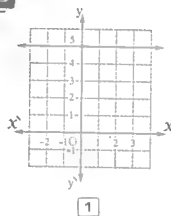
4  $(-2, 0)$     5 2    6 2, 3

7  $-2, 0$     8 negative, positive

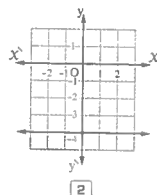
9 downward    10  $X = 0$     11  $-\frac{b}{2a}$

12  $(1, 3)$     13 2    14 5

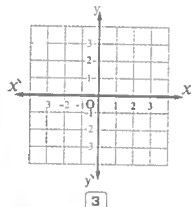
3



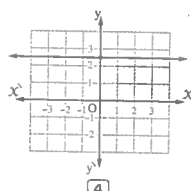
1



2



3

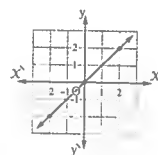


4

4

1  $f(X) = X$

$X$	-2	zero	2
$f(X)$	-2	zero	2

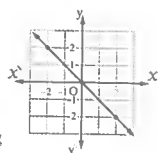


The straight line representing  
 the function intersects

The two coordinate axes at the origin point  $O(0, 0)$

2  $f(X) = -X$

$X$	-2	zero	2
$f(X)$	2	zero	-2



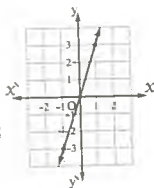
The straight line representing  
 the function intersects

The two coordinate axes at the origin point  $O(0, 0)$

3  $f(x) = 3x$

$x$	-1	zero	1
$f(x)$	-3	zero	3

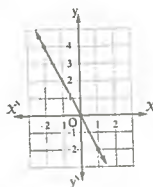
The straight line representing the function intersects the two coordinate axes at the origin point  $O(0, 0)$



4  $f(x) = -2x$

$x$	-2	zero	1
$f(x)$	4	zero	-2

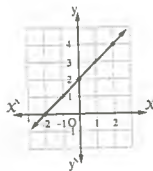
The straight line representing the function intersects the two coordinate axes at the origin point  $O(0, 0)$



5  $f(x) = x + 2$

$x$	-2	zero	2
$f(x)$	zero	2	4

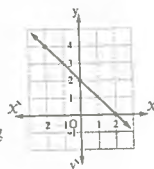
The straight line representing the function intersects the  $x$ -axis at the point  $(-2, 0)$  and the  $y$ -axis at the point  $(0, 2)$



6  $f(x) = 2 - x$

$x$	-2	zero	2
$f(x)$	4	2	zero

The straight line representing the function intersects the  $x$ -axis at the point  $(2, 0)$  and the  $y$ -axis at the point  $(0, 2)$



7  $f(x) = 3x - 1$

$x$	zero	1	2
$f(x)$	-1	2	5

Represent by yourself.

From the graph we find that : the straight line representing the function intersects the  $x$ -axis at the point  $(\frac{1}{3}, 0)$  and the  $y$ -axis at the point  $(0, -1)$

8  $f(x) = -2x + 3$

$x$	-1	zero	1
$f(x)$	5	3	1

Represent by yourself.

From the graph we find that :

The straight line representing the function intersects  $x$ -axis at the point  $(1.5, 0)$  and  $y$ -axis at  $(0, 3)$

9  $f(x) = \frac{1}{2}x$

$x$	-2	zero	2
$f(x)$	-1	zero	1

Represent by yourself.

From the graph we find that :

The straight line representing the function intersects  $x$ -axis and  $y$ -axis at the origin point  $O(0, 0)$

10  $f(x) = 5 - \frac{1}{2}x$

$x$	zero	2	4
$f(x)$	5	4	3

Represent by yourself.

From the graph we find that :

The straight line representing the function intersects  $x$ -axis at the point  $(10, 0)$  and  $y$ -axis at the point  $(0, 5)$

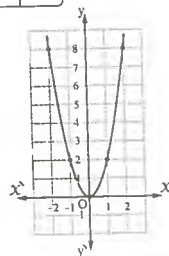
5

1  $f(x) = 2x^2$

$x$	-2	-1	0	1	2
$f(x)$	8	2	0	2	8

From the graph :

- The vertex of the curve is  $(0, 0)$
- The equation of the line of symmetry is  $x = 0$
- The minimum value = zero

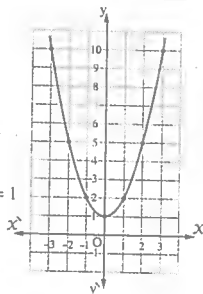


2)  $f(x) = x^2 + 1$

$x$	-3	-2	-1	0	1	2	3
$f(x)$	10	5	2	1	2	5	10

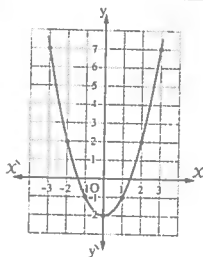
From the graph :

- The vertex of the curve is  $(0, 1)$
- The equation of the line of symmetry is  $x = 0$
- The minimum value is 1



3)  $f(x) = x^2 - 2$

$x$	-3	-2	-1	0	1	2	3
$f(x)$	7	2	-1	-2	-1	2	7



From the graph we find that :

- The vertex of the curve is  $(0, -2)$
- The equation of the line of symmetry is  $x = 0$
- The minimum value is  $-2$

4)  $f(x) = 2 - x^2$

$x$	-3	-2	-1	0	1	2	3
$f(x)$	-7	-2	1	2	1	-2	-7

Represent by yourself.

From the graph we find that :

- The vertex of the curve is  $(0, 2)$
- The equation of the line of symmetry is  $x = 0$
- The maximum value is 2

5)  $f(x) = x^2 - 2x$

$x$	-2	-1	0	1	2	3	4
$f(x)$	8	3	0	-1	0	3	8

Represent by yourself.

From the graph we find that :

- The vertex of the curve is  $(1, -1)$
- The equation of the line of symmetry is  $x = 1$
- The minimum value is  $-1$

6)  $f(x) = x^2 + 2x + 1$

$x$	-4	-3	-2	-1	0	1	2
$f(x)$	9	4	1	0	1	4	9

Represent by yourself.

From the graph we find that :

- The vertex of the curve is  $(-1, 0)$
- The equation of the line of symmetry is  $x = -1$
- The minimum value is 0

7)  $f(x) = (x-2)^2 = x^2 - 4x + 4$

$x$	-1	0	1	2	3	4	5
$f(x)$	9	4	1	0	1	4	9

Represent by yourself.

From the graph we find that :

- The vertex of the curve is  $(2, 0)$
- The equation of the line of symmetry is  $x = 2$
- The minimum value is zero

8)  $f(x) = x^2 - 2x - 3$

$x$	-2	-1	0	1	2	3	4
$f(x)$	5	0	-3	-4	-3	0	5

Represent by yourself.

From the graph we find that :

- The vertex of the curve is  $(1, -4)$
- The equation of the line of symmetry is  $x = 1$
- The minimum value is  $-4$

9)  $f(x) = 3 - 2x - x^2$

$x$	-4	-3	-2	-1	0	1	2
$f(x)$	-5	0	3	4	3	0	-5

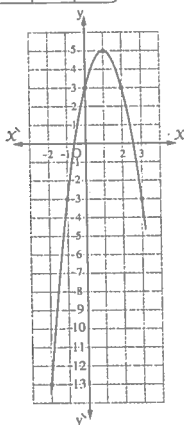
Represent by yourself.

From the graph we find that :

- The vertex of the curve is  $(-1, 4)$
- The equation of the line of symmetry is  $X = -1$
- The maximum value = 4

10  $f(X) = 4X + 3 - 2X^2$

$X$	-2	-1	0	1	2	3
$f(X)$	-13	-3	3	5	3	-3

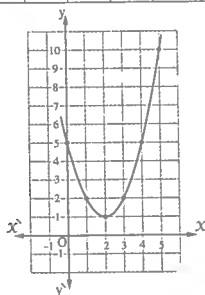


From the graph :

- The vertex of the curve is  $(1, 5)$
- The equation of the axis of symmetry is  $X = 1$   
«notice that : the domain of  $f$  is  $\mathbb{R}$  and the given interval for facilitating representation only»
- The maximum value = 5

11  $f(X) = X^2 - 4X + 5$

$X$	0	1	2	3	4	5
$f(X)$	5	2	1	2	5	10



From the graph :

- The vertex of the curve is  $(2, 1)$
- The equation of the axis of symmetry is  $X = 2$
- The minimum value = 1

12  $f(X) = 1 - 3X + X^2$

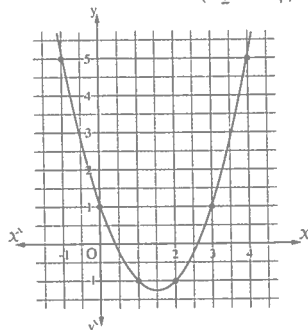
$X$	-1	0	1	2	3	4
$f(X)$	5	1	-1	-1	1	5

∴ The  $X$ -coordinate of the vertex of the curve

$$= -\frac{b}{2a} = -\frac{-3}{2} = \frac{3}{2} = 1\frac{1}{2}$$

$$f\left(\frac{3}{2}\right) = 1 - 3\left(\frac{3}{2}\right) + \frac{9}{4} = -\frac{5}{4} = -1\frac{1}{4}$$

∴ The vertex of the curve is  $\left(1\frac{1}{2}, -1\frac{1}{4}\right)$



- The equation of the axis of symmetry is  $X = 1\frac{1}{2}$
- The minimum value =  $-1\frac{1}{4}$

6

∴ The point  $(0, -3)$  satisfies the relation  $f(X) = aX + b$

$$\therefore -3 = a \times 0 + b \quad \therefore b = -3$$

$$\therefore f(X) = aX - 3$$

∴ the point  $(3, 0)$  satisfies the relation  $f(X) = aX - 3$

$$\therefore 0 = 3 \times a - 3 \quad \therefore 3a = 3 \quad \therefore a = 1$$

$$\therefore f(X) = X - 3 \quad \therefore f(1) = 1 - 3 = -2$$

7

∴ The curve of the function  $f$  intersects the  $X$ -axis at the point  $(-2, b)$

$$\therefore b = 0$$

$$\begin{aligned} \therefore (-2, 0) \text{ satisfies the relation } f(x) &= m - x^2 \\ \therefore m - (-2)^2 &= 0 \quad \therefore m - 4 = 0 \\ \therefore m &= 4 \quad \therefore m^b + 2m = 4^0 + 2 \times 4 = 9 \end{aligned}$$

8

$$\begin{aligned} \text{1} \quad \therefore r(2) &= 9 - 2 = 7, \text{ also } r(3) = 6, r(6) = 3 \\ \therefore \text{The set of images of elements of the set } X &\text{ with the function } r = \{7, 6, 3\} \end{aligned}$$

$$\text{2} \quad r \text{ is not a linear function because each of the domain and the codomain is not the set of real numbers.}$$

9

$$\begin{aligned} \therefore 3f(2) + 3\ell(x) &= 6 \quad \therefore f(2) + \ell(x) = 2 \\ \therefore a + 2^2 + c &= 2 \quad \therefore a + 4 + c = 2 \\ \therefore a + c &= -2 \\ \therefore 2f(0) + 2\ell(7) &= 2[f(0) + \ell(7)] \\ &= 2[a + (0)^2 + c] \\ &= 2[a + c] = 2 \times (-2) = -4 \end{aligned}$$

10

$$\begin{aligned} \text{1} \quad \text{Let } A(x, 0) \\ \therefore A(x, 0) \text{ belongs to the straight line of the function } f \\ \therefore 4 - 2x &= 0 \quad \therefore -2x = -4 \\ \therefore x &= \frac{-4}{-2} = 2 \quad \therefore A(2, 0) \\ \text{let } B(0, y) \\ \therefore B(0, y) \text{ belongs to the straight line of the function } f \\ \therefore 4 - 2 \times 0 &= y \quad \therefore y = 4 \\ \therefore B(0, 4) \end{aligned}$$

$$\text{2} \quad \text{Area of } \triangle AOB = \frac{1}{2} \times 2 \times 4 = 4 \text{ square unit}$$

11

$$\begin{aligned} \text{1} \quad \therefore f \text{ is a constant function, passes through the point } A(2, 3) \text{ and is represented graphically by a straight line parallel to } x\text{-axis} \\ \therefore \text{The rule of the function } f \text{ is } f(x) &= 3 \\ \therefore g(x) \text{ is a linear function and passes through } A(2, 3) \text{ \& } O(0, 0) \\ \therefore \text{The rule of the function } g \text{ is } g(x) &= b x + c \end{aligned}$$

$$\begin{aligned} \therefore (0, 0) &\in \overline{OA} \\ \therefore 0 &= b \times 0 + c \quad \therefore c = 0 \\ \therefore g(x) &= b x \\ \therefore (2, 3) &\in \overline{OA} \quad \therefore 3 = 2 \times b \\ \therefore b &= \frac{3}{2} \quad \therefore g(x) = \frac{3}{2} x \end{aligned}$$

$$\text{2} \quad f(-10) + g(6) = 3 + \frac{3}{2} \times 6 = 12$$

12

$$\begin{aligned} \therefore \overline{AB} \text{ represents the function } f : f(x) &= 4 \\ \therefore \text{the point } B &\in y\text{-axis} \\ \therefore B &= (0, 4) \quad \therefore OB = 4 \text{ length unit} \\ \therefore \text{the area of } \triangle ABO &= 4 \text{ square unit} \\ \therefore \frac{1}{2} AB \times OB &= 4 \quad \therefore \frac{1}{2} AB \times 4 = 4 \\ \therefore \frac{1}{2} AB &= 1 \quad \therefore AB = 2 \\ \therefore A &= (2, 4) \\ \therefore \text{the point } O(0, 0) &\text{ belongs to the straight line representing the function } g : g(x) = n x + k \\ \therefore 0 &= n \times 0 + k \quad \therefore k = 0 \\ \therefore g(x) &= n x \\ \therefore \text{the point } A(2, 4) &\text{ belongs to the straight line representing the function } g : g(x) = n x \\ \therefore 4 &= 2 n \quad \therefore n = 2 \end{aligned}$$

13

$$\begin{aligned} \text{1} \quad \text{Let } A(x, 0) \text{ and } C(-x, 0) \\ \therefore \text{The curve of the function intersects the } x\text{-axis at two points } A \text{ and } C \\ \therefore 0 &= 9 - x^2 \quad \therefore x^2 = 9 \\ \therefore x &= 3 \text{ or } x = -3 \\ \therefore A &= (3, 0) \text{ \& } C = (-3, 0) \\ \text{2} \quad \text{Let } B &= (0, y) \\ \therefore \text{the point } B(0, y) &\text{ belongs to the curve of the function } f \\ \therefore y &= 9 - (0)^2 \quad \therefore y = 9 \\ \therefore b &= (0, 9) \quad \therefore OB = 9 \text{ length units.} \\ \therefore A &= (3, 0) \text{ and } C = (-3, 0) \\ \therefore AC &= 6 \text{ length units.} \\ \therefore \text{The area of } \triangle ABC &= \frac{1}{2} \times 6 \times 9 \\ &= 27 \text{ square units.} \end{aligned}$$

14

- $\therefore AO = 4$  units  $\therefore A(0, 4)$   
 $\therefore A(0, 4)$  belongs to the curve of the function  $f$   
 $\therefore A$  satisfies the equation of the curve  
 $\therefore 4 = m - (0)^2 \therefore m = 4$  (the first req.)  
 $\therefore$  The curve of the function intersects  $X$ -axis at the two points  $B$  and  $C$   
 $\therefore 0 = 4 - x^2 \therefore x^2 = 4$   
 $\therefore x = 2$  or  $-2$   
 $\therefore B = (2, 0), C = (-2, 0)$  (the second req.)  
 $\therefore BC = 4$  units  
 The area of  $\triangle ABC = \frac{1}{2} \times 4 \times 4 = 8$  square units  
 (the third req.)

15

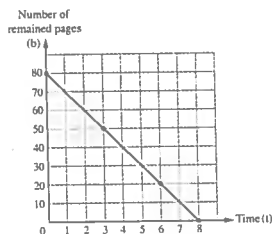
- $\therefore A(0, -7) \therefore OA = 7$  length units  
 $\therefore$  the area of  $\triangle ABC = \frac{1}{2} \times BC \times AO$   
 $\therefore 21 = \frac{1}{2} \times BC \times 7 \therefore BC = \frac{21 \times 2}{7} = 6$  length units  
 $\therefore OB = OC = \frac{6}{2} = 3$  length units  
 $\therefore B = (3, 0)$   
 $\therefore B(3, 0) \in$  the curve of the function  $f$   
 $\therefore t \times 3^2 - 7 = 0 \therefore 9t - 7 = 0$   
 $\therefore 9t = 7 \therefore t = \frac{7}{9}$

16

- 1 The domain of the function  $f = \mathbb{R}$   
 2  $\therefore$  The range of the function  $f$  is the set of images of the elements of the set  $\mathbb{R}$  by the function  $f$   
 $\therefore$  The range of the function  $f = ]-\infty, 4\frac{1}{2}]$   
 3 The equation of the line of symmetry of the curve of the function  $f$  is:  $X = 2$   
 4 The maximum value of  $f = 4\frac{1}{2}$   
 5  $f(1) = 4$   
 6  $\therefore (2, 4\frac{1}{2}) \in$  the curve of the function  $f$   
 $\therefore a(2-2)^2 + k = 4\frac{1}{2} \therefore k = 4\frac{1}{2}$   
 $\therefore (5, 0) \in$  the curve of the function  $f$   
 $\therefore a(5-2)^2 + 4\frac{1}{2} = 0$   
 $\therefore 9a = -4\frac{1}{2} \therefore a = -\frac{4\frac{1}{2}}{9} = -\frac{1}{2}$   
 $\therefore a + k = -\frac{1}{2} + 4\frac{1}{2} = 4$

17

1


 Let the linear relation between  $t$  and  $b$  be:

$$b = at + c$$

 From the graph: The straight line passing through the two points  $(0, 80)$  and  $(8, 0)$ 

 Substituting in the relation by the point  $(0, 80)$ 

$$\therefore 80 = a \times 0 + c \therefore c = 80$$

$$\therefore b = at + 80$$

 Substituting in the relation by the point  $(8, 0)$ 

$$\therefore 0 = a \times 8 + 80 \therefore 8a = -80$$

$$\therefore a = -10 \therefore b = -10t + 80$$

2 8 hours.

3 80 pages.

18

 $\therefore$  The curve of the function intersects

 The  $X$ -axis at two points  $A(1, 0)$  and  $B(4, 0)$ 

$$\therefore f(1) = 0, f(4) = 0 \therefore f(1) = f(4)$$

 $\therefore$  the function is symmetric

$$\therefore \text{The equation of the axis of symmetry is } X = \frac{4+1}{2} = \frac{5}{2}$$

$$\therefore \frac{-2+7}{2} = \frac{5}{2} \therefore f(-2) = f(7)$$

$$\therefore f(-2) + f(-2) = 8 \therefore f(-2) = 4$$

19

 Let  $C = (0, t)$ 
 $\therefore$  the curve of the function  $f$  passes through the point  $C$ 

$$\therefore t = 0^2 - (k-2) \times 0 - k + 4$$

$$\therefore t = 4 - k \therefore B(4-k, 4-k)$$

$$\therefore 4 - k = (4 - k)^2 - (k - 2) \times (4 - k) - k + 4$$

$$\therefore (4 - k)^2 - (k - 2)(4 - k) = 0$$

$$\therefore (4 - k)(4 - k - k + 2) = 0$$

$$\therefore (4 - k)(6 - 2k) = 0$$

$$\therefore 4 - k = 0 \quad \therefore k = 4 \text{ (refused)}$$

$$\text{or } 6 - 2k = 0 \quad \therefore 6 = 2k \quad \therefore k = 3$$

**20**

$$\therefore OB = 5 \text{ OA} \quad \therefore \frac{OB}{OA} = \frac{5}{1}$$

Let  $B(5m, 0)$ ,  $A(-m, 0)$ 

$$\therefore f(5m) = f(-m)$$

$$\therefore -25m^2 + 20m + k - 1 = -m^2 - 4m + k - 1$$

$$\therefore 24m^2 - 24m = 0 \quad \therefore 24m(m - 1) = 0$$

$$\therefore m = 0 \text{ (refused) or } m = 1$$

$$\therefore B(5, 0)$$

By substituting in the rule of the function  $f$ 

$$\therefore 0 = -25 + 20 + k - 1 \quad \therefore k = 6$$

**Answers of exams on unit one****? Model 1****1****1** a**2** d**3** b**4** b**5** c**6** a**2**

$$[a] R = \{(1, 3), (2, 6), (3, 9)\}$$

,  $R$  is a function, its range =  $\{3, 6, 9\}$ 

$$[b] \text{1} X \times (Y \cap Z) = \{(2, 5), (3, 5)\}$$

$$\text{2} (X - Y) \times Z = \{(2, 7), (2, 5)\}$$

$$\text{3} (X - Y) \times (Y - Z) = \{(2, 3)\}$$

**3**

$$[a] \text{1} \text{ The domain of the function } f = \{3, 5, 7\}$$

$$\text{2} \text{ The range of the function } f = \{9, 15, 21\}$$

$$\text{3} \text{ The rule of the function } f \text{ is: } f(x) = 3x$$

**[b]** Represent by yourself.

The straight line representing the function intersects X-axis at the point  $(1.5, 0)$  and y-axis at the point  $(0, 3)$

**4**

$$[a] \text{1} R = \{(-1, 1), (1, 1), (0, 0)\}$$

, represent by yourself

$$\text{2} R \text{ is not a function because } 2 \in X \text{ has no image in } X.$$

**[b]** Represent by yourself.

$$\text{1} \text{ The vertex of the curve is } (3, 0)$$

$$\text{2} \text{ The equation of the line of symmetry is } x = 3$$

$$\text{3} \text{ The minimum value} = 0$$

**5**

$$[a] \text{1} \text{ The function is of the first degree.}$$

$$\text{2} f(0) + g(0) = -4$$

$$[b] \text{1} \text{ The equation of the axis of symmetry is: } x = 0$$

, the maximum value of the function  $f = 4$

$$\text{2} B(-2, 0)$$

$$\text{3} k = 1$$

**Model 2**

**1**

**1** b

**2** b

**3** c

**4** a

**5** c

**6** c

**2** [a] **1**  $R = \{(1, 7), (4, 4), (7, 1), (7, -1)\}$   
 , represent by yourself.

**2**  $R$  is not a function because  $7 \in X$  has two images in  $Y$

[b] **1**  $Y \times X = \{(1, 1), (5, 1), (3, 1), (1, 4), (5, 4), (3, 4)\}$

**2**  $X = \{1, 4\}$

,  $X^2 = \{(1, 1), (1, 4), (4, 1), (4, 4)\}$

**3** [a] Represent by yourself.

**1** The vertex of the curve is  $(-1, 0)$

**2** The equation of the line of symmetry is  $X = -1$

**3** The minimum value = 0

[b]  $m = 2$  ,  $k = 4$

**4** [a]  $\sqrt{3X + 2Y} = 5$

[b] **1**  $(X - Y) \times Z = \{(1, 4), (1, 5)\}$

**2**  $n(X \times Y) + n(Z^2) = 8$

**5** [a] **1** 36

**2**  $\{5, 7\}$

[b]  $f(2) - f\left(\frac{1}{2}\right) = \text{zero}$

## Answers of unit two

## Answers of Exercise 5

1

1 the third proportional 2  $\frac{c}{d}$  3 484 3 5 16 6 3 7  $\pm 4$  8 459  $\frac{3}{7}$  10  $\frac{4}{5}$  11  $\frac{5}{7}$  12  $\frac{4}{9}$ 

2

1 c 2 b 3 a 4 b 5 a

6 a 7 b 8 b 9 d 10 c

11 a 12 c 13 b 14 a

3

1 Let the first proportional be  $X$   $\therefore \frac{X}{\sqrt[3]{8}} = \frac{7}{14\sqrt[3]{2}}$   
 $\therefore X = \frac{7 \times \sqrt[3]{8}}{14\sqrt[3]{2}} = \frac{7 \times 2\sqrt[3]{2}}{14\sqrt[3]{2}} = 1$ 2 Let the third proportional be  $X$ 

$$\therefore \frac{a}{(a+b)} = \frac{X}{(a^2-b^2)}$$

$$\therefore X = \frac{a(a^2-b^2)}{(a+b)} = \frac{a(a+b)(a-b)}{(a+b)} = a(a-b)$$

3 Let the fourth proportional be  $X$ 

$$\therefore \frac{(a+b)}{(a-b)} = \frac{(a-b)}{X} \quad \therefore X = \frac{(a-b)^2}{(a+b)}$$

4

$$1 \therefore \frac{2X-3}{X-5} = \frac{1}{4} \quad \therefore X-5 = 4(2X-3)$$

$$\therefore X-5 = 8X-12 \quad \therefore 7X = 7 \quad \therefore X = 1$$

$$2 \therefore \frac{X-5}{5X+3} = \frac{2}{3} \quad \therefore 3(X-5) = 2(5X+3)$$

$$\therefore 3X-15 = 10X+6 \quad \therefore -7X = 21$$

$$\therefore X = -3$$

$$3 \therefore \frac{X^2-8}{2X^2+1} = \frac{1}{3} \quad \therefore 3X^2-24 = 2X^2+1$$

$$\therefore X^2 = 25 \quad \therefore X = \pm 5$$

$$4 \therefore \frac{X^2+10X}{2X^2-3} = \frac{24}{5} \quad \therefore 5X^2+50X = 48X^2-72$$

$$\therefore 43X^2-50X-72 = 0$$

$$\therefore (X-2)(43X+36) = 0$$

$$\therefore X = 2 \text{ or } X = \frac{-36}{43} \quad (\text{refused})$$

5

$$\therefore 3X-6y = X+3y \quad \therefore 2X = 9y$$

$$\therefore \frac{y}{X} = \frac{2}{9}$$

6

$$\therefore \frac{2X+3}{2X-3} = \frac{2y+5}{2y-5}$$

$$\therefore 4Xy+6y-10X-15 = 4Xy-6y+10X-15$$

$$\therefore 12y = 20X \quad \therefore \frac{X}{y} = \frac{12}{20} = \frac{3}{5}$$

7

$$\therefore X^2-3Xy-4y^2 = 0 \quad \therefore (X+y)(X-4y) = 0$$

$$\therefore X+y = 0 \quad \therefore X = -y \quad \therefore X:y = -1:1$$

$$\text{or } X-4y = 0 \quad \therefore X = 4y \quad \therefore X:y = 4:1$$

8

$$\therefore 3X^2-10Xy+7y^2 = 0 \quad \therefore (X-y)(3X-7y) = 0$$

$$\therefore X = y \text{ (refused) or } 3X-7y = 0 \quad \therefore 3X = 7y$$

$$\therefore X:y = 7:3$$

9

$$\therefore \frac{X}{y} = \frac{2}{3}$$

$$\therefore X = 2m, y = 3m$$

$$\therefore \frac{3X+2y}{6y-X} = \frac{6m+6m}{18m-2m} = \frac{12m}{16m} = \frac{3}{4}$$

10

$$\therefore \frac{a}{b} = \frac{3}{5}$$

$$\therefore a = 3m, b = 5m$$

$$\therefore \frac{7a+9b}{4a+2b} = \frac{21m+45m}{12m+10m} = \frac{66m}{22m} = 3$$

11

$$\therefore \frac{a}{b} = \frac{3}{4}$$

$$\therefore a = 3m, b = 4m$$

$$1 \therefore \frac{4a+b}{2a-b} = \frac{12m+4m}{6m-4m} = \frac{16m}{2m} = 8$$

$$2 \therefore \frac{b^2-a^2}{a^2-b^2} = \frac{16m^2-9m^2}{9m^2-16m^2} = \frac{7m^2}{-7m^2} = -1$$

12

$$\therefore \frac{a}{b} = \frac{1}{3}$$

$$\therefore a = m, b = 3m$$

$$\therefore \frac{c}{d} = \frac{7}{2}$$

$$\therefore c = 7k, d = 2k$$

$$\therefore \frac{2ac+bd}{bc-3ad} = \frac{2 \times m \times 7k + 3m \times 2k}{3m \times 7k - 3 \times m \times 2k} = \frac{14mk+6mk}{21mk-6mk}$$

$$= \frac{20mk}{15mk} = \frac{4}{3}$$

13

$$\begin{aligned}\therefore \frac{7x-3y}{x+y} &= \frac{3}{1} & \therefore 7x-3y &= 3x+3y \\ \therefore 4x &= 6y & \therefore 2x &= 3y \\ \therefore \frac{x}{y} &= \frac{3}{2} & \therefore x &= 3m, y = 2m \\ \therefore \frac{12x+9y}{11x-3y} &= \frac{36m+18m}{33m-6m} = \frac{54m}{27m} = 2:1\end{aligned}$$

14

$$\begin{aligned}\therefore \frac{21x+a}{7x+b} &= \frac{a}{b} & \therefore 21bx+ab &= 7ax+ab \\ \therefore 21bx &= 7ax & \therefore 3b &= a \\ \therefore \frac{a+2b}{2a} &= \frac{3b+2b}{2 \times 3b} = \frac{5b}{6b} = \frac{5}{6}\end{aligned}$$

15

 Let the number be  $x$ 

$$\begin{aligned}\therefore 3+x, 5+x, 8+x, 12+x &\text{ are proportional.} \\ \therefore \frac{3+x}{5+x} &= \frac{8+x}{12+x} \\ \therefore 40+13x+x^2 &= 36+15x+x^2 \\ \therefore 40-36 &= 15x-13x \quad \therefore 4=2x \quad \therefore x=2 \\ \therefore \text{The required number} &= 2\end{aligned}$$

16

 Let the number be  $x$ 

$$\begin{aligned}\therefore 16-x, 21-x, 14-x, 18-x &\text{ are proportional.} \\ \therefore \frac{16-x}{21-x} &= \frac{14-x}{18-x} \\ \therefore (21-x)(14-x) &= (16-x)(18-x) \\ \therefore 294-35x+x^2 &= 288-34x+x^2 \\ \therefore x &= 6 & \therefore \text{The required number} &= 6\end{aligned}$$

17

$$\begin{aligned}\therefore \frac{a+b}{b} &= \frac{c+d}{d} & \therefore d(a+b) &= b(c+d) \\ \therefore a+d &= b+c & \therefore a &= b+c-b \\ \therefore \frac{a}{b} &= \frac{c}{d} \\ \therefore a, b, c, d &\text{ are proportional.}\end{aligned}$$

Another solution :

$$\begin{aligned}\frac{a+b}{b} &= \frac{c+d}{d} & \therefore \frac{a}{b} + \frac{b}{b} &= \frac{c}{d} + \frac{d}{d} \\ \therefore \frac{a}{b} + 1 &= \frac{c}{d} + 1 & \therefore \frac{a}{b} &= \frac{c}{d} \\ \therefore a, b, c, d &\text{ are proportional.}\end{aligned}$$

$$\begin{aligned}\text{2. } \frac{a}{b-a} &= \frac{c}{d-c} & \therefore a(d-c) &= c(b-a) \\ \therefore ad-ac &= cb-ca & \therefore ad &= cb \\ \therefore \frac{a}{b} &= \frac{c}{d} & \therefore a, b, c, d &\text{ are proportional.}\end{aligned}$$

Another solution :

$$\begin{aligned}\therefore \frac{a}{b-a} &= \frac{c}{d-c} & \therefore \frac{b-a}{a} &= \frac{d-c}{c} \\ \therefore \frac{b}{a} - 1 &= \frac{d}{c} - 1 \\ \therefore \frac{b}{a} &= \frac{d}{c} & \therefore \frac{a}{b} &= \frac{c}{d}\end{aligned}$$

$$\begin{aligned}\text{3. } \frac{a-b}{a+b} &= \frac{c-d}{c+d} & \therefore (a-b)(c+d) &= (a+b)(c-d) \\ \therefore ac+ad-bc-bd &= ac-ad+bc-bd \\ \therefore 2ad &= 2bc & \therefore ad &= bc & \therefore \frac{a}{b} &= \frac{c}{d} \\ \therefore a, b, c, d &\text{ are proportional.}\end{aligned}$$

$$\begin{aligned}\text{4. } \frac{a^2-2c^2}{b^2-2d^2} &= \frac{a^2}{b^2} \\ \therefore a^2(b^2-2d^2) &= b^2(a^2-2c^2) \\ \therefore a^2b^2-2a^2d^2 &= a^2b^2-2b^2c^2 \\ \therefore a^2d^2 &= b^2c^2 & \therefore a &= b, c &= d \\ \therefore \frac{a}{b} &= \frac{c}{d} \\ \therefore a, b, c, d &\text{ are proportional.}\end{aligned}$$

18

$$\begin{aligned}\therefore a:b:c &= 5:7:3 & \therefore a &= 5m, b=7m, c=3m \\ \therefore a+b &= 27.6 & \therefore 5m+7m &= 27.6 \\ \therefore 12m &= 27.6 & \therefore m &= 2.3 \\ a &= 5 \times 2.3 = 11.5, b &= 7 \times 2.3 = 16.1, c &= 3 \times 2.3 = 6.9\end{aligned}$$

19

$$\begin{aligned}\therefore a:b:c &= 3:4:5 & \therefore a &= 3m, b=4m, c=5m \\ \therefore \frac{a^2+b^2+c^2}{a(b+c)} &= \frac{9m^2+16m^2+25m^2}{3m(4m+5m)} = \frac{50m^2}{27m^2} = \frac{50}{27}\end{aligned}$$

20

$$\begin{aligned}\therefore 2a &= 3b & \therefore 2a &= 3b & \therefore a &= \frac{3}{2}b \\ \therefore 3b &= 4c & \therefore c &= \frac{3}{4}b \\ \therefore a:b:c &= \frac{3}{2}b:b:\frac{3}{4}b \\ \text{multiplying by 4} & & \therefore a:b:c &= 6b:4b:3b \\ \text{dividing by } b & & \therefore a:b:c &= 6:4:3\end{aligned}$$

21

$$\begin{aligned}\text{1. Let the number be } x & & \therefore \frac{7+x}{11+x} &= \frac{2}{3} \\ \therefore 21+3x &= 22+2x & \therefore x &= 1 \\ \therefore \text{The required number is } & & & \\ \text{2. Let the number be } x & & \therefore \frac{49-3x}{69-3x} &= \frac{2}{3} \\ \therefore 147-9x &= 138-6x & \therefore 3x &= 9 \\ \therefore x &= 3 \\ \therefore \text{The required number} &= 3\end{aligned}$$

3 Let the number be  $x$   $\therefore \frac{7+x^2}{11+x^2} = \frac{4}{5}$

$\therefore 35 + 5x^2 = 44 + 4x^2 \quad \therefore x^2 = 9$

$\therefore x = \pm 3$

$\therefore$  The required number is 3 or -3

4 Let the number be  $x$   $\therefore \frac{5+x^2}{11+x^2} = \frac{3}{5}$

$\therefore 25 + 5x^2 = 33 + 3x^2 \quad \therefore 2x^2 = 8$

$\therefore x^2 = 4$

$\therefore x = 2$  or  $x = -2$  (refused)

$\therefore$  The required number = 2

5 Let the number be  $x$   $\therefore \frac{15-x}{13+x} = \frac{3}{4}$

$\therefore 60 - 4x = 39 + 3x \quad \therefore 7x = 21$

$\therefore x = 3$

$\therefore$  The required number = 3

6 Let the two numbers be  $a$  and  $b$

$\therefore \frac{a}{b} = \frac{3}{7} \quad \therefore a = 3m, b = 7m$

$\therefore \frac{3m-5}{7m-5} = \frac{1}{3} \quad \therefore 9m - 15 = 7m - 5$

$\therefore 2m = 10 \quad \therefore m = 5$

$\therefore$  The two numbers are 15 and 35

7 Let the two numbers be  $a$  and  $b$

$\therefore \frac{a}{b} = \frac{2}{3} \quad \therefore a = 2m, b = 3m$

$\therefore \frac{2m+7}{3m-12} = \frac{5}{3} \quad \therefore 6m + 21 = 15m - 60$

$\therefore 81 = 9m \quad \therefore m = 9$

$\therefore$  The two numbers are 18 and 27

8 Let the two numbers be  $a$  and  $b$

$\therefore \frac{a}{b} = \frac{4}{7} \quad \therefore a = 4m, b = 7m$

$\therefore (4m)^2 - 5(7m) = 39$

$\therefore 16m^2 - 35m - 39 = 0$

$\therefore (m-3)(16m+13) = 0$

$\therefore m = 3$  or  $m = -\frac{13}{16}$  (refused)

$\therefore$  The two numbers are 12 and 21

22

Let the two dimensions of the rectangle be  $a$  and  $b$  in centimetres

$\therefore \frac{a}{b} = \frac{4}{7} \quad \therefore a = 4m, b = 7m$

$\therefore 2(4m + 7m) = 88 \quad \therefore 22m = 88 \quad \therefore m = 4$

$\therefore$  The two dimensions of the rectangle are

$4 \times 4 = 16 \text{ cm.}, 7 \times 4 = 28 \text{ cm.}$

$\therefore$  The area =  $16 \times 28 = 448 \text{ cm}^2$

23

Let the base length =  $a$  cm. and the height =  $b$  cm.

$\therefore \frac{a}{b} = \frac{3}{2} \quad \therefore a = 3m, b = 2m$

$\therefore \frac{1}{2} \times 3m \times 2m = 48 \quad \therefore 3m^2 = 48$

$\therefore m^2 = 16 \quad \therefore m = 4$

$\therefore$  The base length =  $3 \times 4 = 12 \text{ cm.}$

The height =  $2 \times 4 = 8 \text{ cm.}$

24

$\therefore$  The area of the unshaded part from the circle

$= 1 - \frac{5}{6} = \frac{1}{6}$  of the area of the circle

$\therefore$  the area of the unshaded part from the triangle

$= 1 - \frac{2}{3} = \frac{1}{3}$  of the area of the triangle.

$\therefore \frac{1}{6}$  of the area of the circle

$= \frac{1}{3}$  of the area of the triangle

$\therefore$  The area of the circle : the area of the triangle

$= \frac{1}{3} : \frac{1}{6}$  (multiply by 6) = 2 : 1

25

The length of the shadow of tree

The length of the shadow of Islam

The height of the tree

The height of Islam

$\therefore \frac{300}{120} = \frac{\text{The height of the tree}}{180}$

$\therefore$  The height of the tree

$= \frac{300 \times 180}{120} = 450 \text{ cm.} = 4 \frac{1}{2} \text{ m.}$

26

Let the costs of building the school be  $x$  and

Let the costs of building the medical unit =  $y$  and

Let the costs of building the youth centre =  $z$

$\therefore x = \frac{3}{2}y, y = \frac{5}{6}z \quad \therefore z = \frac{6}{5}y$

$\therefore x + y + z = 1.85 \times 10^6$

$\therefore \frac{3}{2}y + y + \frac{6}{5}y = 1.85 \times 10^6 \quad \therefore \frac{37}{10}y = 1.85 \times 10^6$

$\therefore 37y = 1.85 \times 10^7 \quad \therefore y = 5 \times 10^5$

$x = \frac{3}{2} \times 5 \times 10^5 = 7.5 \times 10^5, z = \frac{6}{5} \times 5 \times 10^5 = 6 \times 10^5$

27

 Let the number of boys =  $X$  and the number of girls =  $y$ 
 $\therefore$  The total number of pupils =  $X + y$ 

 The number of succeeded boys =  $X \times \frac{79}{100} = 0.79X$ 

 The number of succeeded girls =  $y \times \frac{89}{100} = 0.89y$ 

 The total number of succeeded pupils =  $0.79X + 0.89y$ 
 $\therefore$  The ratio of success in 3<sup>rd</sup> grade preparatory

$$\frac{0.79X + 0.89y}{X + y} = 0.83$$

$$\therefore (0.79)X + (0.89)y = (0.83)X + (0.83)y$$

$$\therefore (0.89)y - (0.83)y = (0.83)X - (0.79)X$$

$$\therefore (0.06)y = (0.04)X \quad \therefore X : y = 6 : 4 = 3 : 2$$

 $\therefore$  The number of boys : the number of girls = 3 : 2

28

 Let the circumference of the circle be  $a$  cm.  
and the perimeter of the square be  $b$  cm.

$$\therefore \frac{a}{b} = \frac{11}{8} \quad \therefore a = 11 \text{ m}, b = 8 \text{ m}$$

$$\therefore 11 \text{ m} + 8 \text{ m} = 152$$

$$\therefore 19 \text{ m} = 152 \quad \therefore m = 8$$

$$\therefore \text{The circumference of the circle} = 11 \times 8 = 88 \text{ cm.}$$

$$\therefore 2 \times \frac{22}{7} \times r = 88 \quad \therefore r = 14 \text{ cm.}$$

$$\therefore \text{The area of the circle} = \pi r^2 = \frac{22}{7} \times 14 \times 14 = 616 \text{ cm}^2$$

$$\therefore \text{the perimeter of the square} = 8 \times 8 = 64 \text{ cm.}$$

$$\therefore \text{The side length of the square} = \frac{64}{4} = 16 \text{ cm.}$$

$$\therefore \text{The area of the square} = 16 \times 16 = 256 \text{ cm}^2$$

$$\therefore \text{The area of the square : The area of the circle} \\ = \frac{256}{616} = \frac{32}{77}$$

29

 Let the second proportional be  $X$ 
 $\therefore$  The numbers are :  $X - 2$ ,  $X$ ,  $8$  and  $X^2$ 

$$\therefore \frac{X-2}{X} = \frac{8}{X^2} \quad \therefore X^3 - 2X^2 = 8X$$

$$\therefore X^3 - 2X^2 - 8X = 0 \quad \therefore X(X^2 - 2X - 8) = 0$$

$$\therefore X(X-4)(X+2) = 0 \quad \therefore X = 0 \text{ (refused)}$$

 or  $X = 4$  thus, the numbers are : 2, 4, 8 and 16

 or  $X = -2$  thus, the numbers are : -4, -2, 8 and 4

30

$$\therefore X + y = 8 \quad \therefore y = 8 - X \quad (1)$$

$$\therefore y + z = 14 \quad \therefore z = 14 - y = 14 - (8 - X) = 6 + X \quad (2)$$

$$\therefore z + \ell = 24 \quad \therefore \ell = 24 - z = 24 - (6 + X) = 18 - X \quad (3)$$

$$\therefore X, y, z, \ell \text{ are proportional} \quad \therefore \frac{X}{y} = \frac{z}{\ell}$$

$$\text{From (1), (2) and (3)} : \quad \therefore \frac{X}{8-X} = \frac{6+X}{18-X}$$

$$\therefore 18X - X^2 = 48 + 2X - X^2 \quad \therefore 16X = 48$$

$$\therefore X = 3$$

$$\therefore y = 8 - 3 = 5, \quad z = 6 + 3 = 9, \quad \ell = 18 - 3 = 15$$

31

 Let the number be  $X$ , its multiplicative inverse =  $\frac{1}{X}$ 

$$\therefore \frac{2}{3 + \frac{1}{X}} = \frac{3}{5}$$

 Multiplying the two terms of the ratio in the left side by  $X$ 

$$\therefore \frac{2X}{3X+1} = \frac{3}{5} \quad \therefore 10X = 9X + 3$$

$$\therefore X = 3 \quad \therefore \text{The number} = 3$$

### Answers of Exercise 6

1

$$\text{1} \frac{3}{5} \quad \text{2} \frac{3}{5} \quad \text{3} X + y, 6, 9 \quad \text{4} -3$$

$$\text{5} 3 \quad \text{6} 7 \quad \text{7} 3d, 4e$$

2

$$\text{1} b \quad \text{2} a \quad \text{3} b \quad \text{4} c \quad \text{5} d \quad \text{6} c$$

3

$$\therefore \text{Let } \frac{a}{b} = \frac{c}{d} = m \text{ where } m > 0$$

$$\therefore a = bm, c = dm$$

$$\text{1} \text{ L.H.S.} = \frac{3a+c}{5a-2c} = \frac{3bm+dm}{5bm-2dm} = \frac{m(3b+d)}{m(5b-2d)} \\ = \frac{3b+d}{5b-2d} \\ = \text{R.H.S.}$$

 From (1) and (2) :  $\therefore$  The two sides are equal.

$$\text{2} \therefore \frac{3a-2c}{5a+3c} = \frac{3b-2d}{5b+3d} \quad \therefore \frac{3a-2c}{3b-2d} = \frac{5a+3c}{5b+3d} \\ \therefore a = bm, c = dm$$

$$\therefore \text{L.H.S.} = \frac{3bm-2dm}{3b-2d} = \frac{m(3b-2d)}{3b-2d} = m \quad (1)$$

$$\therefore \text{R.H.S.} = \frac{5bm+3dm}{5b+3d} = \frac{m(5b+3d)}{5b+3d} = m \quad (2)$$

 From (1) and (2) :  $\therefore$  The two sides are equal.

$$\text{3} \text{ L.H.S.} = \frac{b^2m^2+d^2m^2}{b^2m+d^2m} = \frac{m^2(b^2+d^2)}{m(b^2+d^2)} = m \quad (1)$$

$$\therefore \text{R.H.S.} = \frac{bm}{b} = m \quad (2)$$

 From (1) and (2) :  $\therefore$  The two sides are equal.

$$\text{[4]} \quad \text{L.H.S.} = \frac{a^2 + c^2}{b^2 + d^2} = \frac{b^2 m^2 + d^2 m^2}{b^2 + d^2} = \frac{m^2 (b^2 + d^2)}{b^2 + d^2} = m^2 \quad (1)$$

$$\text{R.H.S.} = \frac{a c}{b d} = \frac{b m \times d m}{b d} = m^2 \quad (2)$$

From (1) and (2):  $\therefore$  The two sides are equal.

$$\text{[5]} \quad \text{L.H.S.} = \frac{a c}{b d} = \frac{b m \times d m}{b d} = m^2 \quad (1)$$

$$\text{R.H.S.} = \left( \frac{a-c}{b-d} \right)^2 = \left( \frac{b m - d m}{b-d} \right)^2 = \left( \frac{m(b-d)}{b-d} \right)^2 = m^2 \quad (2)$$

From (1) and (2):  $\therefore$  The two sides are equal.

$$\text{[6]} \quad \text{L.H.S.} = \left( \frac{a+b}{c+d} \right)^2 = \left( \frac{b m + d m}{d m + d} \right)^2 = \left( \frac{m(b+1)}{d(m+1)} \right)^2 = \frac{b^2}{d^2} \quad (1)$$

$$\text{R.H.S.} = \frac{2a^2 - 3b^2}{2c^2 - 3d^2} = \frac{2b^2 m^2 - 3b^2}{2d^2 m^2 - 3d^2} = \frac{b^2 (2m^2 - 3)}{d^2 (2m^2 - 3)} = \frac{b^2}{d^2} \quad (2)$$

from (1) and (2):  $\therefore$  The two sides are equal.

$$\text{[7]} \quad \text{L.H.S.} = \sqrt{\frac{3a^3 - 5c^2}{3b^2 - 5d^2}} = \sqrt{\frac{3b^3 m^3 - 5d^3 m^2}{3b^2 - 5d^2}} = \sqrt{\frac{m^2 (3b^2 - 5d^2)}{(3b^2 - 5d^2)}} = m \quad (1)$$

$$\text{R.H.S.} = \frac{a}{b} = \frac{b m}{b} = m \quad (2)$$

From (1) and (2):  $\therefore$  The two sides are equal.

$$\text{[8]} \quad \text{L.H.S.} = \sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \sqrt[3]{\frac{5b^3 m^3 - 3d^3 m^3}{5b^3 - 3d^3}} = \sqrt[3]{\frac{m^3 (5b^3 - 3d^3)}{(5b^3 - 3d^3)}} = \sqrt[3]{m^3} = m \quad (1)$$

$$\text{R.H.S.} = \frac{b m + d m}{b + d} = \frac{m(b+d)}{(b+d)} = m \quad (2)$$

From (1) and (2):  $\therefore$  The two sides are equal.

$$\text{[9]} \quad \text{L.H.S.} = \frac{a^2 - 2ac + c^2}{ac} = \frac{(a-c)^2}{ac} = \frac{(b m - d m)^2}{b m \times d m} = \frac{(m(b-d))^2}{b d m^2} = \frac{m^2 (b-d)^2}{b d m^2} = \frac{(b-d)^2}{b d} \quad (1)$$

$$\text{R.H.S.} = \frac{b^2 - 2bd + d^2}{b d} = \frac{(b-d)^2}{b d} \quad (2)$$

From (1) and (2):  $\therefore$  The two sides are equal.

$$\text{[4]} \quad \text{Let } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = m \text{ where } m > 0$$

$$\therefore a = b m, c = d m, e = f m$$

$$\text{[1]} \quad \text{L.H.S.} = \frac{a+5c}{b+5d} = \frac{b m + 5 d m}{b + 5 d} = \frac{m(b+5d)}{(b+5d)} = m \quad (1)$$

$$\text{R.H.S.} = \frac{c-3e}{d-3f} = \frac{d m - 3 f m}{d - 3 f} = \frac{m(d-3f)}{d-3f} = m \quad (2)$$

From (1) and (2):  $\therefore$  The two sides are equal.

$$\text{[2]} \quad \text{L.H.S.} = \frac{2a+7c-4e}{2b+7d-4f} = \frac{2b m + 7 d m - 4 f m}{2b + 7 d - 4 f} = \frac{m(2b+7d-4f)}{(2b+7d-4f)} = m \quad (1)$$

$$\text{R.H.S.} = \frac{a-8e}{b-8f} = \frac{b m - 8 f m}{b - 8 f} = \frac{m(b-8f)}{b-8f} = m \quad (2)$$

From (1) and (2):  $\therefore$  The two sides are equal.

$$\text{[3]} \quad \text{L.H.S.} = \frac{2a^4 b^2 + 3a^2 e^2 - 5e^4 f}{2b^6 + 3b^2 f^2 - 5f^5} = \frac{(2b^6 m^4 + 3b^2 f^2 m^4 - 5f^5 m^4)}{2b^6 + 3b^2 f^2 - 5f^5} = \frac{m^4 (2b^6 + 3b^2 f^2 - 5f^5)}{2b^6 + 3b^2 f^2 - 5f^5} = m^4 \quad (1)$$

$$\text{R.H.S.} = \frac{a^4}{b^4} = \frac{b^4 m^4}{b^4} = m^4 \quad (2)$$

From (1) and (2):  $\therefore$  The two sides are equal.

$$\text{[4]} \quad \text{L.H.S.} = \sqrt{\frac{5a^2 - 7ce}{5b^2 - 7df}} = \sqrt{\frac{5b^2 m^2 - 7 d f m^2}{5b^2 - 7 d f}} = \sqrt{\frac{m^2 (5b^2 - 7 d f)}{5b^2 - 7 d f}} = \sqrt{m^2} = m \quad (1)$$

$$\text{R.H.S.} = \frac{2a+c}{2b+d} = \frac{2b m + d m}{2b + d} = \frac{m(2b+d)}{2b+d} = m \quad (2)$$

From (1) and (2):  $\therefore$  The two sides are equal.

$$\text{[5]} \quad \therefore \frac{x}{3} = \frac{y}{4} = \frac{z}{5} = m \text{ where } m > 0$$

$$\therefore x = 3 m, y = 4 m, z = 5 m$$

$$\text{[1]} \quad \text{The left side} = \frac{2y-z}{3x-2y+z} = \frac{8m-5m}{9m-8m+5m} = \frac{3m}{6m} = \frac{1}{2}$$

$$\text{[2]} \quad \therefore \sqrt{3x^2 + 3y^2 + z^2} = \sqrt{27m^2 + 48m^2 + 25m^2} = \sqrt{100m^2} = 10m \quad (1)$$

$$2x + y = 6m + 4m = 10m \quad (2)$$

From (1) and (2) we deduce that:

$$\sqrt{3x^2 + 3y^2 + z^2} = 2x + y$$

6

$$\text{Let } \frac{X}{1} = \frac{y}{2} = \frac{z}{3} = m$$

$$\therefore X = m, y = 2m, z = 3m$$

$$\therefore \text{L.H.S.} = \frac{X+y-2z}{X-3z} = \frac{m+2m-6m}{m-9m} = \frac{-3m}{-8m} = \frac{3}{8} = \text{R.H.S.}$$

Another solution :

$$\therefore X = \frac{y}{2} = \frac{z}{3} \quad \therefore y = 2X, z = 3X$$

$$\therefore \text{L.H.S.} = \frac{X+y-2z}{X-3z} = \frac{X+2X-6X}{X-9X} = \frac{-3X}{-8X} = \frac{3}{8} = \text{R.H.S.}$$

7

$$\therefore \frac{a}{2} = \frac{b}{3} = \frac{c}{4}$$

• multiplying the two terms of the 1<sup>st</sup> ratio by 2 and the 2<sup>nd</sup> by -5 and the 3<sup>rd</sup> by 3 and adding the antecedents and consequents of the three ratios.

$$\therefore \frac{2a-5b+3c}{4-15+12} = \text{one of the given ratios.}$$

$$\therefore 2a-5b+3c = \text{one of the given ratios.}$$

8

$$\therefore \frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a-b+5c}{3X}$$

• multiplying the two terms of 1<sup>st</sup> ratio by 2 and the 2<sup>nd</sup> by -1 and the 3<sup>rd</sup> by 5 and adding the antecedents and consequents of the three ratios.

$$\therefore \frac{2a-b+5c}{4-3+20} = \text{one of the given ratios.}$$

$$\therefore \frac{2a-b+5c}{21} = \frac{2a-b+5c}{3X}$$

$$\therefore 3X = 21 \quad \therefore X = 7$$

9

$$\therefore \frac{a}{4X+y} = \frac{b}{X-4y}$$

• adding the antecedents and consequents of the two ratios.

$$\therefore \frac{a+b}{4X+y+X-4y} = \frac{a+b}{5X-3y} = \text{one of the given ratios.} \quad (1)$$

subtracting the antecedents and consequents of 2<sup>nd</sup> ratio from 1<sup>st</sup> ratio.

$$\frac{a-b}{4X+y-X+4y} = \frac{a-b}{3X+5y} = \text{one of the given ratios.} \quad (2)$$

$$\text{From (1) and (2):} \quad \therefore \frac{a+b}{5X-3y} = \frac{a-b}{3X+5y}$$

10

$$\therefore \frac{X+y}{19} = \frac{y+z}{7}$$

• adding the antecedents and consequents of the two ratios.

$$\therefore \frac{X+y+y+z}{19+7} = \frac{X+2y+z}{26} = \text{one of the given ratios.} \quad (1)$$

• subtracting the antecedent and consequent of 2<sup>nd</sup> ratio from the antecedent and consequent of the 1<sup>st</sup> ratio.

$$\therefore \frac{X+y-y-z}{19-7} = \frac{X-z}{12} = \text{one of the given ratios.} \quad (2)$$

$$\text{From (1) and (2):} \quad \therefore \frac{X+2y+z}{26} = \frac{X-z}{12}$$

$$\therefore \frac{X+2y+z}{13} = \frac{X-z}{6}$$

11

$$\therefore \frac{y}{X-z} = \frac{X}{y} = \frac{X+y}{z}$$

• adding the antecedents and consequents of the three ratios  $\therefore \frac{y+X+X+y}{X-z+y+z} = \frac{2(X+y)}{(X+y)} = 2$   
= one of the given ratios.

$$\therefore \text{Each ratio} = 2 \text{ unless } X+y=0$$

$$\therefore \frac{X}{y} = 2 \quad \therefore X = 2y$$

$$\therefore \frac{X+y}{z} = 2 \quad \therefore X+y = 2z \quad \therefore 2y+y = 2z$$

$$\therefore 3y = 2z \quad \therefore z = \frac{3}{2}y$$

$$\therefore X:y:z = 2y:y:\frac{3}{2}y = 4:2:3$$

12

$$\therefore \frac{X}{a-b+c} = \frac{y}{b-c+a} = \frac{z}{c-a+b}$$

• adding the antecedents and consequents of the 1<sup>st</sup> and 2<sup>nd</sup> ratios.

$$\therefore \frac{X+y}{a-b+c+b-c+a} = \frac{X+y}{2a} = \text{one of the given ratios.} \quad (1)$$

• adding the antecedents and consequents of 2<sup>nd</sup> and 3<sup>rd</sup> ratios.

$$\therefore \frac{y+z}{b-c+a+c-a+b} = \frac{y+z}{2b}$$

= one of the given ratios. (2)

From (1) and (2):  $\therefore \frac{X+y}{2a} = \frac{y+z}{2b}$

$$\therefore \frac{X+y}{a} = \frac{y+z}{b}$$

13

$$\therefore \frac{X}{2a+b} = \frac{y}{2b-c} = \frac{z}{2c-a}$$

• multiplying the two terms of the 1<sup>st</sup> ratio by 2 and adding the antecedents and consequents of the 1<sup>st</sup> and the 2<sup>nd</sup> ratios.

$$\therefore \frac{2X+y}{4a+2b+2b-c} = \frac{2X+y}{4a+4b-c}$$

= one of the given ratios. (1)

• multiplying the terms of the 1<sup>st</sup> ratio by 2 and the 2<sup>nd</sup> by 2 and adding the antecedents and consequents of the three ratios

$$\therefore \frac{2X+2y+z}{4a+2b+4b-2c+2c-a} = \frac{2X+2y+z}{3a+6b}$$

= one of the given ratios. (2)

From (1) and (2):

$$\therefore \frac{2X+y}{4a+4b-c} = \frac{2X+2y+z}{3a+6b}$$

14

$$\therefore \frac{a}{2X-y} = \frac{b}{2y-X}$$

• multiplying the terms of the 1<sup>st</sup> ratio by 2 and adding the antecedents and consequents of the two ratios.

$$\therefore \frac{2a+b}{4X-2y+2y-X} = \frac{2a+b}{3X}$$

= one of the given ratios. (1)

• multiplying the terms of the 2<sup>nd</sup> ratio by 2 and adding the antecedents and consequents of the two ratios.

$$\therefore \frac{a+2b}{2X-y+4y-2X} = \frac{a+2b}{3y}$$

= one of the given ratios. (2)

From (1) and (2):  $\therefore \frac{2a+b}{3X} = \frac{a+2b}{3y}$

$$\therefore \frac{2a+b}{a+2b} = \frac{3X}{3y} = \frac{X}{y}$$

15

$$\therefore \frac{a}{2X+y} = \frac{b}{3y-X} = \frac{c}{4X+5y}$$

• multiplying the terms of the 2<sup>nd</sup> ratio by 2 and adding the antecedents and consequents of the two ratios 1<sup>st</sup> and 2<sup>nd</sup>

$$\therefore \frac{a+2b}{2X+y+6y-2X} = \frac{a+2b}{7y}$$

= one of the given ratios. (1)

• multiplying the terms of the 2<sup>nd</sup> ratio by 4 and adding the antecedents and consequents of the 2<sup>nd</sup> and 3<sup>rd</sup> ratios.

$$\therefore \frac{4b+c}{12y-4X+4X+5y} = \frac{4b+c}{17y}$$

= one of the given ratios. (2)

From (1) and (2):  $\therefore \frac{a+2b}{7y} = \frac{4b+c}{17y}$

$$\therefore \frac{a+2b}{4b+c} = \frac{7y}{17y} = \frac{7}{17}$$

16

$$\therefore \frac{a}{2} = \frac{b}{7} = \frac{c}{3}$$

• multiplying the terms of the 2<sup>nd</sup> ratio by 2 and adding the antecedents and consequents of the 1<sup>st</sup> and 2<sup>nd</sup> ratios.

$$\therefore \frac{a+2b}{2+14} = \frac{a+2b}{16} = \text{one of the given ratios. (1)}$$

Subtracting the antecedents and consequents of 3<sup>rd</sup> ratio from the antecedents and consequents of the 2<sup>nd</sup> ratio.

$$\therefore \frac{b-c}{7-3} = \frac{b-c}{4} = \text{one of the given ratios. (2)}$$

From (1) and (2):  $\therefore \frac{a+2b}{16} = \frac{b-c}{4}$

$$\therefore \frac{a+2b}{b-c} = \frac{16}{4} = 4$$

17

$$\therefore \frac{X+y}{7} = \frac{y+z}{5} = \frac{z+X}{8}$$

• adding the antecedents and consequents of the three ratios.

$$\therefore \frac{X+y+y+z+X}{7+5+8} = \frac{2(X+y+z)}{20} = \frac{X+y+z}{10}$$

= one of the given ratios. (1)

• multiplying the terms of the 2<sup>nd</sup> ratio by (-1) and adding the antecedents and consequents of 1<sup>st</sup> and 2<sup>nd</sup> ratios.

$$\therefore \frac{X+y-y-z}{7-5} = \frac{X-z}{2} = \text{one of the given ratios. (2)}$$

From (1) and (2):  $\therefore \frac{X+y+z}{10} = \frac{X-z}{2}$

$$\therefore \frac{X+y+z}{X-z} = \frac{10}{2} = 5$$

18

$$\therefore \frac{a+b}{4} = \frac{b+c}{5} = \frac{c+a}{7}$$

• adding the antecedents and consequents of the three ratios.

$$\therefore \frac{a+b+b+c+c+a}{4+5+7} = \frac{2(a+b+c)}{16} = \frac{a+b+c}{8}$$

$$= \text{one of the given ratios. (1)}$$

, multiplying the terms of 2<sup>nd</sup> ratio by (-1) and adding the antecedents and consequents of the three ratios.

$$\therefore \frac{a+b-b-c+c+a}{4-5+7} = \frac{2a}{6} = \frac{a}{3}$$

$$= \text{one of the given ratios. (2)}$$

From (1) and (2) :

$$\therefore \frac{a+b+c}{8} = \frac{a}{3}$$

19

$$\therefore \frac{X+y}{3} = \frac{y+z}{5} = \frac{z+X}{6}$$

, adding the antecedents and consequents of the three ratios.

$$\therefore \frac{X+y+y+z+z+X}{3+5+6} = \frac{2X+2y+2z}{14} = \frac{X+y+z}{7}$$

$$= \text{one of the given ratios. (1)}$$

, multiplying the terms of 2<sup>nd</sup> ratio by 2 and adding the antecedents and consequents of the three ratios.

$$\therefore \frac{X+y+2y+2z+z+X}{3+10+6} = \frac{2X+3y+3z}{19}$$

$$= \text{one of the given ratios. (2)}$$

$$\text{From (1) and (2) : } \therefore \frac{X+y+z}{7} = \frac{2X+3y+3z}{19}$$

$$\therefore \frac{X+y+z}{2X+3y+3z} = \frac{7}{19}$$

20

Multiplying the terms of 2<sup>nd</sup> ratio by (-1) and adding the antecedents and consequents of the three ratios.

$$\therefore \frac{X+y-y-z+z+X}{5-8+7} = \frac{2X}{2} = \frac{X}{1}$$

$$= \text{one of the given ratios (1)}$$

, multiplying the terms of the 3<sup>rd</sup> ratio by (-1) and adding the antecedents and consequents of the three ratios

$$\therefore \frac{X+y+y+z-z-X}{5+8-7} = \frac{2y}{6} = \frac{y}{3}$$

$$= \text{one of the given ratios (2)}$$

, multiplying the terms of the 1<sup>st</sup> ratio by (-1) and adding the antecedents and consequents of the three ratios.

$$\therefore \frac{X-y+y+z+z+X}{-5+8+7} = \frac{2z}{10} = \frac{z}{5}$$

$$= \text{one of the given ratios (3)}$$

$$\text{From (1), (2) and (3) : } \therefore \frac{X}{2} = \frac{y}{3} = \frac{z}{5}$$

21

$$\therefore \frac{X+y}{25} = \frac{X-y}{11} = \frac{X+y-z}{8}$$

, adding the antecedents and consequents of the 1<sup>st</sup> and 2<sup>nd</sup> ratios.

$$\therefore \frac{2X}{36} = \frac{X}{18} = \text{one of the given ratios. (1)}$$

, subtracting the antecedent and consequent of the 3<sup>rd</sup> ratio from the antecedent and consequent of the 1<sup>st</sup> ratio.

$$\therefore \frac{z}{17} = \text{one of the given ratios. (2)}$$

, subtracting the antecedent and consequent of the 2<sup>nd</sup> ratio from the antecedent and consequent of the 1<sup>st</sup> ratio.

$$\therefore \frac{2y}{14} = \frac{y}{7} = \text{one of the given ratios. (3)}$$

From (1), (2), (3) :

$$\therefore \frac{X}{18} = \frac{y}{7} = \frac{z}{17} \quad \therefore X : y : z = 18 : 7 : 17$$

22

Multiplying the terms of 2<sup>nd</sup> ratio by (-1) and adding the antecedents and consequents of the three ratios.

$$\therefore \frac{a+3b-3b-5c+5c+a}{X+6y-6y-10z+10z+X} = \frac{2a}{2X} = \frac{a}{X} = \text{one of the given ratios (1)}$$

, multiplying the terms of the 3<sup>rd</sup> ratio by (-1) and adding the antecedents and consequents of the three ratios.

$$\therefore \frac{a+3b+3b+5c-5c-a}{X+6y+6y+10z-10z-X} = \frac{6b}{12y} = \frac{b}{2y} = \text{one of the given ratios (2)}$$

$$\text{From (1) and (2) : } \therefore \frac{a}{X} = \frac{b}{2y}$$

$$\therefore \frac{a}{b} = \frac{X}{2y}$$

, multiplying the terms of the 1<sup>st</sup> ratio by (-1) and adding the antecedents and consequents of the three ratios.

$$\therefore \frac{-a-3b+3b+5c+5c+a}{-X-6y+6y+10z+10z+X} = \frac{10c}{20z} = \frac{c}{2z}$$

= one of the given ratios. (3)

$\therefore$  From (1), (2) and (3):

$$\therefore \frac{a}{X} = \frac{b}{2y} = \frac{c}{2z}$$

$$\therefore a : b : c = X : 2y : 2z$$

23

Multiplying the terms of the 1<sup>st</sup> by  $(-2)$  and the 3<sup>rd</sup> by 3 and adding the antecedents and consequents of the two ratios 1<sup>st</sup> and 3<sup>rd</sup>

$$\therefore \frac{3c-2a}{3y+6X-6X-8y} = \frac{3c-2a}{-5y}$$

= one of the given ratios (1)

$\therefore$  multiplying the terms of the 2<sup>nd</sup> ratio by 2 and adding the antecedents and consequents of the two ratios 1<sup>st</sup> and 2<sup>nd</sup>

$$\therefore \frac{a+2b}{3X+4y+10X-4y} = \frac{a+2b}{13X}$$

= one of the given ratios (2)

$$\text{From (1) and (2): } \therefore \frac{3c-2a}{-5y} = \frac{a+2b}{13X}$$

$$\therefore 13X(3c-2a) = -5y(a+2b)$$

$$\therefore 13X(3c-2a) + 5y(a+2b) = 0$$

24

$$\therefore \frac{X}{7} = \frac{y}{3} = m \quad \therefore X = 7m, y = 3m$$

$$\therefore \frac{2X-3y}{X+2y} = \frac{2(7m)-3(3m)}{7m+2(3m)} = \frac{14m-9m}{7m+6m}$$

$$= \frac{5m}{13m} = \frac{5}{13}$$

$$\therefore \frac{10}{26} = \frac{5}{13} \quad (2)$$

$$\therefore \text{From (1) and (2): } \therefore \frac{2X-3y}{X+2y} = \frac{10}{26}$$

$$\therefore (2X-3y), (X+2y), 10, 26 \text{ are proportional.}$$

25

$$\therefore \frac{a}{b} = \frac{3}{5} \quad \therefore b = \frac{5a}{3}$$

$$\therefore \frac{a}{c} = \frac{3}{7} \quad \therefore c = \frac{7a}{3}$$

$$\therefore a+b+c = a + \frac{5a}{3} + \frac{7a}{3} = 5a$$

26

$$\therefore \frac{a}{b} = \frac{2}{3} \quad \therefore b = \frac{3}{2}a$$

$$\therefore \frac{a}{c} = \frac{3}{5} \quad \therefore c = \frac{5}{3}a$$

$$\therefore a+b+c = 75 \quad \therefore a + \frac{3}{2}a + \frac{5}{3}a = 75$$

$$\therefore \frac{25}{6}a = 75 \quad \therefore a = 18$$

$$\therefore b = \frac{3}{2} \times 18 = 27 \quad \therefore c = \frac{5}{3} \times 18 = 30$$

27

$$\therefore \triangle ABC \sim \triangle DEF \quad \therefore \frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC} = \frac{2}{3}$$

Adding the antecedents and the consequents of the three ratios.

$$\therefore \frac{DE+EF+DF}{AB+BC+AC} = \text{one of the given ratios.}$$

$$\therefore \frac{22}{\text{perimeter of } \triangle ABC} = \frac{2}{3}$$

$$\therefore \text{The perimeter of } \triangle ABC = 33 \text{ cm.}$$

28

$$\therefore \frac{a}{X-y+z} = \frac{b}{X+y-z} = \frac{c}{y+z-X}$$

$\therefore$  multiplying the terms of the 1<sup>st</sup> ratio by  $X$  and the 2<sup>nd</sup> by  $y$  and the 3<sup>rd</sup> by  $z$  and adding the antecedents and consequents of the three ratios.

$$\therefore \text{each ratio}$$

$$= \frac{aX+by+cz}{X^2-Xy+Xz+Xy+y^2-z y+zy+z^2-Xz}$$

$$= \frac{aX+by+cz}{X^2+y^2+z^2}$$

29

$$\therefore \frac{2X+y}{X} = \frac{4y+z}{y} = \frac{4z+3X}{z}$$

$\therefore$  adding the antecedents and consequents of the three ratios

$$\therefore \frac{5X+5y+5z}{X+y+z} = \frac{5(X+y+z)}{X+y+z} = 5$$

= one of the given ratios.

$$\therefore \text{each ratio} = 5 \quad \therefore \frac{2X+y}{X} = 5$$

$$\therefore 2X+y = 5X \quad \therefore y = 3X \quad (1)$$

$$\therefore \frac{4z+3X}{z} = 5 \quad \therefore 4z+3X = 5z$$

$$\therefore z = 3X \quad (2)$$

From (1) and (2) :

$$\therefore x : y : z = x : 3x : 3x = 1 : 3 : 3$$

$$\therefore \frac{2x+y+z}{3x-y+2z} = \frac{2x+3x+3x}{3x-3x+6x} = \frac{8x}{6x} = \frac{4}{3}$$

30

$$\therefore \frac{a+2b}{5} = \frac{3b-c}{3} = \frac{c-a}{2} = m$$

$$\therefore a+2b=5m \quad (1), 3b-c=3m \quad (2), c-a=2m \quad (3)$$

Adding (1), (2) and (3)

$$\therefore 5b=10m \quad \therefore b=2m$$

$$\text{from (1) } a+4m=5m \quad \therefore a=m$$

$$\text{from (3) } c-m=2m \quad \therefore c=3m$$

$$\text{1) } a+b-c=m+2m-3m=\text{zero}$$

$$\text{2) } \frac{3b-a}{2b+c} = \frac{6m-m}{4m+3m} = \frac{5m}{7m} = \frac{5}{7}$$

## Answers of Exercise 7

1

$$\text{1) The middle proportion} = \pm \sqrt{3 \times 27}$$

$$= \pm \sqrt{81} = \pm 9$$

$$\text{2) The middle proportion} = \pm \sqrt{9 \times 25} = \pm \sqrt{225} = \pm 15$$

$$\text{3) The middle proportion} = \pm \sqrt{-2 \times -8}$$

$$= \pm \sqrt{16} = \pm 4$$

$$\text{4) The middle proportion} = \pm \sqrt{\frac{1}{5} \times 125}$$

$$= \pm \sqrt{25} = \pm 5$$

$$\text{5) The middle proportion} = \pm \sqrt{2a \times 8a b^2}$$

$$= \pm \sqrt{16a^2 b^2} = \pm 4ab$$

$$\text{6) The middle proportion} = \pm \sqrt{(l^2 - m^2)^2} = \pm (l^2 - m^2)$$

2

1) Let the third proportion be  $c$  :

$$\therefore \frac{6}{12} = \frac{12}{c} \quad \therefore c = \frac{12 \times 12}{6} = 24$$

2) Let the third proportion be  $c$  :

$$\therefore \frac{x^2}{-5x} = \frac{-5x}{c} \quad \therefore c = \frac{-5x \times -5x}{x^2} = 25$$

3) Let the third proportion be  $c$  :

$$\therefore \frac{x^2}{-3x^2} = \frac{-3x^2}{c} \quad \therefore c = \frac{-3x^2 \times -3x^2}{x^2} = 9x^2$$

3

$$\text{1) Let } \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, a = cm^2$$

$$\therefore \frac{a}{c} = \frac{cm^2}{c} = m^2 \quad (1), \frac{b^2}{c^2} = \frac{c^2 m^2}{c^2} = m^2 \quad (2)$$

$$\text{From (1) and (2) : } \therefore \frac{a}{c} = \frac{b^2}{c^2}$$

Another solution :

$$\therefore b^2 = ac \quad \therefore \frac{b^2}{c^2} = \frac{ac}{c^2} = \frac{a}{c} = \text{L.H.S.}$$

$$\text{2) Let } \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, a = cm^2$$

$$\therefore \frac{2a+3b}{2b+3c} = \frac{2cm^2+3cm}{2cm+3c} = \frac{cm(2m+3)}{c(2m+3)} = m \quad (1)$$

$$\therefore \frac{a}{b} = \frac{cm^2}{cm} = m \quad (2)$$

$$\text{From (1) and (2) : } \therefore \frac{2a+3b}{2b+3c} = \frac{a}{b}$$

$$\text{3) Let } \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, a = cm^2$$

$$\therefore \frac{a-b}{b-c} = \frac{cm^2-cm}{cm-c} = \frac{cm(m-1)}{c(m-1)} = m \quad (1)$$

$$\therefore \frac{a+3b}{3c+b} = \frac{cm^2+3cm}{3c+cm} = \frac{cm(m+3)}{c(3+m)} = m \quad (2)$$

$$\text{From (1) and (2) : } \therefore \frac{a-b}{b-c} = \frac{a+3b}{3c+b}$$

$$\text{4) Let } \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, a = cm^2$$

$$\therefore \frac{a^2+b^2}{b^2+c^2} = \frac{c^2 m^4 + c^2 m^2}{c^2 m^2 + c^2} = \frac{c^2 m^2 (m^2 + 1)}{c^2 (m^2 + 1)} = m^2 \quad (1)$$

$$\therefore \frac{a}{c} = \frac{cm^2}{c} = m^2 \quad (2)$$

$$\text{From (1) and (2) : } \therefore \frac{a^2+b^2}{b^2+c^2} = \frac{a}{c}$$

Another solution :

$$\therefore b^2 = ac$$

$$\therefore \frac{a^2+b^2}{b^2+c^2} = \frac{a^2+ac}{ac+c^2} = \frac{a(a+c)}{c(a+c)} = \frac{a}{c} = \text{R.H.S.}$$

$$\text{5) Let } \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, a = cm^2$$

$$\therefore \left( \frac{b-c}{a-b} \right)^2 = \left( \frac{cm-c}{cm^2-cm} \right)^2 = \left( \frac{c(m-1)}{cm(m-1)} \right)^2 = \frac{1}{m^2} \quad (1)$$

$$\therefore \frac{c}{a} = \frac{c}{cm^2} = \frac{1}{m^2} \quad (2)$$

$$\text{From (1) and (2) : } \therefore \left( \frac{b-c}{a-b} \right)^2 = \frac{c}{a}$$

$$\text{6) Let } \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, a = cm^2$$

$$\therefore \frac{a^3+b^3}{b^3+c^3} = \frac{c^3 m^3 + c^3 m}{c^3 m^3 + c^3} = \frac{c^3 m^3 (m^3 + 1)}{c^3 (m^3 + 1)} = m^3 \quad (1)$$

$$\therefore \frac{a^2}{c^2} = \frac{c^2 m^4}{c \times c m} = m^3 \quad (2)$$

$$\text{From (1) and (2):} \quad \therefore \frac{a^3 + b^3}{b^3 + c^3} = \frac{a^2}{c^2}$$

$$[7] \text{ Let } \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, \quad a = cm^2$$

$$\begin{aligned} \therefore \frac{a^3 - 4b^3}{b^3 - 4c^3} &= \frac{c^3 m^6 - 4c^3 m^3}{c^3 m^3 - 4c^3} = \frac{c^3 m^3 (m^3 - 4)}{c^3 (m^3 - 4)} = m^3 \end{aligned} \quad (1)$$

$$\frac{b^3}{c^3} = \frac{c^3 m^3}{c^3} = m^3 \quad (2)$$

$$\text{From (1) and (2):} \quad \therefore \frac{a^3 - 4b^3}{b^3 - 4c^3} = \frac{b^3}{c^3}$$

$$[8] \text{ Let } \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, \quad a = cm^2$$

$$\begin{aligned} \therefore \frac{2c^2 - 3b^2}{2b^2 - 3a^2} &= \frac{2c^2 - 3c^2 m^2}{2c^2 m^2 - 3c^2 m^4} \\ &= \frac{c^2 (2 - 3m^2)}{c^2 m^2 (2 - 3m^2)} = \frac{1}{m^2} \end{aligned} \quad (1)$$

$$\therefore \frac{c}{a} = \frac{c}{cm^2} = \frac{1}{m^2} \quad (2), \quad \frac{c^2}{b^2} = \frac{c^2}{c^2 m^2} = \frac{1}{m^2} \quad (3)$$

$$\text{From (1), (2) and (3):} \quad \therefore \frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{c}{a} = \frac{c^2}{b^2}$$

$$\text{Another solution:} \quad \therefore b^2 = ac$$

$$\therefore \frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{2c^2 - 3ac}{2ac - 3a^2} = \frac{c(2c - 3a)}{a(2c - 3a)} = \frac{c}{a}$$

$$\therefore \frac{c^2}{b^2} = \frac{c^2}{ac} = \frac{c}{a}$$

$$\therefore \frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{c}{a} = \frac{c^2}{b^2}$$

$$[9] \text{ Let } \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, \quad a = cm^2$$

$$\begin{aligned} \therefore \frac{a^2 + ab + b^2}{b^2 + bc + c^2} &= \frac{c^2 m^4 + c^2 m^3 + c^2 m^2}{c^2 m^2 + c^2 m + c^2} \\ &= \frac{c^2 m^2 (m^2 + m + 1)}{c^2 (m^2 + m + 1)} = m^2 \end{aligned} \quad (1)$$

$$\frac{a^2 - b^2}{b^2 - c^2} = \frac{c^2 m^4 - c^2 m^2}{c^2 m^2 - c^2} = \frac{c^2 m^2 (m^2 - 1)}{c^2 (m^2 - 1)} = m^2 \quad (2)$$

$$\text{From (1) and (2):} \quad \therefore \frac{a^2 + ab + b^2}{b^2 + bc + c^2} = \frac{a^2 - b^2}{b^2 - c^2}$$

$$[10] \text{ Let } \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, \quad a = cm^2$$

$$\therefore \frac{2a}{c} = \frac{2cm^2}{c} = 2m^2 \quad (1)$$

$$\therefore \frac{a^2}{b^2} + \frac{b^2}{c^2} = m^2 + m^2 = 2m^2 \quad (2)$$

$$\text{From (1) and (2):} \quad \therefore \frac{2a}{c} = \frac{a^2}{b^2} + \frac{b^2}{c^2}$$

$$\text{Another solution:} \quad \therefore b^2 = ac$$

$$\therefore \frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{a^2}{ac} + \frac{ac}{c^2} = \frac{a}{c} + \frac{a}{c} = \frac{2a}{c} = \text{R.H.S.}$$

$$[11] \text{ Let } \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, \quad a = cm^2$$

$$\begin{aligned} \therefore \frac{a+b+c}{a^{-1}+b^{-1}+c^{-1}} &= \frac{cm^2+cm+c}{c^{-1}m^{-2}+c^{-1}m^{-1}+c^{-1}} \\ &= \frac{c(m^2+m+1)}{c^{-1}m^{-2}(1+m+m^2)} \\ &= c \times c m^2 = c^2 m^2 = b^2 \end{aligned}$$

$$[12] \text{ Let } \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, \quad a = cm^2$$

$$\therefore \frac{ac}{b(b+c)} = \frac{cm^2 \times c}{cm(cm+cm)} = \frac{c^2 m^2}{c^2 m(m+1)} = \frac{m}{m+1} \quad (1)$$

$$\therefore \frac{a}{a+b} = \frac{cm^2}{cm^2+cm} = \frac{cm^2}{cm(m+1)} = \frac{m}{m+1} \quad (2)$$

$$\text{From (1) and (2):} \quad \therefore \frac{ac}{b(b+c)} = \frac{a}{a+b}$$

$$\text{Another solution:} \quad \therefore b^2 = ac$$

$$\begin{aligned} \therefore \frac{ac}{b(b+c)} &= \frac{ac}{b^2+b^2} = \frac{ac}{a+b^2} = \frac{ac}{a+ac} \\ &= \frac{a}{a+b} = \text{R.H.S.} \end{aligned}$$

4

$$[1] \text{ Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = dm, \quad b = dm^2, \quad a = dm^3$$

$$\therefore \frac{a-2b}{b-2c} = \frac{dm^3-2dm^2}{dm^2-2dm} = \frac{dm^2(m-2)}{dm(m-2)} = m \quad (1)$$

$$\therefore \frac{3b+4c}{3c+4d} = \frac{3dm^2+4dm}{3dm+4d} = \frac{dm(3m+4)}{d(3m+4)} = m \quad (2)$$

$$\text{From (1) and (2):} \quad \therefore \frac{a-2b}{b-2c} = \frac{3b+4c}{3c+4d}$$

$$[2] \text{ Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = dm, \quad b = dm^2, \quad a = dm^3$$

$$\therefore \frac{3a+5c}{3b+5d} = \frac{3dm^3+5dm}{3dm^2+5d} = \frac{dm(3m^2+5)}{d(3m^2+5)} = m \quad (1)$$

$$\therefore \frac{a-4c}{b-4d} = \frac{dm^3-4dm}{dm^2-4d} = \frac{dm(m^2-4)}{d(m^2-4)} = m \quad (2)$$

$$\text{From (1) and (2):} \quad \therefore \frac{3a+5c}{3b+5d} = \frac{a-4c}{b-4d}$$

$$[3] \text{ Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = dm, \quad b = dm^2, \quad a = dm^3$$

$$\therefore \frac{3a-5c}{a-b+c} = \frac{3dm^3-5dm}{d^3-dm^2+dm}$$

$$= \frac{dm(3m^2-5)}{dm(m^2-m+1)} = \frac{3m^2-5}{m^2-m+1}$$

$$\therefore \frac{3b-5d}{b-c+d} = \frac{3dm^2-5d}{d^2-dm+d}$$

$$= \frac{d(3m^2-5)}{d(m^2-m+1)} = \frac{3m^2-5}{m^2-m+1}$$

$$\text{From (1) and (2)}: \therefore \frac{3a-5c}{a-b+c} = \frac{3b-5d}{b-c+d}$$

$$\text{[4] Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = dm, \quad b = dm^2, \quad a = dm^3$$

$$\therefore \frac{a-d}{a+b+c} = \frac{dm^3-d}{dm^3+dm^2+dm}$$

$$= \frac{d(m^3-1)}{dm(m^2+m+1)} = \frac{(m-1)(m^2+m+1)}{m(m^2+m+1)}$$

$$= \frac{m-1}{m}$$

$$\therefore \frac{a-2b+c}{a-b} = \frac{dm^3-2dm^2+dm}{dm^3-dm^2}$$

$$= \frac{dm(m^2-2m+1)}{dm^2(m-1)} = \frac{(m-1)^2}{m(m-1)} = \frac{m-1}{m}$$

$$\text{From (1) and (2)}: \therefore \frac{a-d}{a+b+c} = \frac{a-2b+c}{a-b}$$

$$\text{[5] Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = dm, \quad b = dm^2, \quad a = dm^3$$

$$\therefore \frac{c^2-d^2}{a-c} = \frac{d^2m^2-d^2}{dm^3-dm} = \frac{d^2(m^2-1)}{dm(m^2-1)} = \frac{d}{m}$$

$$\therefore \frac{bd}{a} = \frac{d^2m^2}{dm^3} = \frac{d}{m}$$

$$\text{From (1) and (2)}: \therefore \frac{c^2-d^2}{a-c} = \frac{bd}{a}$$

$$\text{[6] Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = dm, \quad b = dm^2, \quad a = dm^3$$

$$\therefore \frac{a^2-3c^2}{b^2-3d^2} = \frac{d^2m^6-3d^2m^2}{d^2m^4-3d^2} = \frac{d^2m^2(m^4-3)}{d^2(m^4-3)} = m^2$$

$$\therefore \frac{b}{d} = \frac{dm^2}{d} = m^2$$

$$\text{From (1) and (2)}: \therefore \frac{a^2-3c^2}{b^2-3d^2} = \frac{b}{d}$$

$$\text{[7] Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = dm, \quad b = dm^2, \quad a = dm^3$$

$$\therefore \frac{ab-cd}{b^2-c^2} = \frac{dm^3 \times dm^2 - dm \times d}{d^2m^4 - d^2m^2} = \frac{d^2m^5 - d^2m}{d^2m^2(m^2-1)}$$

$$= \frac{d^2m(m^4-1)}{d^2m^2(m^2-1)} = \frac{(m^2-1)(m^2+1)}{m(m^2-1)} = \frac{m^2+1}{m} \quad (1)$$

$$\therefore \frac{a+c}{b} = \frac{dm^3+dm}{dm^2} = \frac{dm(m^2+1)}{dm^2} = \frac{m^2+1}{m} \quad (2)$$

$$\text{From (1) and (2)}: \therefore \frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$$

$$\text{[8] Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = dm, \quad b = dm^2, \quad a = dm^3$$

$$\therefore \frac{a}{b+d} = \frac{dm^3}{dm^2+d} = \frac{dm^3}{d(m^2+1)} = \frac{m^3}{m^2+1} \quad (1)$$

$$\therefore \frac{c^3}{c^2d+d^3} = \frac{d^3m^3}{d^2m^2 \times d + d^3} = \frac{d^3m^3}{d^3(m^2+1)} = \frac{m^3}{m^2+1} \quad (2)$$

$$\text{From (1) and (2)}: \therefore \frac{a}{b+d} = \frac{c^3}{c^2d+d^3}$$

$$\text{[9] Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = dm, \quad b = dm^2, \quad a = dm^3$$

$$\therefore \frac{a^2+b^2+c^2}{b^2+c^2+d^2} = \frac{d^2m^6+d^2m^4+d^2m^2}{d^2m^4+d^2m^2+d^2} = \frac{d^2m^2(m^4+m^2+1)}{d^2(m^4+m^2+1)} = m^2 \quad (1)$$

$$\therefore \frac{ac}{bd} = \frac{dm^3 \times dm}{dm^2 \times d} = m^2 \quad (2)$$

$$\text{From (1) and (2)}: \therefore \frac{a^2+b^2+c^2}{b^2+c^2+d^2} = \frac{ac}{bd}$$

$$\text{[10] Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = dm, \quad b = dm^2, \quad a = dm^3$$

$$\therefore \frac{2a+3d}{3a-4d} = \frac{2dm^3+3d}{3dm^3-4d} = \frac{d(2m^3+3)}{d(3m^3-4)} = \frac{2m^3+3}{3m^3-4} \quad (1)$$

$$\therefore \frac{2a^3+3b^3}{3a^3-4b^3} = \frac{2d^3m^9+3d^3m^6}{3d^3m^9-4d^3m^6} = \frac{2d^3m^3(2m^3+3)}{3d^3m^3(3m^3-4)} = \frac{2m^3+3}{3m^3-4} \quad (2)$$

$$\text{From (1) and (2)}: \therefore \frac{2a+3d}{3a-4d} = \frac{2a^3+3b^3}{3a^3-4b^3}$$

$$\text{[11] Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m \text{ where } m > 0$$

$$\therefore c = dm, \quad b = dm^2, \quad a = dm^3$$

$$\therefore \frac{a+5b}{b+5c} = \frac{dm^3+5dm^2}{dm^2+5dm} = \frac{d(m^2(m+5))}{dm(m+5)} = m \quad (1)$$

$$\sqrt{\frac{b}{d}} = \sqrt{\frac{d m^2}{d}} = \sqrt{m^2} = m$$

$$\text{From (1) and (2): } \therefore \frac{a+5b}{b+5c} = \sqrt{\frac{b}{d}}$$

$$(12) \text{ Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = d m, b = d m^2, a = d m^3$$

$$\therefore \sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \sqrt[3]{\frac{5d^3 m^9 - 3d^3 m^3}{5d^3 m^6 - 3d^3}}$$

$$= \sqrt[3]{\frac{d^3 m^3 (5m^6 - 3)}{d^3 (5m^6 - 3)}} = \sqrt[3]{m^3} = m$$

$$\therefore \frac{a+c}{b+d} = \frac{d m^3 + d m}{d m^2 + d} = \frac{d m (m^2 + 1)}{d (m^2 + 1)} = m$$

$$\text{From (1) and (2): } \therefore \sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \frac{a+c}{b+d}$$

$$(13) \text{ Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

where  $m$  is a positive real number.

$$\therefore c = d m, b = d m^2, a = d m^3$$

$$\therefore \left( \frac{a+b}{b+c} \right)^3 = \left( \frac{d m^3 + d m^2}{d m^2 + d m} \right)^3 = \left( \frac{d m^2 (m+1)}{d m (m+1)} \right)^3 = m^3$$

$$\therefore \frac{a}{d} = \frac{d m^3}{d} = m^3$$

$$\text{From (1) and (2): } \therefore \left( \frac{a+b}{b+c} \right)^3 = \frac{a}{d}$$

$$(14) \text{ Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = d m, b = d m^2, a = d m^3$$

$$\therefore \frac{a^2 + d^2}{c(a+c)} = \frac{d^2 m^6 + d^2}{d m (d m^3 + d m)} = \frac{d^2 (m^6 + 1)}{d^2 m^2 (m^2 + 1)}$$

$$= \frac{(m^2 + 1)(m^4 - m^2 + 1)}{m^2 (m^2 + 1)} = \frac{m^4 - m^2 + 1}{m^2}$$

$$\therefore \frac{b}{d} + \frac{d}{b} - 1 = \frac{d m^2}{d} + \frac{d}{d m^2} - 1 = m^2 + \frac{1}{m^2} - 1$$

$$= \frac{m^4 + 1 - m^2}{m^2} = \frac{m^4 - m^2 + 1}{m^2}$$

$$\text{From (1) and (2): } \therefore \frac{a^2 + d^2}{c(a+c)} = \frac{b}{d} + \frac{d}{b} - 1$$

4

$$(1) \text{ c } (2) \text{ c } (3) \text{ d } (4) \text{ c } (5) \text{ b }$$

$$(6) \text{ c } (7) \text{ c } (8) \text{ c } (9) \text{ c } (10) \text{ c }$$

(2)

6

$\therefore a, 3, 9, b$  are in continued proportion

$$\therefore \frac{a}{3} = \frac{3}{9} = \frac{9}{b}$$

$$\therefore a = \frac{3 \times 3}{9} = 1, b = \frac{9 \times 9}{3} = 27$$

7

$\therefore 3, \ell, 12, m$  are in continued proportion

$$\therefore \frac{3}{\ell} = \frac{\ell}{12} = \frac{12}{m}$$

$$\therefore \ell = \pm \sqrt{3 \times 12} = \pm \sqrt{36} = \pm 6$$

$$m = \frac{12 \times 12}{\pm 6} = \pm 24$$

8

$$(1) \text{ 7 } (2) \text{ 4 } (r-1)^2 (3) \text{ 3 } (3X+5y) (4) \text{ 4 } \frac{4}{25}$$

$$(5) \text{ m}^4 (6) \text{ a } (7) \text{ 3 } (8) \text{ -10 }$$

9

Let the number be  $X$

$$\therefore \frac{3-X}{7-X} = \frac{7-X}{19-X}$$

$$\therefore (3-X)(19-X) = (7-X)^2$$

$$\therefore 57 - 22X + X^2 = 49 - 14X + X^2$$

$$\therefore 57 - 49 = -14X + 22X$$

$$\therefore 8 = 8X$$

$$\therefore X = 1$$

$\therefore$  The number is 1

10

$$\therefore a = 4c = 4$$

$$\therefore a = 4, c = 1$$

$\therefore b$  is the middle proportion between  $a$  and  $c$

$$\therefore b^2 = ac$$

$$\therefore b^2 = 4 \times 1 = 4$$

$$\therefore a^2 + b^2 + c^2 = 4^2 + 4 + 1^2 = 16 + 4 + 1 = 21$$

11

$$\text{Let } \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = d m, b = d m^2, a = d m^3$$

$$\therefore \frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{d^2} = m^2 + m^2 + m^2 = 3m^2 \quad (1)$$

$$\frac{a}{c} + \frac{b}{d} + \frac{a c}{b d} = \frac{d m^3}{d m} + \frac{d m^2}{d} + \frac{d m^3 \times d m}{d m^2 \times d}$$

$$= m^2 + m^2 + m^2 = 3m^2 \quad (2)$$

$$\text{From (1) and (2): } \therefore \frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{d^2} = \frac{a}{c} + \frac{b}{d} + \frac{a c}{b d}$$

12

$$\therefore y^2 = Xz$$

 $\therefore X, y, z$  are proportional

$$\therefore \frac{X}{y} = \frac{y}{z} = m$$

$$\therefore y = zm, X = zm^2$$

$$\therefore \frac{X(X-y)}{y(y-z)} = \frac{zm^2(zm^2 - zm)}{zm(zm - z)} = \frac{zm^3 \times zm(m-1)}{zm \times z(m-1)}$$

$$= \frac{z^2 m^3 (m-1)}{z^2 m (m-1)} = m^2 \quad (1)$$

$$\therefore \frac{y^2}{z^2} = \frac{zm^2}{z} = m^2 \quad (2)$$

$$\text{From (1) and (2):} \quad \therefore \frac{X(X-y)}{y(y-z)} = \frac{y^2}{z^2}$$

13

$$\therefore b^2 = ac \quad \therefore \frac{a}{b} = \frac{b}{c} \quad \therefore c^2 = bd \quad \therefore \frac{b}{c} = \frac{c}{d}$$

$$\therefore \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = dm, b = dm^2, a = dm^3$$

$$\therefore \frac{2a+3d}{3a-4d} = \frac{2dm^3+3d}{3dm^3-4d} = \frac{d(2m^3+3)}{d(3m^3-4)} = \frac{2m^3+3}{3m^3-4} \quad (1)$$

$$\begin{aligned} \therefore \frac{2a^3+3b^3}{3a^3-4b^3} &= \frac{2d^3m^9+3d^3m^6}{3d^3m^9-4d^3m^6} \\ &= \frac{d^3m^6(2m^3+3)}{d^3m^6(3m^3-4)} = \frac{2m^3+3}{3m^3-4} \quad (2) \end{aligned}$$

$$\text{From (1) and (2):} \quad \therefore \frac{2a+3d}{3a-4d} = \frac{2a^3+3b^3}{3a^3-4b^3}$$

14

$$\therefore \frac{a^2+b^2}{b^2} = \frac{b^2+c^2}{c^2} \quad \therefore a^2c^2+b^2c^2=b^4+b^2c^2$$

$$\therefore a^2c^2=b^4 \quad \therefore ac=b^2$$

 $\therefore b$  is the middle proportion between  $a$  and  $c$ 

15

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = dm, b = dm^2, a = dm^3$$

$$\therefore (b+c)^2 = (dm^2+dm)^2 = (dm(m+1))^2 = d^2m^2(m+1)^2 \quad (1)$$

$$\begin{aligned} \therefore (a+b)(c+d) &= (dm^3+dm^2)(dm+d) \\ &= dm^2(m+1) \times d(m+1) = d^2m^2(m+1)^2 \quad (2) \end{aligned}$$

$$\therefore (b+c)^2 = (a+b)(c+d)$$

 $\therefore (b+c)$  is the middle proportion between  $(a+b)$  and  $(c+d)$ 

16

$$\text{Let } \frac{5a}{6b} = \frac{6b}{7c} = \frac{7c}{8d} = m \text{ where } m > 0$$

$$\therefore 7c = 8dm, 6b = 8dm^2, 5a = 8dm^3$$

$$\therefore \sqrt[3]{\frac{5a}{8d}} = \sqrt[3]{\frac{8dm^3}{8d}} = \sqrt[3]{m^3} = m \quad (1)$$

$$\begin{aligned} \therefore \sqrt{\frac{5a+6b}{7c+8d}} &= \sqrt{\frac{8dm^3+8dm^2}{8dm+8d}} \\ &= \sqrt{\frac{8dm^2(m+1)}{8d(m+1)}} = \sqrt{m^2} = m \quad (2) \end{aligned}$$

$$\text{From (1) and (2):} \quad \sqrt[3]{\frac{5a}{8d}} = \sqrt{\frac{5a+6b}{7c+8d}}$$

17

$$\therefore \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, a = cm^2$$

$$\begin{aligned} \therefore \frac{a^4+b^4+c^4}{a^4+b^4+c^4} &= \frac{c^4m^8+c^4m^4+c^4}{c^4m^8+c^4m^4+c^4} \\ &= \frac{c^4m^8(1+m^4+m^{-8})}{c^4(m^8+m^4+1)} = c^8m^8 \end{aligned}$$

$$\therefore b^8 = c^8m^8$$

$$\text{From (1) and (2):} \quad \therefore \frac{a^4+b^4+c^4}{a^4+b^4+c^4} = b^8$$

18

$$\text{Let } \frac{X}{y} = \frac{y}{z} = m \quad \therefore y = zm, X = zm^2$$

$$\therefore X+y = 15 \quad \therefore zm+zm^2 = 15$$

$$\therefore zm(1+m) = 15 \quad \therefore 1+m = \frac{15}{zm} \quad (1)$$

$$\therefore y+z = 22.5 \quad \therefore zm+z = 22.5$$

$$\therefore z(m+1) = 22.5 \quad \therefore m+1 = \frac{22.5}{z} \quad (2)$$

$$\text{From (1) and (2):} \quad \therefore \frac{15}{zm} = \frac{22.5}{z}$$

$$\therefore 22.5m = 15 \quad \therefore m = \frac{15}{22.5} = \frac{2}{3}$$

$$\therefore X:y = 2:3$$

19

$$\text{Let } \frac{m(\angle A)}{m(\angle B)} = \frac{m(\angle B)}{m(\angle C)} = e$$

$$\therefore m(\angle B) = m(\angle C) \times e$$

$$\therefore m(\angle A) = m(\angle C) \times e^2$$

$$\therefore m(\angle A) + m(\angle B) + m(\angle C) = 180^\circ$$

$$\therefore m(\angle C) \times e^2 + m(\angle C) \times e + m(\angle C) = 180^\circ$$

$$\therefore 60^\circ e^2 + 60^\circ e + 60^\circ = 180^\circ$$

$$\begin{aligned} \therefore e^2 + e + 1 &= 3 & \therefore e^2 + e - 2 &= 0 \\ \therefore (e + 2)(e - 1) &= 0 & \therefore e &= -2 \text{ (refused) or } e = 1 \\ \therefore m(\angle A) &= 60^\circ \times 1^2 = 60^\circ \\ \therefore m(\angle B) &= 60^\circ \times 1 = 60^\circ \end{aligned}$$

20

$$\begin{aligned} \therefore \frac{a}{b} &= \frac{b}{c} = \frac{c}{d} = 2 & \therefore c &= 2d, b = 4d, a = 8d \\ \therefore aX^2 - 2bX + c &= 0 & \therefore 8dX^2 - 8dX + 2d &= 0 \\ \text{dividing by } 2d & & \therefore 4X^2 - 4X + 1 &= 0 \\ \therefore (2X - 1)^2 &= 0 & \therefore 2X - 1 &= 0 \\ \therefore X &= \frac{1}{2} & \therefore \text{The S.S.} &= \left\{ \frac{1}{2} \right\} \end{aligned}$$

21

$\therefore$  The number 5 is the middle proportion between  $X$  and  $y$

$$\therefore Xy = 25$$

Let the middle proportion between

$$\left(x + \frac{1}{y}\right), \left(y + \frac{1}{x}\right) \text{ be } z$$

$$\begin{aligned} \therefore z^2 &= \left(x + \frac{1}{y}\right) \left(y + \frac{1}{x}\right) = xy + 1 + 1 + \frac{1}{xy} \\ &= xy + \frac{1}{xy} + 2 \end{aligned}$$

$$\text{and from (1)} \quad \therefore z^2 = 25 + \frac{1}{25} + 2 = 27.04$$

$$\therefore z = \pm \sqrt{27.04} = \pm 5.2$$

### Answers of Exercise 8

1

1  $my$  where  $m \neq 0$

2  $\frac{1}{x^2}$

3  $\frac{y_1}{y_2}$

4  $\frac{x_2}{x_1}$

5  $x$

6  $x$

7  $y$

8  $\frac{1}{y}$

9 3

10 5

11  $\frac{1}{2}$

12  $\frac{3}{2}$

13 directly, inversely

2

$$\therefore y \propto x$$

$$\therefore \frac{y_1}{y_2} = \frac{x_1}{x_2}$$

$$\therefore \frac{20}{40} = \frac{7}{x_2}$$

$$\therefore x_2 = \frac{40 \times 7}{20} = 14$$

3

$$\therefore a \propto \frac{1}{b}$$

$$\therefore \frac{a_1}{a_2} = \frac{b_2}{b_1}$$

1  $\frac{12}{a_2} = \frac{1.5}{8}$

$$\therefore a_2 = \frac{8 \times 12}{1.5} = 64$$

2  $\frac{12}{2} = \frac{b_2}{8}$

$$\therefore b_2 = \frac{8 \times 12}{2} = 48$$

4

1  $\therefore y \propto X$

$$\therefore y = mX$$

$$\therefore 14 = 42m$$

$$\therefore m = \frac{1}{3}$$

$$\therefore y = \frac{1}{3}X \text{ (The relation between } X, y)$$

2 As  $X = 60$

$$\therefore y = \frac{1}{3} \times 60 = 20$$

5

1  $\therefore y \propto \frac{1}{X}$

$$\therefore Xy = m$$

$$\therefore 3 \times 2 = m$$

$$\therefore m = 6$$

$$\therefore Xy = 6 \text{ (The relation between } X, y)$$

2 As  $X = 1.5$

$$\therefore y = \frac{6}{1.5} = 4$$

6

$$\therefore y \propto \frac{1}{X}$$

$$\therefore Xy = m$$

$$\therefore m = 3 \times 10 = 30$$

$$\therefore Xy = 30$$

$$\therefore y = \frac{30}{X}$$

$$\text{As } X = 1$$

$$\therefore y = \frac{30}{1} = 30$$

$$\text{As } X = 2$$

$$\therefore y = \frac{30}{2} = 15$$

$$\text{As } X = 3$$

$$\therefore y = \frac{30}{3} = 10$$

$$\text{As } X = 4$$

$$\therefore y = \frac{30}{4} = 7.5$$

$$\text{As } X = 5$$

$$\therefore y = \frac{30}{5} = 6$$

7

$$\therefore y \propto X^2$$

$$\therefore y = mX^2$$

$$\therefore 4 = m(3)^2$$

$$\therefore m = \frac{4}{9}$$

$$\therefore y = \frac{4}{9}X^2 \text{ (The relation between } X \text{ and } y)$$

$$\text{As } X = 9$$

$$\therefore y = \frac{4}{9} \times 9^2 = 36$$

8

$$\therefore y \propto X^3$$

$$\therefore y = mX^3$$

$$\therefore 64 = m(2)^3 \quad \therefore m = 8$$

$$\therefore y = 8x^3 \text{ (The relation between } x \text{ and } y)$$

$$\text{As } x = \frac{1}{2} \quad \therefore y = 8\left(\frac{1}{2}\right)^3 = 1$$

9

$$\therefore y \propto \frac{1}{\sqrt{x}} \quad \therefore \frac{y_1}{y_2} = \frac{\sqrt{x_2}}{\sqrt{x_1}}$$

$$\therefore \frac{2}{y_2} = \frac{\sqrt{32}}{\sqrt{16}} \quad \therefore y_2 = \frac{2 \times 4}{4\sqrt{2}} = \sqrt{2}$$

10

$$\therefore y^2 \propto x^3 \quad \therefore y^2 = m x^3 \quad \therefore y = 3 \text{ As } x = 2$$

$$\therefore 9 = 8m \quad \therefore m = \frac{9}{8} \quad \therefore y^2 = \frac{9}{8} x^3$$

11

$$\therefore y^2 \propto \frac{1}{\sqrt[3]{x}} \quad \therefore \left(\frac{y_1}{y_2}\right)^2 = \sqrt[3]{\frac{x_2}{x_1}}$$

$$\therefore \left(\frac{3}{1.5}\right)^2 = \sqrt[3]{\frac{x_2}{8}} \quad \therefore 4 = \sqrt[3]{\frac{x_2}{2}}$$

$$\therefore \sqrt[3]{x_2} = 8 \quad \therefore x_2 = 512$$

12

$$\therefore y \propto (x+1) \quad \therefore y = m(x+1)$$

$$\therefore y = 2, x = 3 \quad \therefore 2 = m(3+1)$$

$$\therefore m = \frac{1}{2} \quad \therefore y = \frac{1}{2}(x+1)$$

13

$$\therefore \frac{5x-3y}{3x+5y} = 1 \quad \therefore 5x-3y = 3x+5y$$

$$\therefore 2x = 8y \quad \therefore x = 4y$$

$$\therefore y = \frac{1}{4}x \quad \therefore y \propto x$$

14

$$\therefore \frac{a+2b}{6} = \frac{b+3c}{3} \quad \therefore 3a+6b = 6b+18c$$

$$\therefore 3a = 18c \quad \therefore a = 6c \quad \therefore a \propto c$$

15

$$\therefore \frac{21x-y}{7x-z} = \frac{y}{z} \quad \therefore 21xz - zy = 7xy - zy$$

$$\therefore 21xz = 7xy \quad \therefore 3z = y \quad \therefore y \propto z$$

16

$$\therefore x^2y^2 - 6xy + 9 = 0 \quad \therefore (xy-3)^2 = 0$$

$$\therefore xy = 3 \quad \therefore y \propto \frac{1}{x}$$

17

$$\therefore 4a^2 - 12ab + 9b^2 = 0$$

$$\therefore (2a-3b)^2 = 0 \quad \therefore 2a-3b = 0 \quad \therefore 2a = 3b$$

$$\therefore a = \frac{3}{2}b \quad \therefore a \propto b$$

18

$$\therefore x^4y^2 - 14x^2y + 49 = 0$$

$$\therefore (x^2y-7)^2 = 0 \quad \therefore x^2y-7 = 0$$

$$\therefore x^2y = 7 \quad \therefore y \propto \frac{1}{x^2}$$

19

$$\therefore (4x+7y) \propto (x+2y)$$

$$\therefore 4x+7y = m(x+2y)$$

$$\therefore 4x+7y = mx+2my$$

$$\therefore 7y-2my = mx-4x$$

$$\therefore y(7-2m) = x(m-4)$$

$$\therefore y = \frac{m-4}{7-2m}x$$

$$\text{putting } \frac{m-4}{7-2m} = k \in \mathbb{R}^*$$

$$\therefore y = kx \quad \therefore y \propto x$$

20

$$\therefore \left(\frac{a}{y} - \frac{a}{x}\right) \propto x-y \quad \therefore \frac{a}{y} - \frac{a}{x} = m(x-y)$$

$$\therefore \frac{aX-ay}{xy} = m(x-y) \quad \therefore \frac{a(X-y)}{xy} = m(x-y)$$

$$\therefore xy = \frac{a}{m} \text{ (constant)} \quad \therefore x \text{ varies inversely as } y$$

21

The first table represents an inverse variation

 because :  $3 \times 20 = 60$  ,  $5 \times 12 = 60$  ,  $4 \times 15 = 60$ 

$$, 6 \times 10 = 60 \quad \text{i.e. } xy = m$$

The second table represents a direct variation

$$\text{because : } \frac{9}{2} = \frac{18}{4} = \frac{54}{12} = \frac{72}{16} \quad \text{i.e. } \frac{y}{x} = m$$

The third table represents a direct variation

$$\text{because : } \frac{9}{5} = \frac{18}{10} = \frac{27}{15} = \frac{45}{25} \quad \text{i.e. } \frac{y}{x} = m$$

The fourth table does not represent a direct variation or inverse variation because :

$$3 \times 6 \neq -18 \times 1 \text{ or } \frac{6}{3} \neq \frac{-9}{-2}$$

i.e.  $Xy \neq m$   $\therefore$  The variation is not inverse

or  $\frac{y}{x} \neq m$   $\therefore$  The variation is not direct

22

1 The variation is inverse.

2  $\because y \propto \frac{1}{x}$   $\therefore yx = m$   $\therefore m = 12$

3 As  $X = 3$   $\therefore 3y = 12$   $\therefore y = 4$

4 As  $y = 2\frac{2}{5}$   $\therefore (2\frac{2}{5})X = 12$   
 $\therefore \frac{12}{5}X = 12$   $\therefore X = 12 \times \frac{5}{12} = 5$

23

1 Direct variation

2  $\because y \propto X$   $\therefore \frac{y}{X} = m$   $\therefore m = 12$

$\therefore \frac{a}{2} = 12$   $\therefore a = 24$

$\therefore \frac{36}{b} = 12$   $\therefore b = 3$

24

$\because y = z + 5, z \propto \frac{1}{x}$   $\therefore z = \frac{m}{x}$

$\therefore y = \frac{m}{x} + 5$

At  $y = 6, X = 2$

$\therefore 6 = \frac{m}{2} + 5$   $\therefore 1 = \frac{m}{2}$   $\therefore m = 2$

$\therefore y = \frac{2}{x} + 5$

At  $X = 1$   $\therefore y = \frac{2}{1} + 5 = 7$

25

1  $\because y = a + b$   $b \propto X$   $\therefore b = mX$

$\therefore y = a + mX$  At  $y = 3, X = 0$

$\therefore 3 = a + m \times 0$   $\therefore a = 3$

$\therefore y = 3 + mX$  At  $y = 5, X = 3$

$\therefore 5 = 3 + m \times 3$   $\therefore m = \frac{2}{3}$

$\therefore y = 3 + \frac{2}{3}X$  At  $X = 7$

2  $\therefore y = 3 + \frac{2}{3} \times 7 = 7\frac{2}{3}$

26

$\because y = a - 9$   $y \propto \frac{1}{x^2}$   $\therefore y = \frac{m}{x^2}$

$\therefore \frac{m}{x^2} = a - 9$   $\therefore m = x^2(a - 9)$

$\therefore a = 18$  as  $X = \frac{2}{3}$   $\therefore m = \frac{4}{9}(18 - 9)$

$\therefore m = \frac{4}{9} \times 9 = 4$   $\therefore y = \frac{4}{x^2}$

As  $X = 1$   $\therefore y = 4$

27

1  $\because y = 2 + a$   $a \propto \frac{1}{x}$   $\therefore a = \frac{m}{x}$

At  $a = 5$   $\therefore X = 2$

$\therefore 5 = \frac{m}{2}$   $\therefore m = 10$

$\therefore a = \frac{10}{x}$   $\therefore y = 2 + \frac{10}{x}$

2 At  $X = 5$   $\therefore y = 2 + \frac{10}{5} = 4$

28

$\because X = l + 9$   $\therefore l \propto y$   $\therefore l = my$

$\therefore X = my + 9$  As  $X = 24, y = 5$

$\therefore 24 = 5m + 9$   $\therefore 5m = 15$

$\therefore m = 3$   $\therefore l = 3y$

As  $l = 12$   $\therefore 12 = 3y$   $\therefore y = 4$

29

1 d 2 d 3 d 4 c

5 b 6 d 7 c 8 b

9 b 10 a 11 d 12 c

13 a 14 e 15 d

30

$\because h \propto \frac{1}{r^2}$   $\therefore \frac{h_1}{h_2} = \frac{r_2^2}{r_1^2}$

$\therefore \frac{27}{h_2} = \frac{(15.75)^2}{(10.5)^2}$   $\therefore h_2 = \frac{27 \times (10.5)^2}{(15.75)^2} = 12 \text{ cm.}$

31

$\because d \propto t$   $\therefore \frac{d_1}{d_2} = \frac{t_1}{t_2}$

$\therefore \frac{150}{d_2} = \frac{6}{10}$   $\therefore d_2 = \frac{150 \times 10}{6} = 250 \text{ km.}$

32

$$\begin{aligned} \therefore W &\propto R & \therefore \frac{W_1}{W_2} &= \frac{R_1}{R_2} \\ \therefore \frac{14}{W_2} &= \frac{84}{144} & \therefore W_2 &= \frac{14 \times 144}{84} = 24 \text{ kg.} \end{aligned}$$

33

$$\begin{aligned} \therefore n &\propto \frac{1}{X} & \therefore \frac{n_1}{n_2} &= \frac{X_2}{X_1} \\ \therefore \frac{4}{n_2} &= \frac{8}{6} & \therefore n_2 &= \frac{4 \times 6}{8} = 3 \text{ hours.} \end{aligned}$$

34

$$\begin{aligned} \therefore d &\propto t^2 & \therefore \frac{d_1}{d_2} &= \frac{t_1^2}{t_2^2} \\ \therefore \frac{81}{144} &= \frac{1}{t_2^2} & \therefore t_2^2 &= \frac{144 \times \frac{1}{16}}{\frac{81}{16}} = \frac{16}{9} \\ \therefore t_2 &= \frac{4}{3} = 1\frac{1}{3} \text{ hour.} \end{aligned}$$

35

$$\begin{aligned} \therefore v &\propto \frac{1}{r} & \therefore \frac{v_1}{v_2} &= \frac{r_2}{r_1} \\ \therefore \frac{5}{v_2} &= \frac{(2.5)^2}{3^2} & \therefore v_2 &= \frac{5 \times 3^2}{(2.5)^2} = 7.2 \text{ cm/s.} \end{aligned}$$

36

$\therefore$  Let the weight of the body =  $w$ , and the distance from the centre of the earth =  $d$

$$\begin{aligned} \therefore w &\propto \frac{1}{d^2} & \therefore \frac{w_1}{w_2} &= \frac{d_2^2}{d_1^2} \\ \therefore \frac{500}{w_2} &= \frac{(640 + 6390)^2}{(6390)^2} & \therefore w_2 &= 413 \text{ w.kg.} \end{aligned}$$

37

$$\begin{aligned} \therefore X &\propto y, z \propto \ell \quad \therefore X = my, z = k\ell \\ \therefore (X+y)(z+\ell) &= (my+y)(k\ell+\ell) \\ &= y\ell(m+1)(k+1) \quad (1) \\ \therefore (X-y)(z-\ell) &= (my-y)(k\ell-\ell) \\ &= y\ell(m-1)(k-1) \quad (2) \end{aligned}$$

Dividing (1) by (2) :

$$\begin{aligned} \frac{(X+y)(z+\ell)}{(X-y)(z-\ell)} &= \frac{y\ell(m+1)(k+1)}{y\ell(m-1)(k-1)} = \frac{(m+1)(k+1)}{(m-1)(k-1)} \\ &= \text{constant} \end{aligned}$$

$$\therefore (X+y)(z+\ell) \propto (X-y)(z-\ell)$$

38

$$\begin{aligned} \therefore (a+b) &\propto \frac{a}{b} & \therefore (a+b) &= \frac{ma}{b} \quad (1) \\ \therefore (a^2 - ab + b^2) &\propto \frac{b}{a} & \therefore (a^2 - ab + b^2) &= \frac{\ell b}{a} \quad (2) \\ \therefore \text{multiplying (1) by (2)} : & & & \\ \therefore (a+b)(a^2 - ab + b^2) &= \frac{ma}{b} \times \frac{\ell b}{a} \\ \therefore a^3 + b^3 &= \ell m = \text{constant} \end{aligned}$$

**Answers of exams on unit two**

**? Model 1**

**1**

- [1] c      [2] d      [3] a  
[4] d      [5] b      [6] c

**2**

- [a]  $\frac{3}{4}$   
[b] Prove by yourself

**3**

- [a] [1]  $xy = 30$       [2]  $y = 6$   
[b] Prove by yourself

**4**

- [a] 3  
[b]  $y = 3 + \frac{2}{x}$  ,  $y = 4$

**5**

- [a]  $a = 1$  ,  $b = 27$   
[b] 3

**? Model 2**

**1**

- [1] a      [2] c      [3] d  
[4] c      [5] c      [6] c

**2**

- [a] Prove by yourself.

[b] [1]  $y = \frac{16}{x^2} - 1$       [2]  $x = \pm \frac{4}{3}$

**3**

- [a] Prove by yourself.

- [b] Prove by yourself.

**4**

- [a] 3

- [b] Prove by yourself.

**5**

- [a] [1] Prove by yourself.      [2]  $x = 4$

- [b] Prove by yourself.

## Answers of unit three

## Answers of Exercise 9

1

1 primary, secondary

2 primary

3 secondary

4 secondary

5 primary

6 samples

7 samples

8 mass population

9 mass population

10 layer random sample

2

1 c

2 c

3 d

4 b

5 b

3

The primary sources 1 and 2

The secondary sources : 3, 4 and 5

4

Side of comparison	The method	Mass population	Samples
Its definition		It is setup collecting data related to the phenomenon from the all individuals of the statistical society.	It is setup collecting data about the phenomenon under study from some individuals of the statistics society not all the individuals, this by selecting a sample representing to all statistics society.
Advantages		Accuracy , perfect representation of all statistics society	1 It is faster and less cost 2 It is the unique method for collecting data from the large societies (infinite) 3 It is the unique method for collecting data from some limited societies
Disadvantages		Sometimes it needs a long time and more costs	The results are not accurated specially if the sample does not represent the statistics society very well.

5

The method of mass population : 1 and 5

The method of samples : 2, 3 and 4

6

8 Answer by yourself.

9

The total number of students

$$= 4\,000 + 3\,000 + 2\,000 + 1\,000 = 10\,000 \text{ students}$$

The number of the individuals of the first layer in

$$\text{the sample} = \frac{4\,000}{10\,000} \times 500 = 200 \text{ students}$$

The number of the individuals of the second layer in

$$\text{the sample} = \frac{3\,000}{10\,000} \times 500 = 150 \text{ students}$$

The number of the individuals of the third layer in

$$\text{the sample} = \frac{2\,000}{10\,000} \times 500 = 100 \text{ students}$$

The number of the individuals of the fourth layer in

$$\text{the sample} = \frac{1\,000}{10\,000} \times 500 = 50 \text{ students}$$

10

 The total number of cars =  $300 + 500 + 200$ 

$$= 1000 \text{ cars}$$

 The number of individuals of the sample =  $1000 \times 5\%$ 

$$= 50 \text{ cars}$$

The number of the first model in the sample

$$= \frac{300}{1000} \times 50 = 15 \text{ cars}$$

The number of the second model in the sample

$$= \frac{500}{1000} \times 50 = 25 \text{ cars}$$

The number of the third model in the sample

$$= \frac{200}{1000} \times 50 = 10 \text{ cars}$$

11

The number of the second layer

$$= 5\,000 - 1\,500 = 3\,500 \text{ individuals}$$

The number of individuals of all

$$\text{The sample} = \frac{5\,000 \times 140}{3\,500} = 200 \text{ individuals}$$

12

$$\text{The size of all the sample} = \frac{40\,000 \times 240}{12\,000}$$

$$= 800 \text{ individuals}$$

13

Number of the layer	1	2	3	4	Total
Number of individuals of the layer	500	700	350	450	2000
Number of individuals of the layer in the sample	10	14	7	9	40

## Answers to Exercise 10

1

- 1 the range, the standard deviation  
 2 the range      3 zero      4 81

2

- 1 a      2 c      3 b      4 a  
 5 b      6 c      7 c      8 d  
 9 c      10 c      11 b      12 d  
 13 c      14 c      15 c

3

Set A	Set B
40, 45, 50, 52, 55, 58	20, 35, 50, 55, 60, 80
$\frac{40+45+50+52+55+58}{6} = 50$	$\frac{20+35+50+55+60+80}{6} = 50$
51	52.5
$58 - 40 = 18$	$80 - 20 = 60$
6	18.9

The set A is more homogeneous

4

- 1 The mean  $(\bar{x}) = \frac{16+32+5+20+27}{5} = 20$

x	$x - \bar{x}$	$(x - \bar{x})^2$
16	$16 - 20 = -4$	16
32	$32 - 20 = 12$	144
5	$5 - 20 = -15$	225
20	$20 - 20 = 0$	0
27	$27 - 20 = 7$	49
Total		434

 The standard deviation  $(\sigma) = \sqrt{\frac{434}{5}} \approx 9.3$ 

- 2 The mean  $(\bar{x}) = \frac{72+53+61+70+59}{5} = 63$

x	$x - \bar{x}$	$(x - \bar{x})^2$
72	$72 - 63 = 9$	81
53	$53 - 63 = -10$	100
61	$61 - 63 = -2$	4
70	$70 - 63 = 7$	49
59	$59 - 63 = -4$	16
Total		250

 The standard deviation  $(\sigma) = \sqrt{\frac{250}{5}} \approx 7.1$ 

- 3 The mean  $(\bar{x}) = \frac{15 + (-12) + (-9) + 27 + (-6)}{5} = 3$

x	$x - \bar{x}$	$(x - \bar{x})^2$
15	$15 - 3 = 12$	144
-12	$-12 - 3 = -15$	225
-9	$-9 - 3 = -12$	144
27	$27 - 3 = 24$	576
-6	$-6 - 3 = -9$	81
Total		1170

 The standard deviation  $(\sigma) = \sqrt{\frac{1170}{5}} \approx 15.3$ 

- 4 The mean  $(\bar{x}) = \frac{22+20+20+20+18}{5} = 20$

x	$x - \bar{x}$	$(x - \bar{x})^2$
22	$22 - 20 = 2$	4
20	$20 - 20 = 0$	0
20	$20 - 20 = 0$	0
20	$20 - 20 = 0$	0
18	$18 - 20 = -2$	4
Total		8

 The standard deviation  $(\sigma) = \sqrt{\frac{8}{5}} \approx 1.3$ 

5

- The mean of the set (A) =  $\frac{7+8+9+10+11}{5} = 9$

x	$x - \bar{x}$	$(x - \bar{x})^2$
7	$7 - 9 = -2$	4
8	$8 - 9 = -1$	1
9	$9 - 9 = 0$	0
10	$10 - 9 = 1$	1
11	$11 - 9 = 2$	4
Total		10

 The standard deviation  $(\sigma)$  of the set (A) =  $\sqrt{\frac{10}{5}} \approx 1.4$ 

- The mean of the set (B) =  $\frac{21+20+11+19}{4} = 17.75$

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
21	$21 - 17.75 = 3.25$	10.5625
20	$20 - 17.75 = 2.25$	5.0625
11	$11 - 17.75 = -6.75$	45.5625
19	$19 - 17.75 = 1.25$	1.5625
Total		62.75

The standard deviation of the set (B) =  $\sqrt{\frac{62.75}{4}} = 4$

\* The mean of the set (C) =  $\frac{29 + 30 + 30 + 35}{4} = 31$

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
29	$29 - 31 = -2$	4
30	$30 - 31 = -1$	1
30	$30 - 31 = -1$	1
35	$35 - 31 = 4$	16
Total		22

The standard deviation of the set (C) =  $\sqrt{\frac{22}{4}} = 2.3$

∴ The set B is more dispersion

6

1 The mean ( $\bar{x}$ ) =  $\frac{73 + 54 + 62 + 71 + 60}{5} = 64$

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
73	$73 - 64 = 9$	81
54	$54 - 64 = -10$	100
62	$62 - 64 = -2$	4
71	$71 - 64 = 7$	49
60	$60 - 64 = -4$	16
Total		250

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{250}{5}} = 7.07$

2 The mean ( $\bar{x}$ ) =  $\frac{13 + 14 + 17 + 19 + 22}{5} = 17$

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
13	$13 - 17 = -4$	16
14	$14 - 17 = -3$	9
17	$17 - 17 = 0$	0
19	$19 - 17 = 2$	4
22	$22 - 17 = 5$	25
Total		54

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{54}{5}} = 3.286$

3 The mean ( $\bar{x}$ )

$$= \frac{65 + 61 + 70 + 64 + 70 + 76 + 70}{7} = 68$$

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
65	$65 - 68 = -3$	9
61	$61 - 68 = -7$	49
70	$70 - 68 = 2$	4
64	$64 - 68 = -4$	16
70	$70 - 68 = 2$	4
76	$76 - 68 = 8$	64
70	$70 - 68 = 2$	4
Total		150

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{150}{7}} = 4.6$

4 The mean ( $\bar{x}$ )

$$= \frac{23 + 12 + 17 + 13 + 15 + 16 + 8 + 9 + 37 + 10}{10} = 16$$

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
23	$23 - 16 = 7$	49
12	$12 - 16 = -4$	16
17	$17 - 16 = 1$	1
13	$13 - 16 = -3$	9
15	$15 - 16 = -1$	1
16	$16 - 16 = 0$	0
8	$8 - 16 = -8$	64
9	$9 - 16 = -7$	49
37	$37 - 16 = 21$	441
10	$10 - 16 = -6$	36
Total		666

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{666}{10}} \approx 8.2$

7

1 The mean of the marks of pupils

$$= \frac{8 + 9 + 6 + 12 + 10}{5} = 9$$

2

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
8	$8 - 9 = -1$	1
9	$9 - 9 = 0$	0
6	$6 - 9 = -3$	9
12	$12 - 9 = 3$	9
10	$10 - 9 = 1$	1
Total		20

The standard deviation ( $\sigma$ ) of the marks of pupils =  $\sqrt{\frac{20}{5}} = 2$

8

- 1 The mean of maximum degrees ( $\bar{x}$ )

$$= \frac{25 + 26 + 24 + 24 + 22 + 26 + 27 + 26}{8} = 25 \text{ degrees}$$

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
25	$25 - 25 = 0$	0
26	$26 - 25 = 1$	1
24	$24 - 25 = -1$	1
24	$24 - 25 = -1$	1
22	$22 - 25 = -3$	9
26	$26 - 25 = 1$	1
27	$27 - 25 = 2$	4
26	$26 - 25 = 1$	1
<b>Total</b>		18

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{18}{8}} = 1.5$

- 2 The mean of the minimum degrees ( $\bar{x}$ )

$$= \frac{11 + 12 + 10 + 6 + 7 + 16 + 15 + 11}{8} = 11$$

$x$	$x - \bar{x}$	$(x - \bar{x})^2$
11	$11 - 11 = 0$	0
12	$12 - 11 = 1$	1
10	$10 - 11 = -1$	1
6	$6 - 11 = -5$	25
7	$7 - 11 = -4$	16
16	$16 - 11 = 5$	25
15	$15 - 11 = 4$	16
11	$11 - 11 = 0$	0
<b>Total</b>		84

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{84}{8}} \approx 3.2$  degrees

9

Number of children ( $x$ )	Number of families ( $k$ )	$x \times k$
0	8	0
1	16	16
2	50	100
3	20	60
4	6	24
<b>Total</b>	100	200

The mean of ( $\bar{x}$ ) =  $\frac{200}{100} = 2$  children

$x$	$k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
0	8	$0 - 2 = -2$	4	32
1	16	$1 - 2 = -1$	1	16
2	50	$2 - 2 = 0$	0	0
3	20	$3 - 2 = 1$	1	20
4	6	$4 - 2 = 2$	4	24
<b>Total</b>	100			92

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{92}{100}} \approx 1$  child

10

Number of defective units ( $x$ )	Number of boxes ( $k$ )	$x \times k$
zero	3	Zero
1	16	16
2	17	34
3	25	75
4	20	80
5	19	95
<b>Total</b>	100	300

The mean of ( $\bar{x}$ ) =  $\frac{300}{100} = 3$  units

$x$	$k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
Zero	3	$0 - 3 = -3$	9	27
1	16	$1 - 3 = -2$	4	64
2	17	$2 - 3 = -1$	1	17
3	25	$3 - 3 = 0$	0	0
4	20	$4 - 3 = 1$	1	20
5	19	$5 - 3 = 2$	4	76
<b>Total</b>	100			204

The Standard deviation ( $\sigma$ ) =  $\sqrt{\frac{204}{100}} \approx 1.4$  units

11

Number of goals ( $x$ )	Number of players ( $k$ )	$x \times k$
0	2	0
1	4	4
2	5	10
3	8	24
4	7	28
5	4	20
<b>Total</b>	30	86

The mean ( $\bar{x}$ ) =  $\frac{86}{30} \approx 2.9$  goals

$x$	$k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
0	2	$0 - 2.9 = -2.9$	8.41	16.82
1	4	$1 - 2.9 = -1.9$	3.61	14.44
2	5	$2 - 2.9 = -0.9$	0.81	4.05
3	8	$3 - 2.9 = 0.1$	0.01	0.08
4	7	$4 - 2.9 = 1.1$	1.21	8.47
5	4	$5 - 2.9 = 2.1$	4.41	17.64
<b>Total</b>	<b>30</b>			<b>61.5</b>

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{61.5}{30}} \approx 1.4$  goals

12

Age ( $x$ )	Number of children ( $k$ )	$x \times k$
5	1	5
8	2	16
9	3	27
10	3	30
12	1	12
<b>Total</b>	<b>10</b>	<b>90</b>

The mean of ( $\bar{x}$ ) =  $\frac{90}{10} = 9$  years

$x$	$k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
5	1	$5 - 9 = -4$	16	16
8	2	$8 - 9 = -1$	1	2
9	3	$9 - 9 = 0$	0	0
10	3	$10 - 9 = 1$	1	3
12	1	$12 - 9 = 3$	9	9
<b>Total</b>	<b>10</b>			<b>30</b>

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{30}{10}} \approx 1.7$  years

13

Number of student ( $x$ )	Number of classes ( $k$ )	$x \times k$
0	1	0
1	3	3
2	5	10
3	6	18
4	3	12
5	2	10
<b>Total</b>	<b>20</b>	<b>53</b>

The mean of ( $\bar{x}$ ) =  $\frac{53}{20} = 2.65$  student

$x$	$k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
0	1	$0 - 2.65 = -2.65$	7.0225	7.0225
1	3	$1 - 2.65 = -1.65$	2.7225	8.1675
2	5	$2 - 2.65 = -0.65$	0.4225	2.1125
3	6	$3 - 2.65 = 0.35$	0.1225	0.735
4	3	$4 - 2.65 = 1.35$	1.8225	5.4675
5	2	$5 - 2.65 = 2.35$	5.5225	11.045
<b>Total</b>	<b>20</b>			<b>34.55</b>

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{34.55}{20}} \approx 1.3$  student

14

Sets	centres of sets ( $x$ )	Frequency ( $k$ )	$x \times k$
5 -	10	7	70
15 -	20	9	180
25 -	30	11	330
35 -	40	15	600
45 -	50	8	400
<b>Total</b>		<b>50</b>	<b>1580</b>

The mean of ( $\bar{x}$ ) =  $\frac{1580}{50} = 31.6$

$x$	$k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
10	7	$10 - 31.6 = -21.6$	466.56	3265.92
20	9	$20 - 31.6 = -11.6$	134.56	1211.04
30	11	$30 - 31.6 = -1.6$	2.56	28.16
40	15	$40 - 31.6 = 8.4$	70.56	1058.4
50	8	$50 - 31.6 = 18.4$	338.56	2708.48
<b>Total</b>	<b>50</b>			<b>8272</b>

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{8272}{50}} \approx 12.9$

15

Sets	centres of sets ( $x$ )	Frequency ( $k$ )	$x \times k$
0 -	2	3	6
4 -	6	4	24
8 -	10	7	70
12 -	14	2	28
16 - 20	18	9	162
<b>Total</b>		<b>25</b>	<b>290</b>

The mean of ( $\bar{x}$ ) =  $\frac{290}{25} = 11.6$

$x$	$k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
2	3	$2 - 11.6 = -9.6$	92.16	276.48
6	4	$6 - 11.6 = -5.6$	31.36	125.44
10	7	$10 - 11.6 = -1.6$	2.56	17.92
14	2	$14 - 11.6 = 2.4$	5.76	11.52
18	9	$18 - 11.6 = 6.4$	40.96	368.64
<b>Total</b>	<b>25</b>			<b>800</b>

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{800}{25}} \approx 5.7$

16

Sets	centres of sets ( $x$ )	Frequency ( $k$ )	$x \times k$
20 -	25	10	250
30 -	35	12	420
40 -	45	8	360
50 -	55	6	330
60 -	65	3	195
70 -	75	1	75
<b>Total</b>		<b>40</b>	<b>1630</b>

The mean of ( $\bar{x}$ ) =  $\frac{1630}{40} = 40.75$  pounds

$x$	$k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
25	10	$25 - 40.75 = -15.75$	248.0625	2480.625
35	12	$35 - 40.75 = -5.75$	33.0625	396.75
45	8	$45 - 40.75 = 4.25$	18.0625	144.5
55	6	$55 - 40.75 = 14.25$	203.0625	1218.375
65	3	$65 - 40.75 = 24.25$	588.0625	1764.1875
75	1	$75 - 40.75 = 34.25$	1173.0625	1173.0625
<b>Total</b>	<b>40</b>			<b>7177.5</b>

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{7177.5}{40}} \approx 13.4$  pounds

17

Sets	centres of sets ( $x$ )	Frequency ( $k$ )	$x \times k$
5 -	6	3	18
7 -	8	6	48
9 -	10	10	100
11 -	12	12	144
13 -	14	5	70
15 - 17	16	4	64
<b>Total</b>		<b>40</b>	<b>444</b>

The mean of ( $\bar{x}$ ) =  $\frac{444}{40} = 11.1$  km./litre

$x$	$k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
6	3	$6 - 11.1 = -5.1$	26.01	78.03
8	6	$8 - 11.1 = -3.1$	9.61	57.66
10	10	$10 - 11.1 = -1.1$	1.21	12.1
12	12	$12 - 11.1 = 0.9$	0.81	9.72
14	5	$14 - 11.1 = 2.9$	8.41	42.05
16	4	$16 - 11.1 = 4.9$	24.01	96.04
<b>Total</b>	<b>40</b>			<b>295.6</b>

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{295.6}{40}} \approx 2.7$  km./litre

18

Sets	centres of sets ( $x$ )	Frequency ( $k$ )	$x \times k$
5 -	10	19	190
15 -	20	50	1000
25 -	30	85	2550
35 -	40	25	1000
45 -	50	15	750
55 -	60	6	360
<b>Total</b>		<b>200</b>	<b>5850</b>

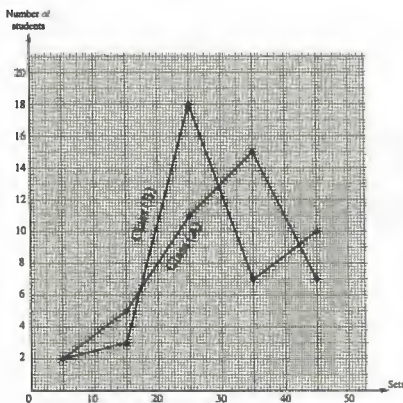
The mean of ( $\bar{x}$ ) =  $\frac{5850}{200} = 29.25$  pounds

$x$	$k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
10	19	$10 - 29.25 = -19.25$	370.5625	7040.6875
20	50	$20 - 29.25 = -9.25$	85.5625	4278.125
30	85	$30 - 29.25 = 0.75$	0.5625	47.8125
40	25	$40 - 29.25 = 10.75$	115.5625	2889.0625
50	15	$50 - 29.25 = 20.75$	430.5625	6458.4375
60	6	$60 - 29.25 = 30.75$	945.5625	5673.375
<b>Total</b>	<b>200</b>			<b>26387.5</b>

The standard deviation ( $\sigma$ ) =  $\sqrt{\frac{26387.5}{200}} \approx 11.5$  pounds

19

## 1 The graph



## 2 With respect to class (A)

Sets	centres of sets ( $x$ )	Frequency ( $k$ )	$x \times k$
0 -	5	2	10
10 -	15	5	75
20 -	25	11	275
30 -	35	15	525
40 - 50	45	7	315
Total		40	1200

The mean of ( $\bar{x}$ ) of class (A) =  $\frac{1200}{40} = 30$  marks

$x$	$k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
5	2	$5 - 30 = -25$	625	1250
15	5	$15 - 30 = -15$	225	1125
25	11	$25 - 30 = -5$	25	275
35	15	$35 - 30 = 5$	25	375
45	7	$45 - 30 = 15$	225	1575
Total	40			4600

The standard deviation ( $\sigma$ ) of class (A)

$$= \sqrt{\frac{4600}{40}} \approx 10.7 \text{ marks}$$

With respect to class (B)

Sets	centres of sets ( $x$ )	Frequency ( $k$ )	$x \times k$
0 -	5	2	10
10 -	15	3	45
20 -	25	18	450
30 -	35	7	245
40 - 50	45	10	450
Total		40	1200

The mean ( $\bar{x}$ ) of class (B) =  $\frac{1200}{40} = 30$  marks

$x$	$k$	$x - \bar{x}$	$(x - \bar{x})^2$	$(x - \bar{x})^2 \times k$
5	2	$5 - 30 = -25$	625	1250
15	3	$15 - 30 = -15$	225	675
25	18	$25 - 30 = -5$	25	450
35	7	$35 - 30 = 5$	25	175
45	10	$45 - 30 = 15$	225	2250
Total	40			4800

The standard deviation of class (B)

$$(\sigma) = \sqrt{\frac{4800}{40}} \approx 11 \text{ marks}$$

3 Class A is the most homogeneous in getting marks.

## Answers of exam on unit three

1

1 a

2 a

3 b

4 b

5 d

6 c

 2  $\bar{x} = 20, \sigma = 9.32$ 

 3  $\bar{x} = 4.88, \sigma \approx 1.7$ 

 4  $\sigma = 2.32$ 

## Answers of accumulative basic skills

1 d

2 c

3 b

4 d

5 c

6 d

7 c

8 d

9 b

10 c

11 b

12 b

13 c

14 a

15 b

16 d

17 c

18 c

19 a

20 a

21 c

22 d

23 d

24 a

25 c

26 c

27 c

28 c

29 c

30 c

31 a

32 d

33 c

34 c

35 c

36 c

**Guide  
Answers**

**of Trigonometry and  
Geometry Exercises**

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## Answers of unit four

## Answers of Exercise 1

1

1  $85.635^\circ$

2  $56^\circ 10' 48''$

3 (a)  $\frac{15}{17}, \frac{8}{17}$

(b)  $\frac{8}{17}, \frac{15}{17}$

(c)  $\frac{15}{8}, \frac{8}{15}$

4  $\frac{12}{25}$

5 C

6  $\frac{3}{5}$

2

1 c

2 b

3 c

4 d

5 c

6 a

7 d

8 d

9 b

3

 Let the measures of the two angles be  $3X$  and  $5X$ 

$\therefore 3X + 5X = 180^\circ$

$\therefore 8X = 180^\circ$

$\therefore X = \frac{180^\circ}{8} = 22.5^\circ$

The measure of the first angle  $= 3 \times 22.5^\circ = 67.5^\circ$   
 $= 67^\circ 30'$

The measure of the second angle  $= 5 \times 22.5^\circ$   
 $= 112.5^\circ = 112^\circ 30'$

4

 Let the measures of the two angles be  $3X$  and  $4X$ 

$\therefore 3X + 4X = 90^\circ$

$\therefore 7X = 90^\circ$

$\therefore X = \frac{90^\circ}{7} = 12\frac{6}{7}^\circ$

$\therefore$  The measure of the greater angle  
 $= 4 \times 12\frac{6}{7}^\circ \approx 51^\circ 25' 43''$

5

 Let the measures of the interior angles of the triangle be  $3X, 4X, 7X$ 

$\therefore 3X + 4X + 7X = 180^\circ$

$\therefore 14X = 180^\circ$

$\therefore X = \frac{180^\circ}{14} = 12\frac{6}{7}^\circ$

The measure of the first angle

$= 3 \times 12\frac{6}{7}^\circ \approx 38^\circ 34' 17''$

The measure of the second angle

$= 4 \times 12\frac{6}{7}^\circ \approx 51^\circ 25' 43''$

The measure of the third angle  $= 7 \times 12\frac{6}{7}^\circ = 90^\circ$

6

$\therefore m(\angle A) = 90^\circ \quad \therefore (BC)^2 = (20)^2 + (15)^2 = 625$

$\therefore BC = 25 \text{ cm.}$

$\therefore \cos C \cos B = \frac{15}{25} \times \frac{20}{25} = \frac{20}{25} \times \frac{15}{25} = 0$

7

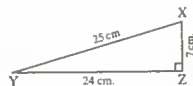
$\therefore m(\angle Z) = 90^\circ$

$\therefore (ZY)^2 = (25)^2 - (7)^2$   
 $= 576$

$\therefore ZY = 24 \text{ cm.}$

1  $\tan X \times \tan Y = \frac{24}{7} \times \frac{7}{24} = 1$

2  $\sin^2 X + \sin^2 Y = \left(\frac{24}{25}\right)^2 + \left(\frac{7}{25}\right)^2 = \frac{625}{625} = 1$



8

$\therefore m(\angle B) = 90^\circ$

$\therefore (AB)^2 = (5)^2 - (4)^2 = 9$

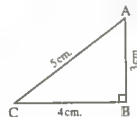
$\therefore AB = 3 \text{ cm.}$

$\therefore \sin^2 A - \cos^2 A$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

$2 \sin^2 A - 1 = 2 \times \left(\frac{4}{5}\right)^2 - 1 = 2 \times \frac{16}{25} - 1 = \frac{7}{25}$

$\therefore \sin^2 A - \cos^2 A = 2 \sin^2 A - 1$



9

$\therefore \frac{AB}{AC} = \frac{3}{5}$

$\therefore AB = 3 \text{ length unit}$

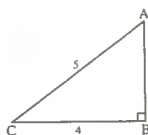
$\therefore AC = 5 \text{ length unit}$

$\therefore m(\angle B) = 90^\circ$

$\therefore (BC)^2 = 5^2 - 3^2 = 16 \quad \therefore BC = 4 \text{ length unit}$

$\therefore \sin A = \frac{BC}{AC} = \frac{4}{5}, \cos A = \frac{AB}{AC} = \frac{3}{5}$

$\therefore \tan A = \frac{BC}{AB} = \frac{4}{3}$



10

$\therefore YZ = 2XY \quad \therefore \frac{YZ}{XY} = \frac{2}{1}$

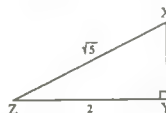
 Let  $YZ = 2 \text{ length unit}$ 

$\therefore XY = 1 \text{ length unit}$

$\therefore XZ = \sqrt{5} \text{ length unit}$

$\therefore \tan Z = \frac{1}{2}, \tan X = 2$

$\cos Z = \frac{2}{\sqrt{5}}, \cos X = \frac{1}{\sqrt{5}}$



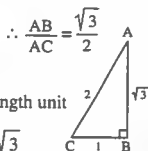
11

$$\therefore 2 AB = \sqrt{3} AC$$

 Let  $AB = \sqrt{3}$  length unit

 $\therefore AC = 2$  length unit  $\therefore BC = 1$  length unit

$$\sin C = \frac{\sqrt{3}}{2}, \cos C = \frac{1}{2}, \tan C = \sqrt{3}$$



12

$$\textcircled{1} \therefore \tan C = \frac{AB}{BC}$$

$$\therefore \frac{3}{4} = \frac{6}{BC}$$

$$\therefore BC = 8 \text{ cm.}$$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2$$

$$\therefore (AC)^2 = 36 + 64 = 100 \quad \therefore AC = 10 \text{ cm.}$$

$$\textcircled{2} \sin A + \cos A = \frac{8}{10} + \frac{6}{10} = \frac{14}{10} = \frac{7}{5}$$

13

 $\therefore \triangle ACD$  is right-angled at D

$$\therefore (AD)^2 = (17)^2 - (15)^2 = 64 \quad \therefore AD = 8 \text{ cm.}$$

$$\therefore 3 \tan C + \sin B = 3 \times \frac{8}{15} + \frac{8}{10} = \frac{12}{5}$$

14

 $\textcircled{1}$  In  $\triangle ABC$  :  $\therefore m(\angle ACB) = 90^\circ$ 

$$\therefore (BC)^2 = 36 + 64 = 100 \quad \therefore BC = 10 \text{ cm.}$$

$$\therefore \overline{AD} \perp \overline{BC} \quad \therefore AD = \frac{6 \times 8}{10} = 4.8 \text{ cm.}$$

$$\therefore (AB)^2 = BD \times BC \quad \therefore 36 = BD \times 10$$

$$\therefore BD = 3.6 \text{ cm.}$$

$$\therefore \tan(\angle BAD) = \frac{3.6}{4.8} = \frac{3}{4}$$

$$\textcircled{2} \cos(\angle DAC) + \cos(\angle DAB) = \frac{4.8}{8} + \frac{4.8}{6} = \frac{7}{5}$$

15

$$\text{From } \triangle ABD : \therefore \cos B = \frac{BD}{AB}$$

$$\therefore \text{from } \triangle ACD : \therefore \cos C = \frac{CD}{AC}$$

$$\therefore AB \cos B + AC \cos C = AB \times \frac{BD}{AB} + AC \times \frac{CD}{AC} = BD + CD = 8 \text{ cm.}$$

16

 In  $\triangle ABD$  :  $\therefore m(\angle A) = 90^\circ$ 

$$\therefore (AD)^2 = (BD)^2 - (AB)^2 = 100 - 36 = 64$$

$$\therefore AD = 8 \text{ cm.}$$

$$\therefore \tan(\angle ADB) = \frac{6}{8} = \frac{3}{4}$$

 $\therefore \overline{AD} \parallel \overline{BC}$ ,  $\overline{BD}$  is a transversal to them

 $\therefore m(\angle ADB) = m(\angle DBC)$  (alternate angles)

$$\therefore \tan(\angle ADB) = \tan(\angle DBC)$$

$$\therefore \tan(\angle DBC) = \frac{3}{4} \quad \therefore \frac{DC}{10} = \frac{3}{4}$$

$$\therefore DC = \frac{10 \times 3}{4} = 7.5 \text{ cm.}$$

17

 Draw  $\overline{AF} \perp \overline{BC}$ ,

 $\overline{DE} \perp \overline{BC}$ 
 $\therefore \overline{AD} \parallel \overline{BC}$ ,

 $\overline{AF} \perp \overline{BC}$ ,  $\overline{DE} \perp \overline{BC}$ 
 $\therefore AFED$  is a rectangle,  $FE = 4 \text{ cm.}$ 

$$\therefore BF + EC = 8 \text{ cm.}$$

$$\therefore BF = EC = 4 \text{ cm.}$$

 ( $\triangle ABF$  and  $\triangle DCE$  are congruent)

 $\therefore$  from  $\triangle ABF$  which is right-angled at F

$$\therefore (AF)^2 = (5)^2 - (4)^2 = 9$$

$$\therefore AF = 3 \text{ cm.}$$

$$\therefore DE = AF = 3 \text{ cm.}$$

 ( $AFED$  is a rectangle)

$$\therefore \frac{5 \tan B \cos C}{\sin^2 C + \cos^2 B} = \frac{5 \times \frac{3}{4} \times \frac{4}{5}}{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = 3$$

18

 Draw  $\overline{DF} \perp \overline{BC}$ 
 $\therefore \overline{AD} \parallel \overline{BC}$ ,  $\overline{AB} \perp \overline{BC}$ 
 $\therefore \overline{DF} \perp \overline{BC}$ 
 $\therefore ABFD$  is a rectangle

$$\therefore BF = AD = 6 \text{ cm.}$$

$$\therefore FC = 4 \text{ cm.}, DF = AB = 3 \text{ cm.}$$

 $\therefore$  from  $\triangle DFC$  which is right-angled at F

$$(DC)^2 = 3^2 + 4^2 = 25 \quad \therefore DC = 5 \text{ cm.}$$

$$\cos(\angle DCB) - \tan(\angle ACB) = \frac{4}{5} - \frac{3}{10} = \frac{1}{2}$$

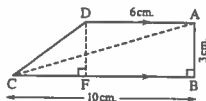
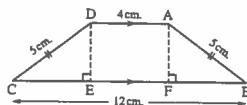
19

$$\therefore 4 DE = 3 EC \quad \therefore \frac{DE}{EC} = \frac{3}{4}$$

$$\therefore \tan C = \frac{DE}{EC} = \frac{3}{4}, \tan C = \frac{AB}{BC}$$

$$\therefore \frac{AB}{BC} = \frac{3}{4} \quad \therefore \frac{9}{BC} = \frac{3}{4} \quad \therefore BC = 12 \text{ cm.}$$

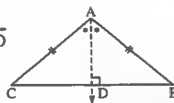
$$\therefore \text{The area of } \triangle ABC = \frac{1}{2} \times 12 \times 9 = 54 \text{ cm}^2$$



20

 Bisect  $\angle A$  by the bisector  $\overline{AD}$ 
 $\therefore \triangle ABC$  is an isosceles triangle

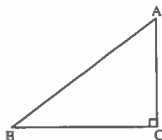
 $\therefore AD \perp BC$ 
 $\therefore \sin \frac{A}{2} = \sin (\angle BAD)$ 
 $\therefore \sin (\angle BAD) = \frac{4}{5}$ 
 $\therefore \angle B, \angle BAD$  are acute angles

 $\therefore \cos B = \sin (\angle BAD) \quad \therefore \cos B = \frac{4}{5}$ 


21

 $\therefore \sin B = \frac{AC}{AB}$ 
 $\therefore \cos B = \frac{BC}{AB}$ 
 $\therefore \sin B + \cos B = \frac{AC}{AB} + \frac{BC}{AB}$   
 $= \frac{AC + BC}{AB}$ 

From triangle inequality

 $\therefore AC + BC > AB \quad \therefore \sin B + \cos B > 1$ 


22

 $\therefore \sin A = \frac{6}{10} = \frac{3}{5}$ 
 $\therefore \frac{BC}{AC} = \frac{3}{5}$ 

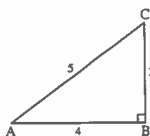
Assuming that :

 $BC = 3$  length unit,  $AC = 5$  length unit

 $\therefore AB = 4$  length unit

 $\therefore \sin A \cos C + \cos A \sin C$ 

$$= \frac{3}{5} \times \frac{3}{5} + \frac{4}{5} \times \frac{4}{5} = \frac{9}{25} + \frac{16}{25} = 1$$



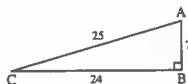
23

 $\therefore 7 \tan A - 24 = 0$ 
 $\therefore 7 \tan A = 24$ 
 $\tan A = \frac{24}{7}$ 

 Assuming that  $BC = 24$  length unit

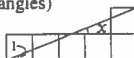
 $\therefore AB = 7$  length unit

 $\therefore AC = 25$  length unit

 $\therefore 1 - \tan A \sin C = 1 - \frac{24}{7} \times \frac{7}{25} = \frac{1}{25}$ 


24

 1  $\therefore m(\angle 1) = x$  (corresponding angles)

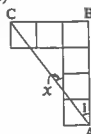
 $\therefore \tan (\angle 1) = \frac{2}{5}$ 
 $\therefore \tan x = \frac{2}{5}$ 

 2  $\therefore m(\angle 1) = x$  (corresponding angles)

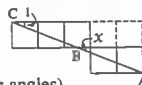
 $\therefore \tan (\angle 1) = \frac{3}{2}$ 
 $\therefore \tan x = \frac{3}{2}$ 

 3  $\therefore m(\angle 1) = x$  (corresponding angles)

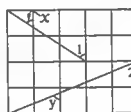
 $\therefore \triangle ABC$  is right-angled at B

 $\therefore (AC)^2 = 3^2 + 4^2 = 25$ 
 $\therefore AC = 5$  length units

 $\therefore \cos (\angle 1) = \frac{4}{5}$ 
 $\therefore \cos x = \frac{4}{5}$ 

 4  $\therefore m(\angle 1) = x$  (corresponding angles)

 $\therefore \tan (\angle 1) = \frac{2}{5}$ 
 $\therefore \tan x = \frac{2}{5}$ 

 5  $\therefore m(\angle 1) = x$  (corresponding angles)

 $\therefore m(\angle 2) = y$  (corresponding angles)

 $\therefore \tan (\angle 1) = \frac{3}{2}, \tan (\angle 2) = \frac{5}{2}$ 
 $\therefore \tan x + \frac{1}{\tan y} = \frac{3}{2} + \frac{2}{5} = \frac{19}{10}$ 

 6  $\therefore m(\angle 1) = y$  (corresponding angles)

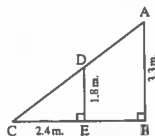
 $\therefore m(\angle 2) = z$  (corresponding angles)

 $\therefore \tan x + \tan y - \tan z$ 
 $= \tan x + \tan (\angle 1) - \tan (\angle 2)$ 
 $= \frac{2}{2} + \frac{2}{3} - \frac{3}{1} = -\frac{4}{3}$ 


25

 $\therefore \tan C = \frac{AB}{BC} = \frac{DE}{EC}$ 
 $\therefore \frac{3.3}{BC} = \frac{1.8}{2.4}$ 
 $\therefore BC = 4.4$  m.

 $\therefore BE = 4.4 - 2.4 = 2$  m.

 $\therefore$  The distance between the man and the base of the lampost = 2 m.


26

 $\therefore (AB)^2 = (3.6)^2 + (4.8)^2 = 36$ 
 $\therefore AB = 6$  km.

 $\therefore m(\angle BAD) + m(\angle DAC) = 90^\circ$ 
 $\therefore \sin (\angle BAD) = \cos (\angle DAC) \quad \therefore \frac{BD}{AB} = \frac{AD}{AC}$ 
 $\therefore \frac{3.6}{6} = \frac{4.8}{AC}$ 
 $\therefore AC = 8$  km.

 $\therefore (DC)^2 = (8)^2 - (4.8)^2 = 40.96 \quad \therefore DC = 6.4$  km.

Try to solve this problem by using Euclidean theorem.

27

 From  $\triangle ABC$  :  $\cos B = \frac{18}{BC}$ 
 $\therefore H$  is the midpoint of  $\overline{BC}$ 
 $\therefore BC = 2 BH$ 
 $\therefore \cos B = \frac{18}{2BH} = \frac{9}{BH}$ 

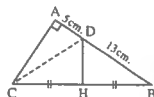
 from  $\triangle BDH$  :  $\cos B = \frac{BH}{13}$ 

 From (1) and (2) :  $\therefore \frac{9}{BH} = \frac{BH}{13}$ 
 $\therefore (BH)^2 = 9 \times 13 \quad \therefore BH = 3\sqrt{13} \text{ cm.}$ 
 $\therefore m(\angle BHD) = 90^\circ$ 
 $\therefore (DH)^2 = (BD)^2 - (BH)^2 = 169 - 117 = 52$ 
 $\therefore DH = 2\sqrt{13} \text{ cm.} \quad \therefore \tan B = \frac{2\sqrt{13}}{3\sqrt{13}} = \frac{2}{3}$ 
**Another Solution :**

 Draw :  $\overline{CD}$ 
**Proof :** In  $\triangle BCD$  :

 $\therefore \overline{DH} \perp \overline{BC}, BH = CH$ 
 $\therefore BD = CD = 13 \text{ cm.}$ 

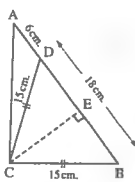
 In  $\triangle ACD$  :  $\therefore m(\angle A) = 90^\circ$ 
 $\therefore (AC)^2 = (CD)^2 - (AD)^2 = 169 - 25 = 144$ 
 $\therefore AC = 12 \text{ cm.}$ 

 In  $\triangle ABC$  :  $\tan B = \frac{AC}{AB} = \frac{12}{18} = \frac{2}{3}$ 


28

 Draw :  $\overline{CE} \perp \overline{BD}$ 

 In  $\triangle CBD$  :  $\therefore CB = CD$ 
 $\therefore \overline{CE} \perp \overline{BD}$ 
 $\therefore E$  is the midpoint of  $\overline{BD}$ 
 $\therefore ED = 9 \text{ cm.}$ 

 In  $\triangle EDC$  which is right-angled at  $E$ 
 $(CE)^2 = (15)^2 - (9)^2 = 144$ 
 $\therefore CE = 12 \text{ cm.}$ 
 $\therefore \tan(\angle BAC) = \frac{EC}{AE} = \frac{12}{15} = \frac{4}{5}$ 


29

 Let  $DE = l \text{ cm.} \quad \therefore AE = (5 - l) \text{ cm.}$ 
 $\therefore m(\angle AEB) = m(\angle DCE)$ 
 $\therefore \tan(\angle AEB) = \tan(\angle DCE)$ 
 $\therefore \frac{2}{5-l} = \frac{l}{5} \quad \therefore 5l - l^2 = 4$ 
 $\therefore l^2 - 5l + 4 = 0 \quad \therefore (l-4)(l-1) = 0$ 
 $\therefore l = 4 \text{ or } l = 1$ 
 $\therefore AE < ED \quad \therefore ED = 4 \text{ cm.}$ 
 $\therefore \tan(\angle CED) = \frac{2}{4} = \frac{1}{2}$ 

30

 $\therefore \angle BAD$  and  $\angle CAD$  are two acute angles

 $\therefore \sin(\angle BAD) = \cos(\angle CAD) \quad \therefore m(\angle CAB) = 90^\circ$ 
 $\therefore \sin(\angle BAD) = \frac{DB}{AB} = \frac{3}{5} \quad \therefore \frac{9}{AB} = \frac{3}{5}$ 
 $\therefore AB = 15 \text{ cm.}$ 
 $\therefore (AD)^2 = (15)^2 - (9)^2 = 144$ 
 $\therefore AD = 12 \text{ cm.}$ 
 $\therefore \overline{AD} \perp \overline{BC}, \overline{CA} \perp \overline{AB}$ 
 $\therefore (AB)^2 = BD \times BC \quad \text{(Euclidean theorem)}$ 
 $\therefore (15)^2 = 9 \times BC \quad \therefore BC = 25 \text{ cm.}$ 
 $\therefore \text{The area of } \triangle ABC = \frac{1}{2} \times 25 \times 12 = 150 \text{ cm}^2$ 

31

 $\therefore \triangle ABC$  is right-angled at  $B$ 
 $\therefore \sin A = \frac{BC}{AC}$ 
 $\therefore \sin^2 A = \frac{(BC)^2}{(AC)^2}$ 
 $\therefore \sin C = \frac{AB}{AC} \quad \therefore \sin^2 C = \frac{(AB)^2}{(AC)^2}$ 
 $\therefore \sin^2 A + \sin^2 C = \frac{(BC)^2}{(AC)^2} + \frac{(AB)^2}{(AC)^2} = \frac{(BC)^2 + (AB)^2}{(AC)^2}$ 
 $\therefore (BC)^2 + (AB)^2 = (AC)^2 \text{ (pythagoras)}$ 
 $\therefore \sin^2 A + \sin^2 C = \frac{(AC)^2}{(AC)^2} = 1$ 


32

 $\therefore \overline{AD} \parallel \overline{BO}, \overline{EZ} \parallel \overline{BO} \quad \therefore \overline{AD} \parallel \overline{EZ}$ 
 $\therefore m(\angle DAY) = m(\angle AZE) \text{ (alternate angles)}$ 
 $\therefore DC = 12 \text{ cm.}, CE = 4 \text{ cm.}$ 
 $\therefore DE = 8 \text{ cm.}$ 

 Let  $EY = x \text{ cm.} \quad \therefore DY = (8 - x) \text{ cm.}$ 
 $\therefore \tan(\angle DAY) = \tan(\angle AZE)$ 
 $\therefore \frac{8-x}{12} = \frac{x}{4} \quad \therefore \frac{8-x}{3} = x$ 
 $\therefore 8 - x = 3x \quad \therefore 8 = 4x$ 
 $\therefore x = 2 \quad \therefore \tan(\angle AZE) = \frac{EY}{EZ} = \frac{2}{4} = \frac{1}{2}$

33

$$\therefore m(\angle AEO) = 90^\circ$$

$$\therefore m(\angle 1) + m(\angle 2) = 90^\circ$$

In  $\triangle ADE$  which is right-angled at D

$$\therefore X + m(\angle 1) = 90^\circ$$

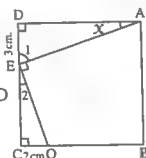
$$\therefore X = m(\angle 2)$$

Let the side length of the square be  $l$

$$\therefore EC = l - 3 \quad \therefore \tan X = \tan(\angle 2)$$

$$\therefore \frac{3}{l} = \frac{2}{l-3} \quad \therefore 2l = 3l - 9$$

$$\therefore l = 9 \quad \therefore \tan X = \frac{DE}{AD} = \frac{3}{9} = \frac{1}{3}$$



34

Draw  $\overline{BM} \perp \overline{AC}$

to intersect it at M

$$\therefore AC = 8 \text{ cm.}$$

$\therefore M$  is the midpoint of  $\overline{AC}$

$$\therefore BM = \frac{1}{2} AC = 4 \text{ cm.}$$

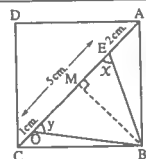
(properties of the square)

$$\therefore AM = \frac{1}{2} AC = 4 \text{ cm.}$$

$$\therefore EM = 2 \text{ cm.} \quad \therefore MC = 4 \text{ cm.}$$

$$\therefore MO = 3 \text{ cm.}$$

$$\therefore \tan X + \tan y = \frac{4}{2} + \frac{4}{3} = 3\frac{1}{3}$$



## Answers of Exercise 2

1

$$[1] \sin 45^\circ - \cos 45^\circ = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$$

$$[2] \cos 60^\circ + \sin 30^\circ = \frac{1}{2} + \frac{1}{2} = 1$$

$$[3] \sin 30^\circ + \cos 60^\circ - \tan 45^\circ = \frac{1}{2} + \frac{1}{2} - 1 = 0$$

$$[4] \sin 60^\circ + \cos 30^\circ + \tan 60^\circ = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \sqrt{3} = 2\sqrt{3}$$

$$[5] \sin^2 45^\circ + \cos^2 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} + \frac{1}{2} = 1$$

$$[6] 4 \cos 30^\circ \tan 60^\circ = 4 \times \frac{\sqrt{3}}{2} \times \sqrt{3} = 6$$

$$[7] \tan^2 60^\circ - 2 \sin 45^\circ \cos 45^\circ = (\sqrt{3})^2 - 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 3 - 1 = 2$$

$$[8] \sin^2 60^\circ - \tan 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 - \sqrt{3} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{3}{2} + \frac{1}{4} = -\frac{1}{2}$$

$$[9] 2 \sin 30^\circ \cos 60^\circ + \sqrt{2} \sin 45^\circ$$

$$= 2 \times \frac{1}{2} \times \frac{1}{2} + \sqrt{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2} + 1 = 1\frac{1}{2}$$

$$[10] (\cos 30^\circ - \cos 60^\circ) (\sin 30^\circ + \sin 60^\circ)$$

$$= \left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right) \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$

$$[11] \frac{\sin 30^\circ}{\cos 60^\circ} - \cos 30^\circ \sin 60^\circ = \frac{\frac{1}{2}}{\frac{1}{2}} - \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = 1 - \frac{3}{4} = \frac{1}{4}$$

$$[12] \frac{\cos^2 60^\circ + \cos^2 30^\circ + \tan^2 45^\circ}{\sin 60^\circ \tan 60^\circ - \sin 30^\circ}$$

$$= \frac{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 1^2}{\frac{\sqrt{3}}{2} \times \sqrt{3} - \frac{1}{2}} = \frac{\frac{1}{4} + \frac{3}{4} + 1}{\frac{3}{2} - \frac{1}{2}} = 2$$

2

$$[1] \text{ The left side} = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\text{The right side} = 2 \sin 30^\circ \cos 30^\circ$$

$$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

$\therefore$  The two sides are equal.

$$[2] \text{ The left side} = \cos 60^\circ = \frac{1}{2}$$

$$\text{The right side} = 2 \cos^2 30^\circ - 1$$

$$= 2 \left(\frac{\sqrt{3}}{2}\right)^2 - 1 = 2 \times \frac{3}{4} - 1 = \frac{1}{2}$$

$\therefore$  The two sides are equal.

$$[3] \text{ The left side} = 2 \cos^2 30^\circ - 1 = 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1$$

$$= 2 \times \frac{3}{4} - 1 = \frac{1}{2}$$

$$\text{The right side} = 1 - 2 \sin^2 30^\circ = 1 - 2 \times \left(\frac{1}{2}\right)^2$$

$$= 1 - 2 \times \frac{1}{4} = \frac{1}{2}$$

$\therefore$  The two sides are equal.

$$[4] \text{ The left side} = \cos 60^\circ = \frac{1}{2}$$

$$\text{The right side} = \cos^2 30^\circ - \sin^2 30^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$

$\therefore$  The two sides are equal.

5 The left side =  $\tan 60^\circ = \sqrt{3}$

$$\begin{aligned} \text{The right side} &= \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} \\ &= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3} \end{aligned}$$

∴ The two sides are equal.

6 The left side =  $\cos^2 60^\circ = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$$\begin{aligned} \text{The right side} &= 5 \sin^2 30^\circ - \tan^2 45^\circ \\ &= 5 \left(\frac{1}{2}\right)^2 - 1^2 = 5 \times \frac{1}{4} - 1 = \frac{1}{4} \end{aligned}$$

∴ The two sides are equal.

7 The left side =  $\sin^3 30^\circ = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$$\begin{aligned} \text{The right side} &= 9 \cos^3 60^\circ - \tan 45^\circ \\ &= 9 \times \left(\frac{1}{2}\right)^3 - 1 \\ &= 9 \times \frac{1}{8} - 1 = \frac{9}{8} - \frac{8}{8} = \frac{1}{8} \end{aligned}$$

∴ The two sides are equal.

8 The left side =  $\frac{\sin 30^\circ \cos 45^\circ + \sin 45^\circ \cos 30^\circ}{\sin 45^\circ \cos 60^\circ + \cos 45^\circ \sin 60^\circ}$

$$\begin{aligned} &= \frac{\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}} \\ &= \frac{\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}}{\frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}} = 1 \end{aligned}$$

The right side =  $\tan^2 45^\circ = 1^2 = 1$

∴ The two sides are equal.

9 The left side =  $\sin 30^\circ = \frac{1}{2}$

$$\begin{aligned} \text{The right side} &= \sqrt{\frac{1 - \cos 60^\circ}{2}} = \sqrt{\frac{1 - \frac{1}{2}}{2}} = \sqrt{\frac{\frac{1}{2}}{2}} \\ &= \sqrt{\frac{1}{4}} = \frac{1}{2} \end{aligned}$$

∴ The two sides are equal.

10

- |      |      |      |      |
|------|------|------|------|
| 1 b  | 2 d  | 3 c  | 4 b  |
| 5 c  | 6 a  | 7 b  | 8 d  |
| 9 c  | 10 c | 11 d | 12 d |
| 13 b | 14 e | 15 d | 16 a |

11

1  $X \times \left(\frac{1}{\sqrt{2}}\right)^2 = (\sqrt{3})^2 \therefore \frac{1}{2} X = 3 \therefore X = 6$

2  $X \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2$

$\therefore \frac{1}{4} X = \frac{3}{4} \therefore X = 3$

3  $X \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times \sqrt{3} = (1)^2 - \left(\frac{1}{2}\right)^2$

$\therefore \frac{\sqrt{3}}{2} X = \frac{3}{4} \therefore X = \frac{\sqrt{3}}{2}$

4  $4X = \left(\frac{\sqrt{3}}{2}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^2 \times (1)^2$

$\therefore 4X = \frac{1}{4} \therefore X = \frac{1}{16}$

12

1  $\therefore \tan X = 4 \times \frac{1}{2} \times \frac{1}{2} \therefore \tan X = 1 \therefore X = 45^\circ$

2  $\therefore \sin X = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} \therefore \sin X = \frac{1}{2}$

$\therefore X = 30^\circ$

3  $\therefore 2 \sin X = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \therefore 2 \sin X = 1$

$\therefore \sin X = \frac{1}{2} \therefore X = 30^\circ$

4  $\therefore 3 \sin X = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{2} + \left(\frac{\sqrt{3}}{2}\right)^2$

$\therefore 3 \sin X = \frac{3}{2} \therefore \sin X = \frac{1}{2} \therefore X = 30^\circ$

5  $6 \times \sin X \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1 - \frac{1}{4}$

$\therefore 6 \times \frac{1}{2} \times \sin X = \frac{3}{4} \therefore 3 \times \sin X = \frac{3}{4}$

$\therefore \sin X = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4} \therefore X = 14^\circ 28' 39''$

6  $\therefore \cos X = \frac{\frac{\sqrt{3}}{2} \times \frac{1}{2}}{1 \times \left(\frac{1}{\sqrt{2}}\right)^2} \therefore \cos X = \frac{\sqrt{3}}{2} \therefore X = 30^\circ$

7  $\therefore \cos (3X + 6^\circ) = \frac{1}{2}$

$\therefore 3X + 6^\circ = 60^\circ$

$\therefore 3X = 54^\circ \therefore X = \frac{54}{3} = 18^\circ$

13

1  $\therefore \left(\frac{1}{\sqrt{2}}\right)^2 = \cos E \times \frac{1}{\sqrt{3}}$

$\therefore \cos E = \frac{1}{2} \div \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{2} \therefore E = 30^\circ$

2  $\therefore \sin E \times \left(\frac{\sqrt{3}}{2}\right)^2 = 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \frac{1}{2}$

$\therefore \sin E \times \frac{3}{4} = \frac{3}{8} \therefore \sin E = \frac{1}{2}$

$\therefore E = 30^\circ$

$$3 \because 3 \tan E - 4 \times \left(\frac{1}{2}\right)^2 = 8 \times \left(\frac{1}{2}\right)^2$$

$$\therefore 3 \tan E - 1 = 2$$

$$\therefore 3 \tan E = 3 \quad \therefore \tan E = 1 \quad \therefore E = 45^\circ$$

7

$$\therefore \tan X = \frac{1}{\sqrt{3}} \quad \therefore X = 30^\circ$$

$$\therefore \sin X \tan \left(\frac{3X}{2}\right) + \cos(2X)$$

$$= \sin 30^\circ \tan \left(\frac{3 \times 30^\circ}{2}\right) + \cos(2 \times 30^\circ)$$

$$= \sin 30^\circ \tan 45^\circ + \cos 60^\circ = \frac{1}{2} \times 1 + \frac{1}{2} = 1$$

8

$$\therefore \sin X = \tan 30^\circ \sin 60^\circ \quad \therefore \sin X = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$$

$$\therefore \sin X = \frac{1}{2} \quad \therefore X = 30^\circ$$

$$\therefore 4 \cos X \tan 2X = 4 \cos 30^\circ \tan 60^\circ$$

$$= 4 \times \frac{\sqrt{3}}{2} \times \sqrt{3} = 6$$

9

Angle Radian	30°	60°	45°	Angle Radian	34° 12'	51° 33' 35"	65° 45' 13"	36° 52' 12"
sin	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	sin	0.562	0.7833	0.9118	0.6
cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	cos	0.827	0.6217	0.4107	0.8
tan	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	1	tan	0.6796	1.2599	2.2203	0.75

10

$$1 \because \tan 32^\circ = \frac{6}{BC}$$

$$\therefore BC = 6 \div \tan 32^\circ \approx 9.6 \text{ cm.}$$

$$2 \because \cos 50^\circ = \frac{AB}{8}$$

$$\therefore AB = 8 \times \cos 50^\circ \approx 5.14 \text{ cm.}$$

$$3 \because \sin 65^\circ = \frac{BC}{12}$$

$$\therefore BC = 12 \times \sin 65^\circ \approx 10.88 \text{ cm.}$$

11

$$1 \because \sin A = \frac{4}{6} \quad \therefore m(\angle A) = 41^\circ 48' 37''$$

$$2 \because \tan C = \frac{10}{6} \quad \therefore m(\angle C) = 59^\circ 2' 10''$$

$$3 \because \cos C = \frac{5}{7} \quad \therefore m(\angle C) = 44^\circ 24' 55''$$

12

 In  $\triangle ACD$  :

$$\therefore m(\angle DAC) = 90^\circ, \quad m(\angle D) = 30^\circ$$

$$\therefore AC = \frac{1}{2} CD = \frac{1}{2} \times 8 = 4 \text{ cm.}$$

$$\therefore \tan B = \frac{AC}{BC} = \frac{4}{3}$$

(First req.)

 In  $\triangle ABC$  :

$$\therefore \tan(\angle BAC) = \frac{3}{4}$$

$$\therefore m(\angle BAC) \approx 36^\circ 52' 12''$$

$$\therefore m(\angle BAD) = 90^\circ + 36^\circ 52' 12''$$

$$= 126^\circ 52' 12''$$

(Second req.)

13

 Draw  $\overline{AD} \perp \overline{BC}$  to cut it at D

$$\therefore \overline{AD} \perp \overline{BC}, \quad AB = AC$$

$$\therefore BD = DC = 5 \text{ cm.}$$

$$\text{In } \triangle ABD : \cos B = \frac{5}{7}$$

$$\therefore m(\angle B) \approx 44^\circ 24' 55''$$

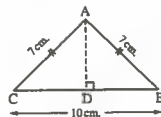
(First req.)

$$\text{In } \triangle ABD : \therefore (AD)^2 = (AB)^2 - (BD)^2$$

$$\therefore (AD)^2 = 49 - 25 = 24 \quad \therefore AD = 2\sqrt{6} \text{ cm.}$$

$$\therefore \text{The area of } \triangle ABC = \frac{1}{2} \times 10 \times 2\sqrt{6}$$

$$= 10\sqrt{6} \text{ cm}^2. \quad (\text{Second req.})$$



14

 Draw  $\overline{AD} \perp \overline{BC}$  to cut it at D

$$\therefore \overline{AD} \perp \overline{BC}, \quad AB = AC$$

$$\therefore BD = DC$$

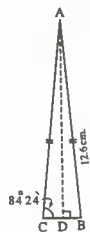
$$\text{In } \triangle ADC : \cos C = \frac{DC}{AC}$$

$$\therefore \cos 84^\circ 24' = \frac{DC}{12.6}$$

$$\therefore DC = 12.6 \times \cos 84^\circ 24' \approx 1.23$$

$$\therefore BC = 2 \times 1.23 = 2.46 \approx 2.5 \text{ cm.}$$

(The req.)



15

 $\therefore \triangle ABC$  is a right-angled triangle at B

$$\therefore m(\angle A) = 2m(\angle C)$$

$$\therefore 2m(\angle C) + m(\angle C) = 90^\circ$$

$$\therefore 3m(\angle C) = 90^\circ$$

$$\therefore m(\angle C) = 30^\circ$$

$$\therefore m(\angle A) = 60^\circ$$

$$\therefore \cos^2 A + \tan^2 C = \cos^2 60^\circ + \tan^2 30^\circ$$

$$= \frac{1}{4} + \frac{1}{3} = \frac{3+4}{12} = \frac{7}{12}$$

16

 $\therefore ABCD$  is a rectangle

$$\therefore m(\angle B) = 90^\circ$$

 In  $\triangle ABC$ :

$$\therefore \sin(\angle ACB) = \frac{15}{25}$$

$$\therefore m(\angle ACB) = 36^\circ 52' 12''$$

(First req.)

 In  $\triangle ABC$ :

$$\therefore (BC)^2 = (AC)^2 - (AB)^2$$

$$\therefore (BC)^2 = 625 - 225 = 400 \quad \therefore BC = 20 \text{ cm.}$$

$$\therefore \text{The area of the rectangle} = 15 \times 20 = 300 \text{ cm}^2.$$

(Second req.)

17

 $\therefore ABCD$  is a rectangle

$$\therefore m(\angle B) = 90^\circ$$

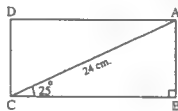
 In  $\triangle ABC$ :

$$\therefore \cos(\angle ACB) = \frac{BC}{AC}$$

$$\therefore \cos 25^\circ = \frac{BC}{24}$$

$$\therefore BC = 24 \cos 25^\circ \approx 21.8 \text{ cm.}$$

(The req.)



18

$$BC = 96 \div 8 = 12 \text{ cm.}$$

$$\therefore AD = BC \quad \therefore AD = 12 \text{ cm.} \quad (\text{First req.})$$

$$\therefore BC = 12 \text{ cm.}, BE = \frac{1}{4}BC \quad \therefore BE = 3 \text{ cm.}$$

 In  $\triangle ABE$ :

$$\therefore m(\angle AEB) = 90^\circ \quad \therefore \tan B = \frac{AE}{BE} = \frac{8}{3}$$

$$\therefore m(\angle B) = 69^\circ 26' 38'' \quad (\text{Second req.})$$

 In  $\triangle AEB$ :

$$\therefore \sin B = \frac{AE}{AB} \quad \therefore \sin 69^\circ 26' 38'' = \frac{8}{AB}$$

$$\therefore AB = \frac{8}{\sin 69^\circ 26' 38''} \approx 8.5 \text{ cm.} \quad (\text{Third req.})$$

Another solution:

$$\text{In } \triangle AEB: \therefore \cos B = \frac{BE}{AB}$$

$$\therefore \cos 69^\circ 26' 38'' = \frac{3}{AB}$$

$$\therefore AB = \frac{3}{\cos 69^\circ 26' 38''} \approx 8.5 \text{ cm.} \quad (\text{Third req.})$$

A third solution:

 $\therefore \triangle ABE$  is right-angled at E

$$\therefore (AB)^2 = (AE)^2 + (BE)^2 \\ = 8^2 + 3^2 = 64 + 9 = 73$$

$$\therefore AB = \sqrt{73} \approx 8.5 \text{ cm.} \quad (\text{Third req.})$$

19

 Draw  $AF \perp BC$ ,  $DE \perp BC$ 

$$\therefore AD \parallel BC, AF \perp BC,$$

$$DE \perp BC$$

$$\therefore AFED \text{ is a rectangle, } FE = 5 \text{ cm.}$$

$$\therefore BF + EC = 6 \text{ cm.}$$

$$\therefore BF = 3 \text{ cm.}$$

 In  $\triangle ABF$ :

$$\therefore \cos B = \frac{BF}{AB} \quad \therefore \cos B = \frac{3}{5}$$

$$\therefore m(\angle B) = 53^\circ 7' 48''$$

$$\therefore m(\angle A) = 180^\circ - (53^\circ 7' 48'')$$

$$= 126^\circ 52' 12''$$

(First req.)

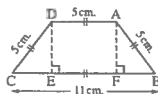
 In  $\triangle ABF$ :

$$\therefore (AF)^2 = (AB)^2 - (BF)^2$$

$$\therefore (AF)^2 = (5)^2 - (3)^2 = 16 \quad \therefore AF = 4 \text{ cm.}$$

$$\therefore \text{the area of the trapezium} = \frac{1}{2}(5 + 11) \times 4 = 32 \text{ cm}^2.$$

(Second req.)



20

 Draw  $DE \perp BC$ 

$$\therefore AD \parallel BC, AB \perp BE$$

$$DE \perp BE$$

$$\therefore ABED \text{ is a rectangle.}$$

$$\therefore BE = 16 \text{ cm.}$$

$$\therefore EC = 25 - 16 = 9 \text{ cm.}, DE = 12 \text{ cm.}$$

$$\text{In } \triangle DEC: \therefore m(\angle DEC) = 90^\circ$$

$$\therefore (DC)^2 = (12)^2 + (9)^2 = 225$$

$$\therefore DC = 15 \text{ cm.}$$

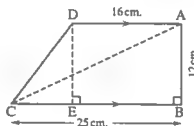
(First req.)

$$\therefore \tan C = \frac{12}{9}$$

$$\therefore m(\angle C) = 53^\circ 7' 48'' \quad (\text{Second req.})$$

$$\therefore \sin(\angle DCB) - \tan(\angle ACB) = \frac{12}{15} - \frac{12}{25} = \frac{8}{25}$$

(Third req.)



21

 In  $\triangle ABC$ :

$$\therefore m(\angle C) = 90^\circ$$

$$\therefore \sin B = \frac{AC}{AB}$$

$$\therefore \sin 60^\circ = \frac{AC}{6}$$

$$\therefore AC = 6 \sin 60^\circ = 3\sqrt{3} \text{ m.}$$



(The req.)

22

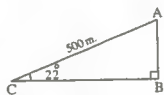
 In  $\triangle ABC$ :

$$\therefore m(\angle B) = 90^\circ$$

$$\therefore \sin C = \frac{AB}{AC}$$

$$\therefore AB = 500 \sin 22^\circ \approx 187 \text{ m.}$$

(The req.)



23

 In  $\triangle ABC$ :

$$\therefore m(\angle B) = 90^\circ$$

$$\therefore \cos A = \frac{AB}{AC}$$

$$\therefore \cos 60^\circ = \frac{4}{AC}$$

$$\therefore AC = \frac{4}{\cos 60^\circ} = 8 \text{ m.}$$

$$\therefore (BC)^2 = 8^2 - 4^2 = 48 \quad \therefore BC = 4\sqrt{3} \text{ m.}$$

$$\therefore \text{The height of the tree} = 8 + 4\sqrt{3} \approx 15 \text{ m.}$$



24

$$\therefore \cos A \times \tan A = \frac{1}{2}$$

$$\therefore \cos A \times \frac{\sin A}{\cos A} = \frac{1}{2}$$

$$\therefore \sin A = \frac{1}{2}$$

$$\therefore A = 30^\circ$$

25

 Draw:  $\overline{AD} \perp \overline{BC}$ 

$$\therefore m(\angle ABD) = 180^\circ - 135^\circ = 45^\circ$$

$$\therefore \tan(\angle ABD) = \frac{AD}{DB}$$

$$\therefore \tan 45^\circ = \frac{AD}{DB} = 1$$

$$\therefore AD = DB = x \text{ cm.}$$

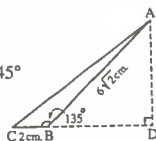
 $\therefore \triangle ADB$  which is right-angled at D

$$\therefore (AB)^2 = x^2 + x^2 \quad \therefore 72 = 2x^2$$

$$\therefore x^2 = 36$$

$$\therefore x = 6 \text{ cm.}$$

$$\therefore \tan C = \frac{AD}{DC} = \frac{6}{8} = \frac{3}{4}$$



## Answers of exams on unit four

## Model 1

1

1 c

2 a

3 c

4 a

5 c

6 d

2

[a] Prove by yourself.

[b] 1  $x = 3$

2  $x = 30^\circ$

3

[a] Prove by yourself.

[b] 1

4

[a] 1  $m(\angle ACB) \approx 36^\circ 52' 12''$

2 The area of the rectangle =  $300 \text{ cm}^2$

[b] The height of the tree  $\approx 5 \text{ m.}$

5

[a]  $30^\circ$

[b]  $\frac{2\sqrt{3}}{3}$

## Model 2

1

1 c

2 c

3 d

4 d

5 d

6 c

2

[a]  $x = 30^\circ$

[b]  $33^\circ 45'$ ,  $56^\circ 15'$

3

[a] 1  $\cos B = \frac{3}{5}$

2  $m(\angle B) = 53^\circ 7' 48''$

3  $\sin(90^\circ - B) = \frac{3}{5}$

[b] Prove by yourself.

4

[a] 1  $CB = 8 \text{ cm.}$ ,  $AC = 10 \text{ cm.}$

2  $\sin A + \cos A = \frac{7}{5}$

[b] 1  $\frac{56}{15}$  2  $\frac{56}{65}$

5

[a] 1 Prove by yourself.

2  $m(\angle C) \approx 22^\circ 37' 12''$

[b] 1

## Answers of unit five

## Answers of Exercise 3

1

$$1 \quad AB = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{9+16} = 5 \text{ length unit.}$$

$$2 \quad AB = \sqrt{(5-2)^2 + (-5+1)^2} = \sqrt{9+16} = 5 \text{ length unit.}$$

$$3 \quad AB = \sqrt{(3+2)^2 + (-5-7)^2} \\ = \sqrt{25+144} = 13 \text{ length unit.}$$

$$4 \quad AB = \sqrt{(3+2)^2 + (0-5)^2} \\ = \sqrt{25+25} = 5\sqrt{2} \text{ length unit.}$$

$$5 \quad AB = \sqrt{(15-6)^2 + (0-0)^2} = \sqrt{9^2} = 9 \text{ length unit.}$$

$$8 \quad AB = \sqrt{(6-0)^2 + (0-8)^2} \\ = \sqrt{36+64} = 10 \text{ length unit.}$$

2

1 d

2 c

3 b

4 a

5 b

6 c

7 c

8 d

9 c

10 d

11 a

12 c

3

$$AB = \sqrt{(1-3)^2 + (2-1)^2} = \sqrt{4+1} = \sqrt{5} \text{ length unit.}$$

$$BC = \sqrt{(5-1)^2 + (4-2)^2} = \sqrt{16+4} = \sqrt{20} \\ = 2\sqrt{5} \text{ length unit.}$$

$$\therefore BC = 2AB$$

4

$$AB = \sqrt{(1-4)^2 + (1-3)^2} = \sqrt{9+4} = \sqrt{13} \text{ length unit}$$

$$BC = \sqrt{(-5-1)^2 + (-3-1)^2} = \sqrt{36+16} = \sqrt{52} \\ = 2\sqrt{13} \text{ length unit}$$

$$AC = \sqrt{(4+5)^2 + (3+3)^2} = \sqrt{81+36} = \sqrt{117} \\ = 3\sqrt{13} \text{ length unit}$$

$$\therefore AC = AB + BC$$

$$\therefore A, B \text{ and } C \text{ are collinear.}$$

5

$$\therefore CA = \sqrt{(3+2)^2 + (4-2)^2} = \sqrt{25+4} = \sqrt{29} \text{ length unit}$$

$$CB = \sqrt{(3-1)^2 + (4+1)^2} = \sqrt{4+25} = \sqrt{29} \text{ length unit}$$

$$\therefore CA = CB$$

$$\therefore C \text{ lies on the axis of symmetry of } \overline{AB}$$

6

$$1 \quad AB = \sqrt{(3-1)^2 + (-2-4)^2} \\ = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10} \text{ length unit.}$$

$$BC = \sqrt{(-3-3)^2 + (16+2)^2} \\ = \sqrt{36+324} = \sqrt{360} = 6\sqrt{10} \text{ length unit.}$$

$$AC = \sqrt{(-3-1)^2 + (16-4)^2} \\ = \sqrt{16+144} = \sqrt{160} = 4\sqrt{10} \text{ length unit.}$$

$$\therefore BC = AB + AC$$

$$\therefore A, B \text{ and } C \text{ are collinear points.}$$

$$2 \quad AB = \sqrt{(-3-7)^2 + (6-0)^2} = \sqrt{100+36} \\ = 2\sqrt{34} \text{ length unit.}$$

$$BC = \sqrt{(22+3)^2 + (9-6)^2} = \sqrt{625+9} \\ = \sqrt{634} \text{ length unit.}$$

$$AC = \sqrt{(22-7)^2 + (9-0)^2} = \sqrt{225+81} \\ = 3\sqrt{34} \text{ length unit.}$$

$$\therefore BC \neq AB + AC$$

$$\therefore A, B \text{ and } C \text{ are non-collinear points}$$

$$3 \quad AB = \sqrt{(3+1)^2 + (-14-4)^2} \\ = \sqrt{16+324} = 2\sqrt{85} \text{ length unit.}$$

$$BC = \sqrt{(-5-3)^2 + (-6+14)^2} \\ = \sqrt{64+64} = 8\sqrt{2} \text{ length unit.}$$

$$AC = \sqrt{(-5+1)^2 + (-6-4)^2} = \sqrt{16+100} \\ = 2\sqrt{29} \text{ length unit}$$

$$\therefore AB \neq BC + AC$$

$$\therefore A, B \text{ and } C \text{ are non-collinear points.}$$

7

$$\therefore AB = \sqrt{(-2-3)^2 + (4+1)^2} = \sqrt{25+25} = \sqrt{50} \\ = 5\sqrt{2} \text{ length unit}$$

$$BC = \sqrt{(3-4)^2 + (-1-5)^2} = \sqrt{1+36} \\ = \sqrt{37} \text{ length unit}$$

$$AC = \sqrt{(-2-4)^2 + (4-5)^2} = \sqrt{36+1} \\ = \sqrt{37} \text{ length unit}$$

$$\therefore BC = AC$$

$$\therefore \triangle ABC \text{ is an isosceles triangle.}$$

8

$$1 \quad \therefore AB = \sqrt{(4-2)^2 + (-2-1)^2} \\ = \sqrt{4+9} = \sqrt{13} \text{ length unit.}$$

$$\begin{aligned} \therefore BC &= \sqrt{(7-4)^2 + (5+2)^2} = \sqrt{9+49} \\ &= \sqrt{58} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} \therefore AC &= \sqrt{(7-2)^2 + (5-1)^2} \\ &= \sqrt{25+16} = \sqrt{41} \text{ length unit.} \end{aligned}$$

$$\therefore (BC)^2 > (AB)^2 + (AC)^2$$

$\therefore A, B$  and  $C$  are vertices of an obtuse-angled triangle at  $A$

$$\boxed{2} \quad AB = \sqrt{(-1-3)^2 + (1-5)^2} = \sqrt{16+16} = \sqrt{32} \text{ length unit.}$$

$$\begin{aligned} \therefore BC &= \sqrt{(5+1)^2 + (-5-1)^2} = \sqrt{36+36} \\ &= \sqrt{72} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} \therefore AC &= \sqrt{(5-3)^2 + (-5-5)^2} = \sqrt{4+100} \\ &= \sqrt{104} \text{ length unit.} \end{aligned}$$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2$$

$\therefore A, B$  and  $C$

are vertices of a right-angled triangle at  $B$

$$\boxed{3} \quad AB = \sqrt{(3-4)^2 + (-1-4)^2} = \sqrt{1+25} = \sqrt{26} \text{ length unit.}$$

$$\begin{aligned} \therefore BC &= \sqrt{(-2-3)^2 + (4+1)^2} \\ &= \sqrt{25+25} = \sqrt{50} \text{ length unit.} \end{aligned}$$

$$\therefore AC = \sqrt{(-2-4)^2 + (4-4)^2} = \sqrt{36} = 6 \text{ length units.}$$

$\therefore \overline{BC}$  is the longest side,

$$\therefore (BC)^2 < (AB)^2 + (AC)^2$$

$\therefore A, B$  and  $C$  are vertices of an acute-angled triangle.

$$\boxed{4} \quad AB = \sqrt{(6-0)^2 + (0-0)^2} = \sqrt{36+0} = 6 \text{ length units.}$$

$$\begin{aligned} \therefore BC &= \sqrt{(0-6)^2 + (8-0)^2} = \sqrt{36+64} = \sqrt{100} \\ &= 10 \text{ length units} \end{aligned}$$

$$\therefore AC = \sqrt{(0-0)^2 + (8-0)^2} = \sqrt{0+64} = 8 \text{ length units}$$

$$\therefore (BC)^2 = (AB)^2 + (AC)^2$$

$\therefore A, B$  and  $C$  are vertices of a right-angled triangle at  $A$

$$\boxed{5} \quad AB = \sqrt{(2-1)^2 + (1+1)^2} = \sqrt{1+4} = \sqrt{5} \text{ length unit.}$$

$$\begin{aligned} \therefore BC &= \sqrt{(-3-2)^2 + (-2-1)^2} = \sqrt{25+9} \\ &= \sqrt{34} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} \therefore AC &= \sqrt{(1+3)^2 + (-1+2)^2} = \sqrt{16+1} \\ &= \sqrt{17} \text{ length unit.} \end{aligned}$$

$$\therefore (BC)^2 > (AB)^2 + (AC)^2 \quad \therefore A, B \text{ and } C$$

are vertices of an obtuse-angled triangle at  $A$

9

$$\begin{aligned} AB &= \sqrt{(-1-5)^2 + (7+5)^2} = \sqrt{36+144} \\ &= \sqrt{180} \text{ length unit} \end{aligned}$$

$$\begin{aligned} \therefore BC &= \sqrt{(15+1)^2 + (15-7)^2} = \sqrt{256+64} \\ &= \sqrt{320} \text{ length unit} \end{aligned}$$

$$\begin{aligned} \text{and } CA &= \sqrt{(5-15)^2 + (-5-15)^2} = \sqrt{100+400} \\ &= \sqrt{500} \text{ length unit} \end{aligned}$$

$$\therefore (CA)^2 = (AB)^2 + (BC)^2$$

$\therefore \triangle ABC$  is right angled at  $B$

$$\begin{aligned} \therefore \text{Area of } \triangle ABC &= \frac{1}{2} \times AB \times BC \\ &= \frac{1}{2} \times \sqrt{180} \times \sqrt{320} \\ &= 120 \text{ square units} \end{aligned}$$

10

$$\begin{aligned} \therefore AB &= \sqrt{(7-5)^2 + (2\sqrt{3}-0)^2} \\ &= \sqrt{4+12} = 4 \text{ length unit.} \end{aligned}$$

$$\begin{aligned} \therefore BC &= \sqrt{(3-7)^2 + (2\sqrt{3}-2\sqrt{3})^2} \\ &= \sqrt{16} = 4 \text{ length unit.} \end{aligned}$$

$$\begin{aligned} \therefore AC &= \sqrt{(3-5)^2 + (2\sqrt{3}-0)^2} \\ &= \sqrt{4+12} = 4 \text{ length unit.} \end{aligned}$$

$\therefore \triangle ABC$  is an equilateral triangle

Let  $M$  be the midpoint of the base  $\overline{AB}$

$$\begin{aligned} \therefore \text{The height } MC &= \sqrt{(4)^2 - (2)^2} = \sqrt{16-4} = \sqrt{12} \\ &= 2\sqrt{3} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} \therefore \text{the area of } \triangle ABC &= \frac{1}{2} \times AB \times MC \\ &= \frac{1}{2} \times 4 \times 2\sqrt{3} \\ &= 4\sqrt{3} \text{ square unit.} \end{aligned}$$

11

$\therefore ABCO$  is a rectangle

$\therefore AC = BO$  (properties of the rectangle)

$$\begin{aligned} \therefore AC &= \sqrt{(9-0)^2 + (12-0)^2} = \sqrt{81+144} \\ &= 15 \text{ length units.} \end{aligned}$$

12

$$\boxed{1} \quad AB = \sqrt{(0+1)^2 + (5-1)^2} = \sqrt{1+16} = \sqrt{17} \text{ length unit.}$$

$$\therefore BC = \sqrt{(5-0)^2 + (6-5)^2} = \sqrt{25+1} = \sqrt{26} \text{ length unit.}$$

$$\therefore CD = \sqrt{(4-5)^2 + (2-6)^2} = \sqrt{1+16} = \sqrt{17} \text{ length unit.}$$

$$\therefore DA = \sqrt{(-1-4)^2 + (1-2)^2} = \sqrt{25+1} = \sqrt{26} \text{ length unit.}$$

$$\therefore AB = CD, BC = DA$$

$\therefore$  ABCD is a parallelogram.

$$\begin{aligned} \text{② } AB &= \sqrt{(5+2)^2 + (-3-4)^2} = \sqrt{49+49} \\ &= \sqrt{98} = 7\sqrt{2} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(7-5)^2 + (1+3)^2} = \sqrt{4+16} \\ &= \sqrt{20} = 2\sqrt{5} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(0-7)^2 + (8-1)^2} = \sqrt{49+49} \\ &= \sqrt{98} = 7\sqrt{2} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} DA &= \sqrt{(0+2)^2 + (8-4)^2} = \sqrt{4+16} \\ &= \sqrt{20} = 2\sqrt{5} \text{ length unit.} \end{aligned}$$

$$\therefore AB = CD, BC = DA$$

$\therefore$  ABCD is a parallelogram.

13

$$\begin{aligned} \therefore AB &= \sqrt{(0-4)^2 + (1-5)^2} = \sqrt{16+16} \\ &= \sqrt{32} = 4\sqrt{2} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(4-1)^2 + (5-8)^2} = \sqrt{9+9} \\ &= \sqrt{18} = 3\sqrt{2} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(1+3)^2 + (8-4)^2} = \sqrt{16+16} \\ &= \sqrt{32} = 4\sqrt{2} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} AD &= \sqrt{(0+3)^2 + (1-4)^2} = \sqrt{9+9} \\ &= \sqrt{18} = 3\sqrt{2} \text{ length unit.} \end{aligned}$$

$$\therefore AB = CD, BC = AD$$

$\therefore$  the figure ABCD is a parallelogram

$$\begin{aligned} \therefore AC &= \sqrt{(0-1)^2 + (1-8)^2} = \sqrt{1+49} \\ &= \sqrt{50} = 5\sqrt{2} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} BD &= \sqrt{(4+3)^2 + (5-4)^2} = \sqrt{49+1} \\ &= \sqrt{50} = 5\sqrt{2} \text{ length unit.} \end{aligned}$$

$$\therefore AC = BD = 5\sqrt{2} \text{ length unit.}$$

$\therefore$  The figure ABCD is a rectangle its diagonal length =  $5\sqrt{2}$  length unit.

14

$$\therefore AB = \sqrt{(3-0)^2 + (3-3)^2} = \sqrt{9+0} = 3 \text{ length unit.}$$

$$BC = \sqrt{(0-0)^2 + (3-0)^2} = \sqrt{0+9} = 3 \text{ length unit.}$$

$$CD = \sqrt{(0-3)^2 + (0-0)^2} = \sqrt{9+0} = 3 \text{ length unit.}$$

$$DA = \sqrt{(3-3)^2 + (0-3)^2} = \sqrt{0+9} = 3 \text{ length unit.}$$

$$\therefore AB = BC = CD = DA \quad \therefore \text{ABCD is a rhombus}$$

$$\begin{aligned} \therefore AC &= \sqrt{(3-0)^2 + (3-0)^2} = \sqrt{9+9} = \sqrt{18} \\ &= 3\sqrt{2} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} BD &= \sqrt{(0-3)^2 + (3-0)^2} = \sqrt{9+9} = \sqrt{18} \\ &= 3\sqrt{2} \text{ length unit.} \end{aligned}$$

$$\therefore AC = BD$$

$\therefore$  The figure ABCD is a square, the length of its diagonal =  $3\sqrt{2}$  length unit, its area =  $3 \times 3 = 9$  square unit.

15

$$AB = \sqrt{(6-5)^2 + (-2-3)^2} = \sqrt{1+25} = \sqrt{26} \text{ length unit.}$$

$$\begin{aligned} BC &= \sqrt{(1-6)^2 + (-1+2)^2} \\ &= \sqrt{25+1} = \sqrt{26} \text{ length unit.} \end{aligned}$$

$$CD = \sqrt{(0-1)^2 + (4+1)^2} = \sqrt{1+25} = \sqrt{26} \text{ length unit.}$$

$$DA = \sqrt{(5-0)^2 + (3-4)^2} = \sqrt{25+1} = \sqrt{26} \text{ length unit.}$$

$$\therefore AB = BC = CD = DA$$

$\therefore$  The figure ABCD is a rhombus

$$\begin{aligned} \therefore AC &= \sqrt{(1-5)^2 + (-1-3)^2} \\ &= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} BD &= \sqrt{(0-6)^2 + (4+2)^2} \\ &= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} \therefore \text{the area of the rhombus} &= \frac{1}{2} AC \times BD \\ &= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} \\ &= 24 \text{ square units.} \end{aligned}$$

16

$$AB = \sqrt{(3+2)^2 + (3-5)^2} = \sqrt{25+4} = \sqrt{29} \text{ length unit.}$$

$$BC = \sqrt{(-4-3)^2 + (2-3)^2} = \sqrt{49+1} = \sqrt{50} \text{ length unit.}$$

$$CA = \sqrt{(-2+4)^2 + (5-2)^2} = \sqrt{4+9} = \sqrt{13} \text{ length unit.}$$

$\therefore BC$  is the greatest distance

$$BC < AB + CA$$

$\therefore A, B, C$  are non-collinear points

$$\begin{aligned} AD &= \sqrt{(-2+9)^2 + (5-4)^2} \\ &= \sqrt{49+1} = \sqrt{50} \text{ length unit.} \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(-9+4)^2 + (4-2)^2} \\ &= \sqrt{25+4} = \sqrt{29} \text{ length unit} \end{aligned}$$

$$\therefore AB = CD, BC = AD$$

$\therefore$  ABCD is a parallelogram

17

$$AB = \sqrt{(-3-2)^2 + (0-4)^2} = \sqrt{25+16} = \sqrt{41} \text{ length unit.}$$

$$BC = \sqrt{(-7+3)^2 + (5-0)^2} = \sqrt{16+25} = \sqrt{41} \text{ length unit.}$$

$$CD = \sqrt{(-2+7)^2 + (9-5)^2} = \sqrt{25+16} = \sqrt{41} \text{ length unit.}$$

$$DA = \sqrt{(2+2)^2 + (4-9)^2} = \sqrt{16+25} = \sqrt{41} \text{ length unit.}$$

$\therefore$  ABCD is a rhombus

$$\begin{aligned} \therefore AC &= \sqrt{(-7-2)^2 + (5-4)^2} \\ &= \sqrt{81+1} = \sqrt{82} \text{ length unit.} \end{aligned}$$

$$BD = \sqrt{(-2+3)^2 + (9-0)^2} = \sqrt{1+81} = \sqrt{82}$$

$$\therefore AC = BD$$

$\therefore$  The figure ABCD is a square.

18

$$\begin{aligned} \therefore MA &= \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9} \\ &= \sqrt{25} = 5 \text{ length units} \end{aligned}$$

$$\begin{aligned} MB &= \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16} \\ &= \sqrt{25} = 5 \text{ length units} \end{aligned}$$

$$\begin{aligned} \text{and } MC &= \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16} \\ &= \sqrt{25} = 5 \text{ length units} \end{aligned}$$

$$\therefore MA = MB = MC$$

$\therefore$  A, B and C lie on the circle M which its radius length is 5 length units

$$\begin{aligned} \therefore \text{circumference of the circle} &= 2\pi r \\ &= 2 \times 3.14 \times 5 \\ &= 31.4 \text{ length units} \end{aligned}$$

19

$$\therefore \sqrt{(x-6)^2 + (5-1)^2} = 2\sqrt{5}$$

$$\therefore \sqrt{(x-6)^2 + 16} = 2\sqrt{5} \text{ "squaring the two sides"}$$

$$\therefore (x-6)^2 + 16 = 20$$

$$\therefore x^2 - 12x + 36 + 16 - 20 = 0$$

$$\therefore x^2 - 12x + 32 = 0$$

$$\therefore (x-4)(x-8) = 0$$

$$\therefore x = 4 \text{ or } x = 8$$

20

$$\therefore \sqrt{(a+2)^2 + (7-3)^2} = 5 \text{ "squaring the two sides"}$$

$$\therefore (a+2)^2 + (4)^2 = 25 \quad \therefore a^2 + 4a + 4 + 16 = 25$$

$$\therefore a^2 + 4a - 5 = 0 \quad \therefore (a-1)(a+5) = 0$$

$$\therefore a = 1 \text{ or } a = -5$$

$$\therefore \sqrt{(3a-1-a)^2 + (-5-7)^2} = 13 \text{ "squaring the two sides"}$$

$$\therefore (2a-1)^2 + (-12)^2 = 169$$

$$\therefore 4a^2 - 4a + 1 + 144 = 169$$

$$\therefore 4a^2 - 4a - 24 = 0 \text{ "dividing by 4 for both sides"}$$

$$\therefore a^2 - a - 6 = 0 \quad \therefore (a+2)(a-3) = 0$$

$$\therefore a = -2 \text{ or } a = 3$$

21

$$BC = \sqrt{(5-3)^2 + (1-2)^2} = \sqrt{4+1} = \sqrt{5} \text{ length unit.}$$

$$\therefore AB = \sqrt{5} \text{ length unit.}$$

$$\therefore \sqrt{(x-3)^2 + (3-2)^2} = \sqrt{5} \text{ "squaring the two sides"}$$

$$\therefore (x-3)^2 + (1)^2 = 5 \quad \therefore x^2 - 6x + 9 + 1 = 5$$

$$\therefore x^2 - 6x + 5 = 0 \quad \therefore (x-5)(x-1) = 0$$

$$\therefore x = 5 \text{ or } x = 1$$

22

$$\begin{aligned} \therefore AB &= \sqrt{(-6-9)^2 + (0-0)^2} = \sqrt{225} \\ &= 15 \text{ length units.} \end{aligned}$$

let  $C = (0, y)$

$$\therefore AC = \sqrt{(0-9)^2 + (y-0)^2} = \sqrt{81+y^2}$$

$$\therefore AB = AC$$

$$\therefore 15 = \sqrt{81+y^2} \text{ "squaring the two sides"}$$

$$\therefore 225 = 81 + y^2 \quad \therefore y^2 = 225 - 81 = 144$$

$$\therefore y = 12$$

or  $y = -12$  (refused because the point C lies on the positive part of y-axis)

$$\therefore C(0, 12)$$

$$\begin{aligned} \therefore CO &= \sqrt{(0-0)^2 + (12-0)^2} = \sqrt{144} \\ &= 12 \text{ length units.} \end{aligned}$$

23

$\therefore$  The axis of symmetry of  $\overline{CD}$  passes through the point A

$$\therefore CA = DA$$

$$\therefore \sqrt{(6-3)^2 + (m-1)^2} = \sqrt{(6+3)^2 + (m-7)^2}$$

"squaring the two sides"

$$\therefore 3^2 + (m-1)^2 = 9^2 + (m-7)^2$$

$$\therefore 9 + m^2 - 2m + 1 = 81 + m^2 - 14m + 49$$

$$\therefore -2m + 14m = 81 + 49 - 9 - 1$$

$$\therefore 12m = 120 \quad \therefore m = \frac{120}{12} = 10$$

24

 $\therefore A \in \text{the } x\text{-axis}$ 

 Let  $A = (x, 0)$ 

$$\therefore AO = AB \quad \therefore x = \sqrt{(x+9)^2 + (0-15)^2}$$

$$\therefore x = \sqrt{x^2 + 18x + 81 + 225}$$

$$\therefore x^2 = x^2 + 18x + 306 \quad \therefore 18x = -306$$

$$\therefore x = -17 \quad \therefore AB = 17 \text{ length units}$$

25

 $\therefore f(x) = x$ 
 $\therefore C \text{ is } (x, x)$ 
 $\therefore AO = 4 \text{ length units}$ 
 $\therefore A(4, 0)$ 
 $\therefore OB = 6 \text{ length units}$ 
 $\therefore B(0, 6)$ 
 $\therefore AC = BC$ 

$$\therefore \sqrt{(x-4)^2 + (x-0)^2} = \sqrt{(x-0)^2 + (x-6)^2}$$

"squaring the two sides"

$$\therefore x^2 - 8x + 16 + x^2 = x^2 + x^2 - 12x + 36$$

$$\therefore 4x = 20 \quad \therefore x = 5 \quad \therefore C(5, 5)$$

26

 The point that represents Basem's house is  $(1, 9)$ 

 The point that represents Eslam's house is  $(3, 10)$ 

 The point that represents school is  $(10, 2)$ 

 The point that represents railway station is  $(4, 0)$ 

1 The distance between Basem's house and the

$$\begin{aligned} \text{school} &= \sqrt{(10-1)^2 + (2-9)^2} \\ &= \sqrt{81 + 49} = \sqrt{130} \text{ km.} \end{aligned}$$

 $\therefore$  the distance between Eslam's house and the

$$\begin{aligned} \text{school} &= \sqrt{(10-3)^2 + (2-10)^2} \\ &= \sqrt{49 + 64} = \sqrt{113} \text{ km.} \end{aligned}$$

 $\therefore$  Eslam's house is nearer to the school.

2 The distance between Basem's house and the

$$\text{school} = \sqrt{130} \text{ km.}$$

 $\therefore$  the distance between Basem's house and

$$\begin{aligned} \text{railway station} &= \sqrt{(1-4)^2 + (9-0)^2} \\ &= \sqrt{9 + 81} = \sqrt{90} \text{ km.} \end{aligned}$$

 $\therefore$  the distance between the school and railway station

$$= \sqrt{(10-4)^2 + (2-0)^2} = \sqrt{36 + 4} = \sqrt{40} \text{ km.}$$

 $\therefore$  The square of the distance between Basem's house and the school equals the sum of the squares of the distance between Basem's house and railway, and the distance between the school and railway.

 $\therefore$  Basem's house, the school and railway station make a right-angled triangle at the railway station.

 $\therefore$  The way (School - railway station) is perpendicular to the way (Basem's house - railway station)

27

$$AB = \sqrt{(x-4)^2 + (2+2)^2} = \sqrt{(x-4)^2 + 16} \text{ length unit.}$$

$$\therefore BC = \sqrt{(x-3)^2 + (2-5)^2} = \sqrt{(x-3)^2 + 9} \text{ length unit.}$$

$$\therefore CA = \sqrt{(4-3)^2 + (-2-5)^2} = \sqrt{1 + 49} = \sqrt{50} \text{ length unit.}$$

 $\therefore \triangle ABC$  is right-angled at B

$$\therefore (CA)^2 = (AB)^2 + (BC)^2$$

$$\therefore (x-4)^2 + 16 + (x-3)^2 + 9 = 50$$

$$\therefore x^2 - 8x + 16 + 16 + x^2 - 6x + 9 + 9 = 50$$

$$\therefore 2x^2 - 14x = 0 \text{ "dividing by 2"}$$

$$\therefore x^2 - 7x = 0 \quad \therefore x(x-7) = 0$$

$$\therefore x = 0 \text{ or } x = 7 \quad \text{At } x = 0$$

$$\therefore AB = \sqrt{32} = 4\sqrt{2} \text{ length unit}$$

$$\therefore BC = \sqrt{18} = 3\sqrt{2} \text{ length unit}$$

$$\begin{aligned} \therefore \text{the area of } \triangle ABC &= \frac{1}{2} AB \times BC \\ &= \frac{1}{2} \times 4\sqrt{2} \times 3\sqrt{2} \\ &= 12 \text{ square units.} \end{aligned}$$

 At  $x = 7 \therefore AB = 5 \text{ length units.}$ 
 $\therefore BC = 5 \text{ length units.}$ 

$$\begin{aligned} \therefore \text{the area of } \triangle ABC &= \frac{1}{2} AB \times BC = \frac{1}{2} \times 5 \times 5 \\ &= 12.5 \text{ square units.} \end{aligned}$$

### Answers of Exercise 4

1

$$1 \text{ The midpoint of } \overline{AB} = \left( \frac{3+7}{2}, \frac{5+1}{2} \right) = (5, 3)$$

$$2 \text{ The midpoint of } \overline{AB} = \left( \frac{5-1}{2}, \frac{-3+3}{2} \right) = (2, 0)$$

$$3 \text{ The midpoint of } \overline{AB} = \left( \frac{-5+5}{2}, \frac{4-4}{2} \right) = (0, 0)$$

$$4 \text{ The midpoint of } \overline{AB} = \left( \frac{0+8}{2}, \frac{4+0}{2} \right) = (4, 2)$$

$$5 \text{ The midpoint of } \overline{AB} = \left( \frac{2+6}{2}, \frac{4+0}{2} \right) = (4, 2)$$

$$6 \text{ The midpoint of } \overline{AB} = \left( \frac{7-1}{2}, \frac{-6+0}{2} \right) = (3, -3)$$

2

$$\therefore (X, 0) = \left( \frac{1+2}{2}, \frac{-5+5}{2} \right) = \left( \frac{3}{2}, 0 \right)$$

$$\therefore X = \frac{3}{2}$$

3

$$\therefore (5, 3) = \left( \frac{15-5}{2}, \frac{y-2}{2} \right) \quad \therefore \frac{y-2}{2} = 3$$

$$\therefore y - 2 = 6 \quad \therefore y = 8$$

4

Let B (X, y)

$$\therefore (6, -4) = \left( \frac{5+X}{2}, \frac{-3+y}{2} \right)$$

$$\therefore \frac{5+X}{2} = 6 \quad \therefore 5+X = 12$$

$$\therefore X = 7, \quad \frac{-3+y}{2} = -4$$

$$\therefore -3+y = -8$$

$$\therefore y = -5 \quad \therefore B(7, -5)$$

5

$$\text{[1]} \therefore (X, y) = \left( \frac{1+3}{2}, \frac{5+7}{2} \right)$$

$$\therefore X = \frac{1+3}{2} = 2, y = \frac{5+7}{2} = 6$$

$$\text{[2]} \therefore (X, -3) = \left( \frac{-3+9}{2}, \frac{y+11}{2} \right)$$

$$\therefore X = \frac{-3+9}{2} = 3, \quad \frac{y+11}{2} = -3$$

$$\therefore y + 11 = -6 \quad \therefore y = -17$$

$$\text{[3]} \therefore (-3, y) = \left( \frac{X+9}{2}, \frac{-6-11}{2} \right)$$

$$\therefore \frac{X+9}{2} = -3 \quad \therefore X+9 = -6$$

$$\therefore X = -15 \quad y = \frac{-6-11}{2} = -8.5$$

$$\text{[4]} \therefore (4, 6) = \left( \frac{X+6}{2}, \frac{3+y}{2} \right)$$

$$\therefore \frac{X+6}{2} = 4 \quad \therefore X+6 = 8 \quad \therefore X = 2$$

$$\frac{3+y}{2} = 6 \quad \therefore 3+y = 12 \quad \therefore y = 9$$

6

[1] d

[2] d

[3] a

[4] c

[5] b

[6] n

[7] c

7

$$\text{[1]} (-5, 2)$$

$$\text{[2]} 4, 6$$

$$\text{[3]} (-2, 2)$$

$$\text{[4]} (3, 2), (7, 0)$$

$$\text{[5]} (0, 4), (0, 6)$$

8

 Let D be the midpoint of  $\overline{AB}$ 

$$\therefore D = \left( \frac{1+9}{2}, \frac{-6+2}{2} \right) = (5, -2)$$

 Let E be the midpoint of  $\overline{AD}$ 

$$\therefore E = \left( \frac{1+5}{2}, \frac{-6-2}{2} \right) = (3, -4)$$

 Let X be the midpoint of  $\overline{BD}$ 

$$\therefore X = \left( \frac{9+5}{2}, \frac{2-2}{2} \right) = (7, 0)$$

9

$$\therefore (0, 0) = \left( \frac{X-2-2}{2}, \frac{y+2}{2} \right) \quad \therefore \frac{X-4}{2} = 0$$

$$\therefore X - 4 = 0 \quad \therefore X = 4$$

$$\therefore \frac{y+2}{2} = 0 \quad \therefore y + 2 = 0$$

$$\therefore y = -2 \quad \therefore (X, y) = (4, -2)$$

10

$$\therefore (2a-3, a-b) = \left( \frac{7+3}{2}, \frac{-1+7}{2} \right) = (5, 3)$$

$$\therefore 2a-3 = 5 \quad \therefore 2a = 8 \quad \therefore a = 4$$

$$a-b = 3 \quad \therefore 4-b = 3 \quad \therefore b = 1$$

11

Let : A (X, y)

$$\therefore (5, 7) = \left( \frac{X+8}{2}, \frac{y+11}{2} \right)$$

$$\therefore \frac{X+8}{2} = 5 \quad \therefore X+8 = 10$$

$$\therefore X = 2, \quad \frac{y+11}{2} = 7 \quad \therefore y+11 = 14$$

$$\therefore y = 3 \quad \therefore A(2, 3)$$

$$\therefore r = MA = \sqrt{(5-2)^2 + (7-3)^2}$$

$$= \sqrt{9+16} = 5 \text{ length unit.}$$

$$\therefore \text{The circumference of the circle} = 2\pi r$$

$$= 2 \times 3.14 \times 5 = 31.4 \text{ length unit}$$

12

 $\therefore D$  is the midpoint of  $\overline{AB}$ 

$$\therefore D = \left( \frac{1+5}{2}, \frac{3+1}{2} \right) = (3, 2)$$

 $\therefore E$  is the midpoint of  $\overline{AC}$ 

$$\therefore E = \left( \frac{1+3}{2}, \frac{3+7}{2} \right) = (2, 5)$$

$$\therefore DE = \sqrt{(3-2)^2 + (2-5)^2} = \sqrt{10} \text{ length unit} \quad (1)$$

$$\begin{aligned} \therefore BC &= \sqrt{(5-3)^2 + (1-7)^2} \\ &= \sqrt{40} = 2\sqrt{10} \text{ length unit} \end{aligned} \quad (2)$$

From (1) and (2):

$$\therefore DE = \frac{1}{2} BC \quad (\text{Q.E.D.})$$

13

 Let  $A(x, 0)$ ,  $B(0, y)$ 

$$\therefore (3, 4) = \left( \frac{x+0}{2}, \frac{y+0}{2} \right)$$

$$\therefore \frac{x}{2} = 3 \quad \therefore x = 6$$

$$\therefore A(6, 0) \quad \therefore OA = 6 \text{ length unit}$$

$$\therefore \frac{y}{2} = 4 \quad \therefore y = 8$$

$$\therefore B(0, 8) \quad \therefore OB = 8 \text{ length unit}$$

$$\begin{aligned} \therefore AB &= \sqrt{(6-0)^2 + (0-8)^2} = \sqrt{36+64} = \sqrt{100} \\ &= 10 \text{ length unit} \end{aligned}$$

$$\begin{aligned} \therefore \text{The perimeter of } \triangle OAB &= 6 + 8 + 10 \\ &= 24 \text{ length unit} \end{aligned}$$

14

 Let  $D(x_1, 0)$  and  $B(0, y_1)$ 
 $\therefore D$  is the midpoint of  $\overline{AB}$ 

$$\therefore (x_1, 0) = \left( \frac{2+0}{2}, \frac{4+y_1}{2} \right)$$

$$\therefore x_1 = \frac{2}{2} = 1, \quad \frac{4+y_1}{2} = 0$$

$$\therefore 4+y_1 = 0 \quad \therefore y_1 = -4 \quad \therefore B(0, -4)$$

 Let  $E(0, y_2)$  and  $C(x_2, 0)$ 
 $\therefore E$  is the midpoint of  $\overline{AC}$ 

$$\therefore (0, y_2) = \left( \frac{2+x_2}{2}, \frac{4+0}{2} \right)$$

$$\therefore \frac{2+x_2}{2} = 0 \quad \therefore 2+x_2 = 0 \quad \therefore x_2 = -2$$

$$\therefore y_2 = 2 \quad \therefore C(-2, 0)$$

$$\begin{aligned} \therefore BC &= \sqrt{(-2-0)^2 + (0+4)^2} = \sqrt{4+16} = \sqrt{20} \\ &= 2\sqrt{5} \text{ length units.} \end{aligned}$$

 In  $\triangle ABC$ :

 $\therefore D$  is the midpoint of  $\overline{AB}$ ,  $E$  is the midpoint of  $\overline{AC}$ 

$$\therefore DE = \frac{1}{2} BC = \frac{1}{2} \times 2\sqrt{5} = \sqrt{5} \text{ length unit}$$

15

 $\therefore \overline{AD}$  is a median in  $\triangle ABC$ 
 $\therefore D$  is the midpoint of  $\overline{BC}$ 

$$\therefore D = \left( \frac{3-3}{2}, \frac{2+6}{2} \right) = (0, 4)$$

 $\therefore M$  is the midpoint of  $\overline{AD}$  and let  $A(x, y)$ 

$$\therefore (0, 6) = \left( \frac{x+0}{2}, \frac{y+4}{2} \right)$$

$$\therefore \frac{x}{2} = 0 \quad \therefore x = 0, \quad \frac{y+4}{2} = 6$$

$$\therefore y+4 = 12 \quad \therefore y = 8 \quad \therefore A(0, 8)$$

16

$$\begin{aligned} \therefore \text{The midpoint of } \overline{AC} &= \left( \frac{-1+6}{2}, \frac{-1+0}{2} \right) \\ &= \left( \frac{5}{2}, -\frac{1}{2} \right) \end{aligned}$$

$$\begin{aligned} \therefore \text{The midpoint of } \overline{BD} &= \left( \frac{2+3}{2}, \frac{3-4}{2} \right) \\ &= \left( \frac{5}{2}, -\frac{1}{2} \right) \end{aligned}$$

 $\therefore$  The midpoint of  $\overline{AC}$  is the same midpoint of  $\overline{BD}$ 
 $\therefore \overline{AC}$  and  $\overline{BD}$  bisect each other.

17

$$\therefore \text{The midpoint of } \overline{AC} = \left( \frac{3+0}{2}, \frac{-2-7}{2} \right) = \left( \frac{3}{2}, -\frac{9}{2} \right)$$

$$\begin{aligned} \therefore \text{the midpoint of } \overline{BD} &= \left( \frac{-5+8}{2}, \frac{0-9}{2} \right) \\ &= \left( \frac{3}{2}, -\frac{9}{2} \right) \end{aligned}$$

 $\therefore$  The midpoint of  $\overline{AC}$  is the same midpoint of  $\overline{BD}$ 
 $\therefore$  The two diagonals bisect each other

 $\therefore$  The points  $A, B, C$  and  $D$  are vertices of a parallelogram.

18

 [1] Let  $E$  be the point of intersection of the two diagonals

$$\therefore \text{The coordinates of } E = \left( \frac{3-1}{2}, \frac{2-2}{2} \right) = (1, 0)$$

$$\begin{aligned} [2] AC &= \sqrt{(-1-3)^2 + (-2-2)^2} = \sqrt{16+16} = \sqrt{32} \\ &= 4\sqrt{2} \text{ length units} \end{aligned}$$

$$BD = \sqrt{(-2-4)^2 + (3+3)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

$$\begin{aligned} \text{The area of the rhombus} &= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} \\ &= 24 \text{ square unit} \end{aligned}$$

19

$$\therefore \text{The midpoint of } \overline{AC} = \left( \frac{3+0}{2}, \frac{2-3}{2} \right) = \left( 1\frac{1}{2}, -\frac{1}{2} \right)$$

$$\therefore \text{The point of intersection of the two diagonals is } \left( 1\frac{1}{2}, -\frac{1}{2} \right) \quad (\text{Q.E.D.1})$$

and let D (X, y)

$$\therefore \text{The midpoint of } \overline{AC} = \text{the midpoint of } \overline{BD}$$

$$\therefore \left( 1\frac{1}{2}, -\frac{1}{2} \right) = \left( \frac{X+4}{2}, \frac{y-5}{2} \right) \quad \therefore \frac{X+4}{2} = 1\frac{1}{2}$$

$$\therefore X+4=3 \quad \therefore X=-1$$

$$\therefore \frac{y-5}{2} = -\frac{1}{2} \quad \therefore y-5=-1 \quad \therefore y=4$$

$$\therefore D(-1, 4) \quad (\text{Q.E.D.2})$$

20

$$AB = \sqrt{(2-6)^2 + (-4-0)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \text{ length unit}$$

$$BC = \sqrt{(-4-2)^2 + (2+4)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2} \text{ length unit}$$

$$CA = \sqrt{(6+4)^2 + (0-2)^2} = \sqrt{100+4} = \sqrt{104} = 2\sqrt{26} \text{ length unit}$$

$$\therefore (AB)^2 + (BC)^2 = (4\sqrt{2})^2 + (6\sqrt{2})^2 = 32 + 72 = 104 = (CA)^2$$

$$\therefore \Delta ABC \text{ is a right-angled at B} \quad (\text{Q.E.D.1})$$

Let E be the midpoint of AC

$$\therefore \text{The coordinates of E} = \left( \frac{6+4}{2}, \frac{0+2}{2} \right) = (1, 1)$$

 $\therefore$  In the rectangle the two diagonals bisect each other

 $\therefore$  E is the midpoint of BD

Let D (X, y)

$$\therefore (1, 1) = \left( \frac{X+2}{2}, \frac{y-4}{2} \right) \quad \therefore \frac{X+2}{2} = 1$$

$$\therefore X+2=2 \quad \therefore X=0$$

$$\therefore \frac{y-4}{2} = 1 \quad \therefore y-4=2$$

$$\therefore y=6 \quad \therefore D(0, 6) \quad (\text{Q.E.D.2})$$

21

$$AB = \sqrt{(3-5)^2 + (-2-3)^2} = \sqrt{4+25} = \sqrt{29} \text{ length unit}$$

$$BC = \sqrt{(-2-3)^2 + (-4+2)^2} = \sqrt{25+4} = \sqrt{29} \text{ length unit}$$

$$CA = \sqrt{(5+2)^2 + (3+4)^2} = \sqrt{49+49} = \sqrt{98} \text{ length unit}$$

$$\therefore (AB)^2 + (BC)^2 = 29 + 29 = 58, (AC)^2 = 98$$

$$\therefore (AC)^2 > (AB)^2 + (BC)^2$$

$$\therefore \Delta ABC \text{ is an obtuse-angled at B} \quad (\text{Q.E.D.1})$$

Let E be the midpoint of AC

$$\therefore \text{The coordinates of E} = \left( \frac{5-2}{2}, \frac{3-4}{2} \right) = (1.5, -0.5)$$

 $\therefore$  In the rhombus the two diagonals bisect each other

 $\therefore$  E is the midpoint of BD

Let D (X, y)

$$\therefore (1.5, -0.5) = \left( \frac{X+3}{2}, \frac{y-2}{2} \right)$$

$$\therefore \frac{X+3}{2} = 1.5 \quad \therefore X+3=3 \quad \therefore X=0$$

$$\therefore \frac{y-2}{2} = -0.5 \quad \therefore y-2=-1$$

$$\therefore y=1 \quad \therefore D(0, 1)$$

$$\therefore AC = \sqrt{98} = 7\sqrt{2}$$

$$BD = \sqrt{(3-0)^2 + (-2-1)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

The area of the rhombus ABCD

$$= \frac{1}{2} \times 7\sqrt{2} \times 3\sqrt{2} = 21 \text{ square unit} \quad (\text{Q.E.D.2})$$

22

$$AB = \sqrt{(3+3)^2 + (4-0)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13} \text{ length unit}$$

$$BC = \sqrt{(1-3)^2 + (-6-4)^2} = \sqrt{4+100} = \sqrt{104} = 2\sqrt{26} \text{ length unit}$$

$$CA = \sqrt{(-3-1)^2 + (0+6)^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13} \text{ length unit}$$

$$\therefore AB = AC$$

$$\therefore \Delta ABC \text{ is an isosceles triangle and its vertex is A} \quad (\text{Q.E.D.1})$$

 Let D be the midpoint of BC (the base of  $\Delta ABC$ )

$$\therefore D = \left( \frac{3+1}{2}, \frac{4-6}{2} \right) = (2, -1)$$

$$\therefore AD = \sqrt{(2+3)^2 + (-1-0)^2} = \sqrt{25+1} = \sqrt{26} \text{ length unit}$$

$$\therefore \text{The length of the drawn line segment perpendicular to } \overline{BC} \text{ from A} = \sqrt{26} \text{ length unit} \quad (\text{Q.E.D.2})$$

23

$$\therefore AB = \sqrt{(3-1)^2 + (1-1)^2} = \sqrt{4+0} = 2 \text{ length units.}$$

$$BC = \sqrt{(1-3)^2 + (3-1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ length units.}$$

$$\therefore CA = \sqrt{(1-1)^2 + (1-3)^2} = \sqrt{0+4} = 2 \text{ length units}$$

$$\therefore AB = CA$$

$\therefore \triangle ABC$  is an isosceles triangle and its vertex is A  
(Q.E.D.1)

Let the point E be the midpoint of  $\overline{BC}$

$$\therefore E = \left( \frac{3+1}{2}, \frac{1+3}{2} \right) = (2, 2)$$

$$\therefore AE = \sqrt{(2-1)^2 + (2-1)^2} = \sqrt{1+1} = \sqrt{2} \text{ length unit}$$

$\therefore \triangle ABC$  is an isosceles triangle  
and E is the midpoint of  $\overline{BC}$

$$\therefore \overline{AE} \perp \overline{BC}$$

$$\begin{aligned} \therefore \text{The area of } \triangle ABC &= \frac{1}{2} BC \times AE \\ &= \frac{1}{2} \times 2\sqrt{2} \times \sqrt{2} \\ &= 2 \text{ square units.} \end{aligned} \quad (\text{Q.E.D. 2})$$

24

$\therefore$  In the parallelogram the two diagonals bisect each other

$\therefore$  Let M be the point of intersection of the two diagonals

$$\therefore \text{The coordinates of M} = \left( \frac{3-4}{2}, \frac{4-3}{2} \right) = \left( -\frac{1}{2}, \frac{1}{2} \right)$$

Let  $D(x_1, y_1)$

$$\therefore \left( -\frac{1}{2}, \frac{1}{2} \right) = \left( \frac{x_1+2}{2}, \frac{y_1-1}{2} \right)$$

$$\therefore \frac{x_1+2}{2} = -\frac{1}{2}$$

$$\therefore x_1 + 2 = -1$$

$$\therefore x_1 = -3$$

$$\therefore \frac{y_1-1}{2} = \frac{1}{2}$$

$$\therefore y_1 - 1 = 1$$

$$\therefore y_1 = 2$$

$\therefore$  The coordinates of D are  $(-3, 2)$

$$\therefore AE = 2 AD$$

$\therefore$  D is the midpoint of  $\overline{AE}$

Let  $E(x_2, y_2)$

$$\therefore (-3, 2) = \left( \frac{x_2+3}{2}, \frac{y_2+4}{2} \right)$$

$$\therefore \frac{x_2+3}{2} = -3$$

$$\therefore x_2 + 3 = -6$$

$$\therefore x_2 = -9$$

$$\therefore \frac{y_2+4}{2} = 2$$

$$\therefore y_2 + 4 = 4$$

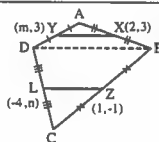
$$\therefore y_2 = 0$$

$\therefore$  The coordinates of E are  $(-9, 0)$

25

In  $\triangle ABD$ :

$\therefore X, Y$  are the midpoints  
of  $\overline{AB}, \overline{AD}$



$$\therefore \overline{XY} \parallel \overline{BD}, XY = \frac{1}{2} BD \quad (1)$$

$\therefore$  similarly in  $\triangle CBD$ :

$$\overline{ZL} \parallel \overline{BD}, ZL = \frac{1}{2} BD \quad (2)$$

$$\text{From (1), (2):} \quad \therefore \overline{XY} \parallel \overline{ZL}, XY = ZL$$

$\therefore$  The figure  $XYLZ$  is parallelogram

$\therefore$  The midpoint of  $\overline{XL}$  is the same the midpoint of  $\overline{ZY}$

$$\therefore \left( \frac{2-4}{2}, \frac{3+n}{2} \right) = \left( \frac{m+1}{2}, \frac{3-1}{2} \right)$$

$$\therefore \frac{2-4}{2} = \frac{m+1}{2}$$

$$\therefore m = -3$$

$$\therefore \frac{3+n}{2} = \frac{3-1}{2}$$

$$\therefore n = -1$$

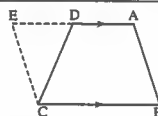
$$\therefore m + n = -3 + (-1) = -4$$

26

Let  $E(x, y) \in \overline{AD}$

such that  $ABCE$

is a parallelogram



$\therefore$  The midpoint of  $\overline{AC}$  = the midpoint of  $\overline{BE}$

$$\therefore \left( \frac{6-2}{2}, \frac{4-4}{2} \right) = \left( \frac{4+x}{2}, \frac{-2+y}{2} \right)$$

$$\therefore \frac{4+x}{2} = 2$$

$$\therefore x = 0, \quad \frac{-2+y}{2} = 0$$

$$\therefore y = 2$$

$$\therefore E(0, 2)$$

$\therefore AE = BC$  (properties of the parallelogram)

$$BC = 2 AD$$

$$\therefore AE = 2 AD$$

$\therefore$  D is the midpoint of  $\overline{AE}$

$$\therefore D = \left( \frac{6+0}{2}, \frac{4+2}{2} \right) = (3, 3)$$

## Answers of Exercise 5

27

$$\boxed{1} \text{ zero} \quad \boxed{2} \text{ undefined} \quad \boxed{3} \frac{2}{3} \quad \boxed{4} -2$$

$$\boxed{5} -\frac{1}{4} \quad \boxed{6} -3 \quad \boxed{7} \frac{1}{3}$$

28

$$\boxed{1} a \quad \boxed{2} b \quad \boxed{3} c \quad \boxed{4} b$$

$$\boxed{5} c \quad \boxed{6} a \quad \boxed{7} a \quad \boxed{8} d$$

$$\boxed{9} c \quad \boxed{10} d \quad \boxed{11} c \quad \boxed{12} b$$

$$\boxed{13} a \quad \boxed{14} d$$

3

$$\boxed{1} \frac{1}{\sqrt{3}} \quad \boxed{2} -\frac{1}{\sqrt{3}} \quad \boxed{3} -\sqrt{3}$$

4

$$\boxed{1} \text{ zero} \quad \boxed{2} \frac{1}{\sqrt{3}} \quad \boxed{3} 1 \quad \boxed{4} 1.5399$$

$$\boxed{5} \sqrt{3} \quad \boxed{6} \text{ undefined} \quad \boxed{7} 17.3432 \quad \boxed{8} -1$$

5

$$\boxed{1} 16^\circ 41' 57'' \quad \boxed{2} 20^\circ 10' 6''$$

$$\boxed{3} 45^\circ 41' 46'' \quad \boxed{4} 38^\circ 39' 35''$$

6

$$\therefore m_1 = \frac{6-2}{5-4} = 4, m_2 = \frac{1-5}{-1-0} = 4 \quad \therefore m_1 = m_2$$

$\therefore$  The two straight lines are parallel

7

$\therefore$  The slope of  $\overline{AC} = \frac{-2-4}{-3+3} = \frac{-6}{\text{zero}}$

$\therefore$  The straight line  $\overline{AC} \parallel y\text{-axis}$ ,

The slope of the straight line  $\overline{BD} = \frac{2-2}{-3-1} = \text{zero}$

$\therefore$  The straight line  $\overline{BD} \parallel x\text{-axis}$

$\therefore \overline{AC} \perp \overline{BD}$

8

$$\therefore m_1 = \frac{3+1}{6-2} = 1, m_2 = \tan 45^\circ = 1$$

$\therefore m_1 = m_2$

$\therefore$  The two straight lines are parallel.

9

$$\therefore m_1 = \frac{2\sqrt{3}-3\sqrt{3}}{5-4} = -\sqrt{3}, m_2 = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$\therefore m_1 \times m_2 = -1$

$\therefore$  The two straight lines are perpendicular

10

$\therefore$  The slope of  $\overline{AD} = \frac{1-5}{2-1} = -4$

The slope of  $\overline{BC} = \frac{7-3}{4-(x-1)} = \frac{4}{5-x}$

$\therefore$  The slope of  $\overline{AD} =$  the slope of  $\overline{BC}$

$\therefore -4 = \frac{4}{5-x} \quad \therefore 5-x = -1 \quad \therefore x = 6$

11

$\therefore \triangle XYZ$  is right-angled at  $Y$

$\therefore \overline{XY} \perp \overline{YZ}$ , the slope of  $\overline{XY} = \frac{5-2}{3-4} = -3$

$\therefore$  The slope of  $\overline{YZ} = \frac{1}{3}$

$\therefore$  the slope of  $\overline{YZ} = \frac{a-2}{-5-4} = \frac{a-2}{-9} = \frac{1}{3}$

$\therefore a-2 = -3 \quad \therefore a = -1$

12

$\therefore m = \frac{7-5}{x-3} \quad \therefore m$  is undefined

$\therefore x-3 = 0 \quad \therefore x = 3$

13

$\therefore m = \frac{y-2}{-5-4} \quad \therefore m$  is zero

$\therefore y-2 = 0 \quad \therefore y = 2$

14

$\boxed{1} \therefore m_1 = \frac{k-1}{2-3} = \frac{k-1}{-1}, m_2 = \tan 45^\circ = 1$

$\therefore L_1 \parallel L_2 \quad \therefore m_1 = m_2$

$\therefore \frac{k-1}{-1} = 1 \quad \therefore k-1 = -1$

$\therefore k = \text{zero}$

$\boxed{2} \therefore m_1 = \frac{k-1}{-1}, m_2 = 1$

$\therefore L_1 \perp L_2 \quad \therefore m_1 m_2 = -1$

$\therefore \frac{k-1}{-1} \times 1 = -1 \quad \therefore k-1 = 1 \quad \therefore k = 2$

15

Let the measure of the positive angle be  $\theta$

$\therefore m = \frac{-5-3}{2-4} = 4 \quad \therefore \tan \theta = 4$

$\therefore \theta = 75^\circ 57' 50''$

16

Let the measure of the positive angle be  $\theta$

$m = \frac{-2-0}{2-0} = -1$

$\therefore \tan$  (supplementary of  $\theta$ ) = 1

$\therefore$  supplementary of  $\theta = 45^\circ$

$\therefore \theta = 180^\circ - 45^\circ = 135^\circ$

17

Let the measure of the positive angle be  $\theta$

$\therefore$  The slope of the given straight line =  $\frac{-1-5}{4+2} = -1$

$\therefore$  The two straight lines are perpendicular

$\therefore$  The slope of the required straight line = 1

$\therefore \tan \theta = 1 \quad \therefore \theta = 45^\circ$

18

$$\therefore \text{The slope of } \overline{AB} = \frac{3-1}{2-1} = \frac{2}{1} = 2$$

$$\therefore \text{the slope of } \overline{BC} = \frac{-1-3}{0-2} = \frac{-4}{-2} = 2$$

$$\therefore \text{The slope of } \overline{AB} = \text{the slope of } \overline{BC}$$

$$\therefore \overline{AB} \parallel \overline{BC}$$

$\therefore$  B is a common point between the two straight lines  $\overline{AB}$  and  $\overline{BC}$

$\therefore$  A, B, C are collinear points

19

$$\therefore m_1 = \frac{3-1}{A-0} = \frac{2}{A}, m_2 = \frac{5-1}{2-0} = \frac{4}{2} = 2$$

$$\therefore m_1 = m_2 \quad \therefore \frac{2}{A} = 2 \quad \therefore A = 1$$

20

$$\therefore \text{The slope of } \overline{AB} = \frac{5-7}{-1-1} = 1$$

$$\therefore \text{the slope of } \overline{BC} = \frac{2-5}{4+1} = \frac{-3}{5}$$

$$\therefore \text{The slope of } \overline{AB} \neq \text{the slope of } \overline{BC}$$

$$\therefore C \notin \overline{AB}$$

21

$$\therefore \text{The slope of } \overline{AB} = m_1 = \frac{3+1}{2+1} = \frac{4}{3}$$

$$\therefore \text{the slope of } \overline{BC} = m_2 = \frac{0-3}{6-2} = \frac{-3}{4}$$

$$\therefore m_1 \times m_2 = \frac{4}{3} \times \frac{-3}{4} = -1$$

$$\therefore \overline{AB} \perp \overline{BC} \quad \therefore \triangle ABC \text{ is a right-angled at B}$$

22

$$\therefore \text{The slope of } \overline{AB} = \frac{5-1}{0+1} = 4$$

$$\therefore \text{the slope of } \overline{CD} = \frac{6-2}{5-4} = 4$$

$$\therefore \text{The slope of } \overline{AB} = \text{the slope of } \overline{CD}$$

$$\therefore \overline{AB} \parallel \overline{CD}$$

$$\therefore \text{The slope of } \overline{AC} = \frac{2-1}{4+1} = \frac{1}{5}$$

$$\therefore \text{the slope of } \overline{BD} = \frac{6-5}{5-0} = \frac{1}{5}$$

$$\therefore \text{The slope of } \overline{AC} = \text{the slope of } \overline{BD}$$

$$\therefore \overline{AC} \parallel \overline{BD}$$

From (1) and (2):

$\therefore$  The figure ABDC is a parallelogram

23

$$\therefore \text{The slope of } \overline{AB} = \frac{1-3}{5+1} = -\frac{1}{3}$$

$$\therefore \text{the slope of } \overline{CD} = \frac{6-4}{0-6} = -\frac{1}{3}$$

$$\therefore \overline{AB} \parallel \overline{CD} \quad (1)$$

$$\therefore \text{The slope of } \overline{AD} = \frac{6-3}{0+1} = 3$$

$$\therefore \text{the slope of } \overline{BC} = \frac{4-1}{6-5} = 3$$

$$\therefore \overline{AD} \parallel \overline{BC} \quad (2)$$

From (1) and (2): we deduce that

ABCD is a parallelogram

$$\therefore \text{the slope of } \overline{AC} \times \text{the slope of } \overline{BC} = -\frac{1}{3} \times 3 = -1$$

$$\therefore \overline{AB} \perp \overline{BC}$$

$\therefore$  The figure ABCD is a rectangle

24

$$\therefore \text{The slope of } \overline{AB} = \frac{4-3}{6-1} = \frac{1}{5}$$

$$\therefore \text{the slope of } \overline{CD} = \frac{8-9}{2-7} = \frac{1}{5}$$

$$\therefore \overline{AB} \parallel \overline{CD} \quad (1)$$

$$\therefore \text{The slope of } \overline{AD} = \frac{8-3}{2-1} = 5$$

$$\therefore \text{the slope of } \overline{BC} = \frac{9-4}{7-6} = 5$$

$$\therefore \overline{AD} \parallel \overline{BC} \quad (2)$$

From (1) and (2): we deduce that

ABCD is a parallelogram.

$$\therefore \text{The slope of } \overline{AC} = \frac{9-3}{7-1} = 1$$

$$\therefore \text{the slope of } \overline{BD} = \frac{8-4}{2-6} = -1$$

$$\therefore \text{The slope of } \overline{AC} \times \text{the slope of } \overline{BD} = 1 \times -1 = -1$$

$$\therefore \overline{AC} \perp \overline{BD}$$

$\therefore$  The figure ABCD is a rhombus.

25

$$(1) \quad \therefore \text{The slope of } \overline{AB} = \frac{3+1}{2+1} = \frac{4}{3}$$

$$\therefore \text{the slope of } \overline{CD} = \frac{-4-0}{3-6} = \frac{4}{3}$$

$$\therefore \overline{AB} \parallel \overline{CD} \quad (1)$$

$$\therefore \text{The slope of } \overline{AD} = \frac{-4+1}{3+1} = -\frac{3}{4}$$

$$(2) \quad \therefore \text{the slope of } \overline{BC} = \frac{0-3}{6-2} = -\frac{3}{4}$$

$$\therefore \overline{AD} \parallel \overline{BC} \quad (2)$$

From (1) and (2): we deduce that

ABCD is a parallelogram.

$$\therefore \text{The slope of } \overline{AB} \times \text{the slope of } \overline{BC} = \frac{4}{3} \times \frac{-3}{4} = -1$$

$$\therefore \overline{AB} \perp \overline{BC}$$

$\therefore$  The figure ABCD is a rectangle.

$$\therefore \text{the slope of } \overline{AC} = \frac{0+1}{6+1} = \frac{1}{7}$$

$$\therefore \text{the slope of } \overline{BD} = \frac{-4-3}{3-2} = -7$$

$$\therefore \text{The slope of } \overline{AC} \times \text{The slope of } \overline{BD} = \frac{1}{7} \times -7 = -1$$

$$\therefore \overline{AC} \perp \overline{BD} \quad \therefore \text{ABCD is a square}$$

26

$$\therefore \text{The slope of } \overline{AB} = \frac{2+2}{3-9} = -\frac{2}{3}$$

$$\therefore \text{The slope of } \overline{AB} = \text{the slope of } \overline{CD}$$

$$\therefore \frac{-x+3}{x-4} = \frac{2}{-3} \quad \therefore -3(-x+3) = 2(x-4)$$

$$\therefore 3x-9 = 2x-8 \quad \therefore x = 1$$

$$\therefore \text{The coordinates of C} = (1, -1)$$

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$$\therefore \text{The slope of } \overline{AB} = \frac{0-3}{7-4} = -1$$

$$\therefore \text{the slope of } \overline{BC} = \frac{-2-0}{1-7} = \frac{1}{3}$$

$$\therefore \text{The slope of } \overline{AB} \neq \text{the slope of } \overline{BC}$$

$\therefore$  A, B and C are not collinear

$\therefore$  A, B and C are vertices of a triangle (Q.E.D.1)

$$\therefore \text{The slope of } \overline{CD} = \frac{2+2}{1-1} = \frac{0}{0} \text{ (undefined)}$$

$$\therefore \text{the slope of } \overline{AD} = \frac{3-2}{4-1} = \frac{1}{3}$$

$$\therefore \text{The slope of } \overline{AB} \neq \text{the slope of } \overline{CD}$$

$$\therefore \text{the slope of } \overline{BC} = \text{the slope of } \overline{AD}$$

$$\therefore \overline{BC} \parallel \overline{AD}$$

$\therefore$  The figure ABCD is a trapezoid (Q.E.D.2)

$$AD = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{9+1} = \sqrt{10} \text{ length unit}$$

$$BC = \sqrt{(7-1)^2 + (0+2)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10} \text{ length unit}$$

$$\therefore AD : BC = 1 : 2 \quad \text{(Q.E.D.3)}$$

28

Let the measure of the positive angle be  $\theta$

$$\therefore \sin \theta = \frac{3}{5}$$

$$\therefore \theta = 36^\circ 52' 11.63'' \quad \therefore \tan \theta = \frac{3}{4}$$

$$\therefore \text{The slope of the straight line} = \frac{3}{4}$$

29

$$\therefore \overline{AB} \parallel \overline{CD}$$

$$\therefore \text{The slope of } \overline{AB} = \text{the slope of } \overline{CD}$$

$$\therefore \frac{3-1}{3-1} = \frac{y+3x}{x-0} \quad \therefore y+3x = x$$

$$\therefore y+2x = 0 \quad (1)$$

$$\therefore \overline{AB} \perp \overline{BC}$$

$$\therefore \text{The slope of } \overline{AB} \times \text{the slope of } \overline{BC} = -1$$

$$\frac{3-1}{3-1} \times \frac{-3x-3}{0-3} = -1 \quad \therefore \frac{-3x-3}{-3} = -1$$

$$\therefore -3x-3 = 3 \quad \therefore -3x = 6 \quad \therefore x = -2$$

$$\text{And from (1) : } y+2x = 0 \quad \therefore y = 4$$

30

1 The two diagonals of the rhombus

bisect each other

$\therefore$  Let M be the intersection point of the diagonals

$$\therefore M = \left( \frac{3-1}{2}, \frac{2-2}{2} \right) = (1, 0)$$

$$\therefore \text{the slope of } \overline{MA} = \frac{2-0}{3-1} = \frac{2}{2} = 1$$

$$\therefore \text{the slope of } \overline{MB} = \frac{k-0}{4-1} = \frac{k}{3}$$

$\therefore$  the two diagonals of the rhombus are perpendicular

$$\therefore \overline{MA} \perp \overline{MB} \quad \therefore 1 \times \frac{k}{3} = -1$$

$$\therefore k = -3$$

$$2 \text{ Let : } D(x, y) \quad \therefore (1, 0) = \left( \frac{4+x}{2}, \frac{-3+y}{2} \right)$$

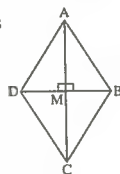
$$\therefore \frac{4+x}{2} = 1 \quad \therefore 4+x = 2$$

$$\therefore x = -2, \quad \frac{-3+y}{2} = 0$$

$$\therefore -3+y = 0 \quad \therefore y = 3$$

$$\therefore D(-2, 3)$$

$$\therefore BD = \sqrt{(4+2)^2 + (-3-3)^2} = \sqrt{36+36} = 6\sqrt{2} \text{ length units}$$

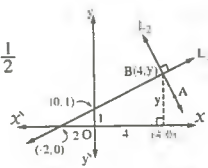


31

$\therefore$  The slope of the straight line  $L_1 = \frac{1-0}{0+2} = \frac{1}{2}$

The slope of the

$$\text{straight line } L_1 = \frac{y-0}{4+2} = \frac{y}{6}$$



- $\therefore \frac{y}{6} = \frac{1}{2}$        $\therefore y = 3$        $\therefore B(4, 3)$   
 $\therefore$  the straight line  $L_1 \perp$  the straight line  $L_2$   
 $\therefore$  the slope of the straight line  $L_1 = \frac{1}{2}$   
 $\therefore$  The slope of the straight line  $L_2 = -2$   
 $\therefore A(5, m), B(4, 3)$  lie on the straight line  $L_2$   
 $\therefore \frac{m-3}{5m-4} = -2$        $\therefore -10m + 8 = m - 3$   
 $\therefore -11m = -11$        $\therefore m = 1$

### Answers of Exercise 6

- 1** The slope = 5 and the intercepted part = 3 units from the negative part of y-axis  
**2**  $\therefore 2y = 4 - x$  (dividing by 2)  $\therefore y = -\frac{1}{2}x + 2$   
 $\therefore$  The slope =  $-\frac{1}{2}$  and the intercepted part = 2 units from the positive part of y-axis  
**3**  $\therefore 2x - 3y - 6 = 0$   
 $\therefore 3y = 2x - 6$  (dividing by 3)  
 $\therefore y = \frac{2}{3}x - 2$   
 $\therefore$  The slope =  $\frac{2}{3}$  and the intercepted part = 2 units from the negative part of y-axis  
**4**  $\therefore y + x - 1 = 0$        $\therefore y = -x + 1$   
 $\therefore$  The slope = -1 and the intercepted part = one unit from the positive part of y-axis  
**5**  $\therefore \frac{x}{2} + 3y = 6$  (multiplying by 2)  
 $\therefore x + 6y = 12$        $\therefore 6y = -x + 12$   
 $\therefore y = -\frac{1}{6}x + 2$   
 $\therefore$  The slope =  $-\frac{1}{6}$  and the intercepted part = 2 units from the positive part of y-axis.  
**6**  $\therefore \frac{x}{2} + \frac{y}{3} = 1$  (multiplying by 3)  
 $\therefore \frac{3x}{2} + y = 3$        $\therefore y = -\frac{3x}{2} + 3$   
 $\therefore$  The slope =  $-\frac{3}{2}$  and the intercepted part = 3 units from the positive part of y-axis

2

- 1**  $y = 2x + 7$       **2**  $y = -x + 3$   
**3**  $y = 2\frac{1}{2}x - 1$       **4**  $y = -\frac{3}{4}x - 2\frac{1}{2}$   
**5**  $y = -2$       **6**  $y = -\frac{1}{3}x + 2$   
**7**  $y = -2x$

3

- 1**  $\therefore$  The slope =  $\tan 135^\circ = -1$   
 $\therefore$  the straight line passes through the origin point  
 $\therefore y = -x$   
**2**  $\therefore$  The slope =  $\tan 45^\circ = 1$        $\therefore y = x + c$   
 $\therefore$  The straight line passes through the point  $(3, 2)$   
 $\therefore 2 = 3 + c$        $\therefore c = -1$   
 $\therefore$  The equation is:  $y = x - 1$   
**3**  $\therefore$  The slope of the given straight line =  $\frac{2}{3}$   
 $\therefore$  The slope of the required straight line =  $\frac{2}{3}$   
 and intercepted from the negative part of y-axis 3 units  
 $\therefore$  The equation of the required straight line is:  
 $y = \frac{2}{3}x - 3$   
**4**  $\therefore$  The slope of the given straight line =  $\frac{3}{4}$   
 $\therefore$  The slope of the required straight line =  $-\frac{4}{3}$   
 and it intercepts from the positive part of y-axis 6 units  
 $\therefore$  The equation of the required straight line is  
 $y = -\frac{4}{3}x + 6$   
**5**  $\therefore$  The slope of the given straight line =  $\frac{7-1}{2+2} = \frac{3}{2}$   
 $\therefore$  The slope of the required straight line =  $-\frac{2}{3}$   
 and intercepts from the positive part of y-axis 5 units  
 $\therefore$  The equation of the required straight line is  
 $y = -\frac{2}{3}x + 5$   
**6**  $\therefore$  The straight line passes through the two points  $(4, 0), (0, 9)$   
 $\therefore$  The slope of the straight line =  $-\frac{9-0}{0-4} = -\frac{9}{4}$   
 and the intercepted part = 9 units from the positive part of y-axis  
 $\therefore$  The equation of the straight line is:  
 $y = -\frac{9}{4}x + 9$   
**7**  $\therefore$  The slope = 2       $\therefore y = 2x + c$   
 $\therefore$  the straight line passes through the point  $(2, -1)$   
 $\therefore -1 = 2 \times 2 + c$        $\therefore c = -5$   
 $\therefore y = 2x - 5$   
**8**  $\therefore$  The slope of the given straight line =  $\frac{1}{2}$   
 $\therefore$  The slope of the required straight line = -2  
 $\therefore$  The equation of the required straight line is:

$y = -2X + c$   
 $\therefore$  The straight line passes through the point  $(-2, 3)$   
 $\therefore 3 = -2 \times (-2) + c \quad \therefore c = -1$   
 $\therefore$  The equation of the required straight line is :  
 $y = -2X - 1$

**9**  $\therefore$  The slope of the given straight line  $= -\frac{1}{2}$   
 $\therefore$  The slope of the required straight line  $= -\frac{1}{2}$   
 $\therefore$  The equation of the required straight line is  
 $y = -\frac{1}{2}X + c$   
 $\therefore$  The straight line passes through the point  $(3, -5)$   
 $\therefore -5 = -\frac{1}{2} \times 3 + c \quad \therefore c = -3\frac{1}{2}$   
 $\therefore$  The equation of the required straight line is  
 $y = -\frac{1}{2}X - 3\frac{1}{2}$

**10**  $\therefore$  The slope of the given straight line  
 $= \frac{2-6}{-1-5} = \frac{2}{3}$   
 $\therefore$  The slope of the required straight line  $= \frac{2}{3}$   
 $\therefore$  The equation of the required straight line is  
 $y = \frac{2}{3}X + c$   
 $\therefore$  The straight line passes through the point  $(3, 2)$   
 $\therefore 2 = \frac{2}{3} \times 3 + c \quad \therefore c = 0$   
 $\therefore$  The equation of the straight line is  $y = \frac{2}{3}X$

**11**  $\therefore$  The slope of  $\overline{AB} = \frac{-4+3}{5-2} = -\frac{1}{3}$   
 $\therefore$  The slope of the required straight line  $= 3$   
 $\therefore$  The equation of the required straight line is :  
 $y = 3X + c$   
 $\therefore$  The straight line passes through the point  $(1, 2)$   
 $\therefore 2 = 3 \times 1 + c \quad \therefore c = -1$   
 $\therefore$  The equation of the required straight line is :  
 $y = 3X - 1$

**12**  $\therefore$  The slope of the given straight line  $= \tan 45^\circ = 1$   
 $\therefore$  The slope of the required straight line  $= -1$   
 $\therefore$  The equation of the required straight line is :  
 $y = -X + c$   
 $\therefore$  The straight line passes through the point  $(2, -2)$   
 $\therefore -2 = -2 + c \quad \therefore c = 0$   
 $\therefore$  The equation of the required straight line is :  
 $y = -X$

**13**  $\therefore$  The slope of the straight line  $= \frac{1+1}{1-2} = -2$   
 $\therefore$  The equation of the straight line  
 $\text{is } y = -2X + c$

$\therefore$  The straight line passes through the point  $(1, 1)$   
 $\therefore 1 = -2 \times 1 + c \quad \therefore c = 3$   
 $\therefore$  The equation of the straight line is  
 $y = -2X + 3$

**14**  $\therefore$  The slope of the straight line  $= \frac{-1-2}{-2-4} = \frac{1}{2}$

$\therefore$  The equation of the straight line is  $y = \frac{1}{2}X + c$   
 $\therefore$  The straight line passes through  
 the point  $(4, 2)$   
 $\therefore 2 = \frac{1}{2} \times 4 + c \quad \therefore c = \text{zero}$   
 $\therefore$  The equation of the straight line is  $y = \frac{1}{2}X$   
 $\therefore$  The intercepted part of y-axis is zero  
 $\therefore$  The straight line passes through the origin point

**15**  $\therefore \frac{y-1}{x} = \frac{1}{3} \quad \therefore y-1 = \frac{1}{3}X$   
 $\therefore y = \frac{1}{3}X + 1$

$\therefore$  The slope of the required straight line  $= \frac{1}{3}$   
 $\therefore$  The equation of the required straight line is  
 $y = \frac{1}{3}X + c \quad \therefore c = -3$   
 $\therefore$  The equation of the required straight line is  
 $y = \frac{1}{3}X - 3$

**16**  $\therefore$  The slope of  $\overline{AB} = \frac{1-6}{2+3} = -1$   
 $\therefore$  The slope of the required straight line  $= 1$   
 $\therefore$  The equation of the required straight line is  
 $y = X + c$   
 $\therefore$  The required straight line passes through  
 the point  $A(-3, 6) \quad \therefore 6 = -3 + c \quad \therefore c = 9$   
 $\therefore$  The equation of the required straight line is  
 $y = X + 9$

**17**  $\therefore$  The slope of  $\overline{AB} = \frac{5-3}{3-1} = 1$

$\therefore$  The slope of the required straight line  $= -1$   
 $\therefore$  The equation of the required straight line is  
 $y = -X + c$   
 $\therefore$  the coordinates of the midpoint of  $\overline{AB}$   
 $= \left( \frac{1+3}{2}, \frac{3+5}{2} \right) = (2, 4)$   
 $\therefore$  the required straight line passes through the  
 midpoint of  $\overline{AB}$   
 $\therefore 4 = -2 + c \quad \therefore c = 6$   
 $\therefore$  The equation of the required straight line is  
 $y = -X + 6$

$$(18) \therefore 2y = 4x - 5 \quad \therefore y = 2x - \frac{5}{2} \quad \therefore m = 2$$

$\therefore$  The slope of the required straight line = 2

$\therefore$  The equation of the required straight line is

$$y = 2x + c$$

$$\therefore \text{The midpoint of } \overline{AB} = \left( \frac{4+2}{2}, \frac{8+4}{2} \right) = (1, 6)$$

$\therefore (1, 6)$  satisfies its equation

$$\therefore 6 = 2 \times 1 + c \quad \therefore c = 4$$

$\therefore$  The equation of the required straight line is

$$y = 2x + 4$$

$$(19) \therefore \text{The slope of the given straight line} = 2$$

$\therefore$  The slope of the required straight line =  $-\frac{1}{2}$

$\therefore$  The equation of the required straight line is

$$y = -\frac{1}{2}x + c$$

$$\therefore \text{The midpoint of } \overline{AB} = \left( \frac{3+1}{2}, \frac{6+4}{2} \right) = (1, 5)$$

$\therefore$  The required straight line passes through the midpoint of  $\overline{AB}$

$$\therefore 5 = -\frac{1}{2} \times 1 + c \quad \therefore c = 5\frac{1}{2}$$

$\therefore$  The equation of the required straight line is

$$y = -\frac{1}{2}x + 5\frac{1}{2}$$

$$(20) \therefore \text{The required straight line intercepts from the positive part of } x\text{-axis 4 units}$$

$\therefore$  The required straight line passes through the point  $(4, 0)$

$\therefore$  The slope of the required straight line

$$= \frac{0-3}{4-2} = -\frac{3}{2}$$

$\therefore$  The equation of the required straight line is:

$$y = -\frac{3}{2}x + c$$

$\therefore$  the required straight line passes through the point  $(2, 3)$

$$\therefore 3 = -\frac{3}{2} \times 2 + c \quad \therefore c = 6$$

$\therefore$  The equation of the required straight line is:

$$y = -\frac{3}{2}x + 6$$

4

$$(1) \frac{3}{4}, -\frac{4}{3}$$

$$(3) y = 2x + 3$$

$$(5) y = 5$$

$$(7) y = -5$$

$$(9) y = 2x - 3$$

$$(2) (0, 2)$$

$$(4) y = 0, x = 0$$

$$(6) x = -3$$

$$(8) y = 2x$$

$$(10) (1, 4), (6)$$

$$(2) \frac{3}{4}$$

$$(3) y = \frac{3}{4}x + 3$$

5

$$(1) d$$

$$(2) b$$

$$(3) d$$

$$(4) d$$

$$(5) a$$

$$(6) a$$

$$(7) c$$

$$(8) d$$

$$(9) a$$

$$(10) c$$

$$(11) c$$

$$(12) d$$

$$(13) b$$

$$(14) a$$

$$(15) b$$

$$(16) d$$

6

$$\therefore \text{The slope of } \overline{AB} = \frac{2-1}{1-3} = -\frac{1}{2}$$

$$\therefore \text{the slope of the other straight line} = \frac{-2}{4} = -\frac{1}{2}$$

$\therefore$  The slope of  $\overline{AB}$  is the slope of the other straight line

$\therefore$  The two straight lines are parallel.

7

The slope of the straight line whose equation:

$$2x + y + 8 = 0 \text{ is } -\frac{2}{1} = -2$$

$$\text{and the slope of } \overline{AB} = \frac{1-3}{-2-2} = \frac{1}{2}$$

$$\therefore -2 \times \frac{1}{2} = -1$$

$\therefore$  The two straight lines are perpendicular.

8

$$y = 2, x = -3$$

9

$$\therefore \text{The slope of the straight line} = \frac{-3}{-2} = \frac{3}{2}$$

$$\therefore \text{The slope of the straight line} = \tan \theta$$

$$\therefore \tan \theta = \frac{3}{2} \therefore \theta = 56^\circ 18' 36''$$

Put  $x = 0$

$$\therefore 3 \times 0 - 2y + 6 = 0 \therefore -2y = -6 \therefore y = 3$$

$\therefore$  The coordinates of the intersection point with  $y$ -axis is  $(0, 3)$

10

$$(1) \text{ At } y = 0$$

$$\therefore 2x - 3 \times 0 - 6 = 0$$

$$\therefore 2x = 6$$

$$\therefore x = 3$$

$\therefore$  The straight line cuts the  $x$ -axis at the point  $A(3, 0)$

$$\therefore \text{At } x = 0$$

$$\therefore 2 \times 0 - 3y - 6 = 0$$

$$\therefore -3y = 6$$

$$\therefore y = -2$$

$\therefore$  The straight line cuts the  $y$ -axis at the point

$$B(0, -2)$$

2] Let D be the midpoint of  $\overline{AB}$

$$\therefore D = \left( \frac{3+0}{2}, \frac{0-2}{2} \right) = \left( \frac{3}{2}, -1 \right)$$

$\therefore$  the straight line is parallel to the y-axis

$\therefore$  Its slope is undefined

$\therefore$  the straight line passes through the point

$$D \left( \frac{3}{2}, -1 \right)$$

$\therefore$  The equation of the straight line is :  $X = \frac{3}{2}$

11

$$\therefore m_1 = \frac{1-(-1)}{5-2} = \frac{2}{3}, m_2 = \frac{-a}{3}$$

$\therefore$  the two straight lines are parallel

$$\therefore m_1 = m_2$$

$$\therefore \frac{2}{3} = \frac{-a}{3}$$

$$\therefore a = -2$$

12

$$\therefore m_1 = \frac{-3-2}{6-5} = -5$$

$$\therefore m_2 = \frac{1}{5}$$

$$\therefore m_2 = \frac{a}{5}$$

$$\therefore \frac{a}{5} = \frac{1}{5}$$

$$\therefore a = 1$$

13

$\therefore$  The slope of the straight line L =  $\frac{4}{3}$

$\therefore$  The slope of  $\overline{AB} = \frac{4}{3}$

$$\therefore \text{The slope of } \overline{AB} = \frac{y+3}{5-2} \therefore \frac{y+3}{3} = \frac{4}{3}$$

$$\therefore y+3=4$$

$$\therefore y=1$$

14

$$\therefore m_1 = \frac{-\text{Coefficient of } X}{\text{Coefficient of } y} = 2k-1$$

$$\therefore m_2 = \tan 45^\circ = 1$$

$\therefore$  the two straight lines are parallel

$$\therefore m_1 = m_2$$

$$\therefore 2k-1=1$$

$$\therefore 2k=2$$

$$\therefore k=1$$

15

$$\therefore \text{The slope of } \overline{XY} = \frac{6+2}{-5-3} = -1$$

$\therefore$  The slope of the axis of symmetry of  $\overline{XY} = 1$

$\therefore$  The equation of the axis of symmetry of  $\overline{XY}$  is :

$$y = X + c$$

$\therefore$  the midpoint of  $\overline{XY}$

$$= \left( \frac{3-5}{2}, \frac{-2+6}{2} \right) = (-1, 2)$$

$\therefore (-1, 2)$  satisfies the equation :  $y = X + c$

$$\therefore 2 = -1 + c$$

$$\therefore c = 3$$

$\therefore$  The equation of the axis of symmetry of  $\overline{XY}$  is :

$$y = X + 3$$

16

Let D be the midpoint of  $\overline{BC}$

$$\therefore D = \left( \frac{3+1}{2}, \frac{7-3}{2} \right) = (2, 2)$$

$$\therefore \text{The slope of } \overline{AD} = \frac{-6-2}{5-2} = -\frac{8}{3}$$

$\therefore$  The equation of  $\overline{AD}$  is  $y = -\frac{8}{3}X + c$

$$\therefore D \in \overline{AD}$$

$\therefore (2, 2)$  satisfies its equation

$$\therefore 2 = -\frac{8}{3} \times 2 + c$$

$$\therefore c = \frac{22}{3}$$

$\therefore$  The equation of  $\overline{AD}$  is  $y = -\frac{8}{3}X + \frac{22}{3}$

17

$$\therefore \text{The slope of } \overline{BC} = \frac{1+1}{-2-5} = -\frac{2}{7}$$

$\therefore$  The slope of the perpendicular straight line to it =  $\frac{7}{2}$

$\therefore$  The equation of the perpendicular to  $\overline{BC}$  is

$$y = \frac{7}{2}X + c$$

$\therefore A \in$  the perpendicular to  $\overline{BC}$

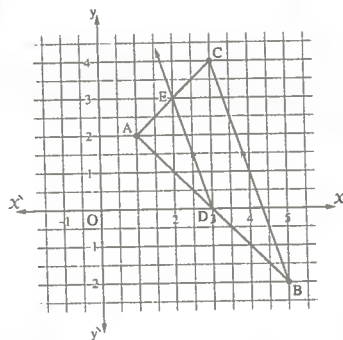
$\therefore (0, 6)$  satisfies the equation

$$\therefore 6 = \frac{7}{2} \times 0 + c$$

$$\therefore c = 6$$

$\therefore$  The equation of the perpendicular to  $\overline{BC}$  from the point A is  $y = \frac{7}{2}X + 6$

18



In  $\triangle ABC$  :

$\therefore D$  is the midpoint of  $\overline{AB}$ ,  $\overline{DE} \parallel \overline{BC}$

$\therefore E$  is the midpoint of  $\overline{AC}$ ,  $DE = \frac{1}{2} BC$

$$\begin{aligned}\therefore DE &= \frac{1}{2} \sqrt{(5-3)^2 + (-2-4)^2} \\ &= \frac{1}{2} \sqrt{4+36} = \frac{1}{2} \sqrt{40} \\ &= \frac{1}{2} \times 2\sqrt{10} = \sqrt{10} \text{ length unit}\end{aligned}$$

$$\therefore \text{The slope of } \overline{BC} = \frac{4+2}{3-5} = -3$$

$$\therefore \text{The slope of } \overline{DE} = -3$$

$$\therefore \text{The equation of } \overline{DE} \text{ is } y = -3x + c$$

$$\therefore D \text{ is the midpoint of } \overline{AB} = \left( \frac{1+5}{2}, \frac{2-2}{2} \right) = (3, 0)$$

$$\therefore (3, 0) \text{ satisfies its equation}$$

$$\therefore 0 = -3 \times 3 + c \quad \therefore c = 9$$

$$\therefore \text{The equation of } \overline{DE} \text{ is } y = -3x + 9$$

19

$$\therefore \text{The slope of } \overline{AC} = \frac{6-4}{-1-5} = -\frac{2}{6} = -\frac{1}{3}$$

 $\therefore$  the two diagonals of the square are perpendicular

$$\therefore \text{The slope of } \overline{BD} = 3$$

$$\therefore \text{The equation of } \overline{BD} \text{ is } y = 3x + c$$

$$\therefore \text{The coordinates of the midpoint of } \overline{AC}$$

$$= \left( \frac{5-1}{2}, \frac{6+4}{2} \right) = (2, 5)$$

$$\therefore (2, 5) \text{ satisfies the equation of } \overline{BD}$$

$$\therefore 5 = 2 \times 3 + c \quad \therefore c = -1$$

$$\therefore \text{The equation of } \overline{BD} \text{ is } y = 3x - 1$$

20

$$\therefore \text{The slope of } \overline{AC} = \frac{3-0}{1-6} = -\frac{3}{5}$$

$$\therefore \overline{AC} \perp \overline{BD}$$

$$\therefore \text{The slope of } \overline{BD} = \frac{5}{3}$$

$$\therefore \text{The equation of } \overline{BD} \text{ is } y = \frac{5}{3}x + c$$

 $\therefore$  The two diagonals of the rhombus bisect each other

$$\therefore \text{midpoint of } \overline{AC} = \left( \frac{1+6}{2}, \frac{3-0}{2} \right) = (3.5, 1.5)$$

$$\therefore (3.5, 1.5) \text{ satisfies the equation of } \overline{BD}$$

$$\therefore 1.5 = \frac{5}{3} \times 3.5 + c \quad \therefore c = -4\frac{1}{3}$$

$$\therefore \text{The equation of } \overline{BD} \text{ is } y = \frac{5}{3}x - 4\frac{1}{3}$$

21

$$\therefore \text{The slope of } \overline{AB} = \frac{-3-3}{-1-2} = 2$$

$$\therefore \text{The equation of } \overline{AB} \text{ is } y = 2x + c$$

$$\therefore \overline{AB} \text{ passes through the point } (2, 3)$$

$$\therefore 3 = 2 \times 2 + c$$

$$\therefore c = -1$$

$$\therefore \text{The equation of } \overline{AB} \text{ is } y = 2x - 1$$

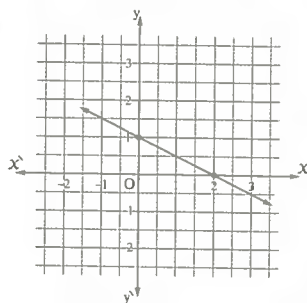
 by substituting in the equation of  $\overline{AB}$  by  $x = 2k + 1$ 

$$\therefore y = 2(2k + 1) - 1 = 4k + 2 - 1 = 4k + 1$$

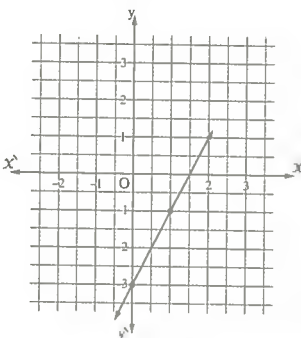
$$\therefore \text{The point } C(2k + 1, 4k + 1) \text{ satisfies the equation of } \overline{AB} \quad \therefore C \in \overline{AB}$$

22

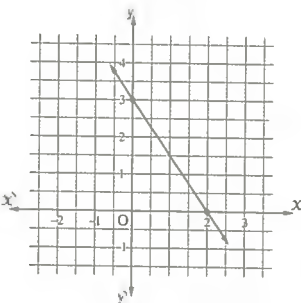
1



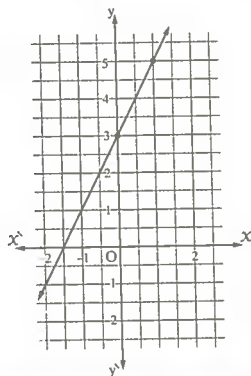
2



3



- 23 The slope of the straight line = 2 and the length of the intercepted part from y-axis = 3 units



24

- 1 The straight line passes through the two points (0, 3) and (4, 5)
- ∴ The slope =  $\frac{5-3}{4-0} = \frac{1}{2}$
- 2 3 units from the positive part of y-axis
- 3 The equation is  $y = \frac{1}{2}x + 3$
- 4 6 units from the negative part of x-axis
- 5 The area of the triangle =  $\frac{1}{2} \times 3 \times 6 = 9$  square units

25

- 1 The slope of the straight line =  $\frac{3-1}{2-1} = 2$
- ∴ The equation of the straight line is  $y = 2x + c$
- ∴ The point (1, 1) ∈ the straight line
- ∴  $1 = 2 \times 1 + c$  ∴  $c = -1$
- ∴ The equation of the straight line is  $y = 2x - 1$
- 2 One unit of the negative part of y-axis
- 3 The point (3, a) satisfies the equation
- ∴  $a = 2 \times 3 - 1 = 5$

26

- ∴  $\overline{AB}$  cuts two equal parts of the two axes
- ∴  $OA = OB$
- ∴ In  $\triangle AOB$  which is right at O
- $m(\angle ABO) = m(\angle BAO) = 45^\circ$

- ∴  $\overline{AB}$  makes with the positive direction of the x-axis an angle of measure  $135^\circ$
- ∴ The slope of  $\overline{AB} = \tan 135^\circ = -1$
- ∴  $k =$  the slope of  $\overline{AB} = -1$
- ∴  $y = -x + c$
- ∴  $(2, 3) \in \overline{AB}$
- ∴  $3 = -2 + c$  ∴  $c = 5$  (First req.)
- ∴  $OA = OB = 5$  length units
- ∴ The area of  $\triangle ABO = \frac{1}{2} \times 5 \times 5 = 12.5$  square units (Second req.)

27

- ∴  $\triangle ABO$  is equilateral
- ∴ C is the midpoint of  $\overline{AB}$
- ∴  $\overline{OC} \perp \overline{AB}$
- ∴  $m(\angle BOC) = 30^\circ$
- ∴  $\tan(\angle BOC) = \tan 30^\circ = \frac{1}{\sqrt{3}}$
- ∴ The equation of  $\overline{OC}$  is  $y = \frac{1}{\sqrt{3}}x + c$
- ∴  $O \in \overline{OC}$  ∴  $c = 0$
- ∴ The equation of  $\overline{OC}$  is  $y = \frac{1}{\sqrt{3}}x$

28

- Let: D (X, y)
- ∴ the midpoint of  $\overline{AB} =$  The midpoint of  $\overline{OD}$
- ∴ the midpoint of  $\overline{AB} = \left(\frac{6+2}{2}, \frac{6+2}{2}\right) = (4, 4)$
- ∴ the midpoint of  $\overline{OD} = \left(\frac{0+X}{2}, \frac{0+y}{2}\right)$
- ∴  $(4, 4) = \left(\frac{X}{2}, \frac{y}{2}\right)$  ∴  $\frac{X}{2} = 4$  ∴  $X = 8$
- ∴  $\frac{y}{2} = 4$  ∴  $y = 8$
- ∴ D (8, 8) (First req.)
- ∴  $\overline{OD}$  passes through the origin point
- ∴ Its equation is  $y = mX$  (m is the slope)
- ∴ the slope of  $\overline{OD} = \frac{8-0}{8-0} = 1$
- ∴ The equation of  $\overline{OD}$  is  $y = X$  (Second req.)
- ∴ the slope of  $\overline{OD} = \tan(\angle DOC)$
- ∴  $\tan(\angle DOC) = 1$  ∴  $m(\angle DOC) = 45^\circ$
- ∴  $m(\angle DOE) = 180^\circ - 45^\circ = 135^\circ$  (Third req.)

29

Let the point A (0, y)

 ∴ The straight line  $L_1$  passes through the point A (0, y)

$$\therefore 2 \times 0 - y + 2 = 0 \quad \therefore y = 2 \quad \therefore A(0, 2)$$

$$\therefore m_1 = \frac{-2}{-1} = 2, \quad L_1 \perp L_2$$

$$\therefore m_1 m_2 = -1 \quad \therefore 2 \times m_2 = -1$$

$$\therefore m_2 = \frac{-1}{2}$$

$$\therefore \text{The equation of } L_2 \text{ is : } y = \frac{-1}{2}x + c$$

 ∴ the straight line  $L_2$  passes through the point A (0, 2)

$$\therefore 2 = \frac{-1}{2} \times 0 + c \quad \therefore c = 2$$

$$\therefore \text{The equation of } L_2 \text{ is : } y = \frac{-1}{2}x + 2$$

30

$$\text{1 First : } \therefore \tan(\angle ABO) = \frac{4}{3}$$

$$\therefore m(\angle ABO) \approx 53^\circ 7' 48''$$

 From  $\triangle ABO$ :

$$m(\angle BAO) = 180^\circ - (90^\circ + 53^\circ 7' 48'') \\ = 36^\circ 52' 12''$$

Second : Let the point (x, 0)

$$\therefore \tan(\angle ABO) = \frac{4}{3}$$

$$\therefore \frac{8}{x} = \frac{4}{3} \quad \therefore 4x = 24$$

$$\therefore x = 6 \quad \therefore B(6, 0)$$

$$\text{2 First : The slope of } \overline{AB} = \frac{0-8}{6-0} = \frac{-8}{6} = \frac{-4}{3}$$

 Second : ∴ The required straight line is perpendicular to  $\overline{AB}$ 

$$\therefore \text{The slope of } \overline{AB} = -\frac{4}{3}$$

$$\therefore \text{The slope of the required straight line is } = \frac{3}{4}$$

$$\therefore \text{The equation of the required straight line is :}$$

$$y = \frac{3}{4}x + c$$

∴ the straight line passes through the point

$$O(0, 0)$$

$$\therefore 0 = \frac{3}{4} \times 0 + c \quad \therefore c = 0$$

∴ The equation of the required straight line is :

$$y = \frac{3}{4}x$$

31

$$\text{1 } O = (0, 0)$$

$$\therefore A = (x, 0), B = (0, y), C = (4, 3)$$

 where C is the midpoint of  $\overline{AB}$ 

$$\therefore \frac{x+0}{2} = 4$$

$$\therefore x = 8$$

$$\therefore \frac{y+0}{2} = 3$$

$$\therefore y = 6$$

$$\therefore A(8, 0), B(0, 6)$$

$$\text{2 } OA = 8 \text{ length units, } OB = 6 \text{ length units}$$

$$\therefore CA = \sqrt{(8-4)^2 + (0-3)^2} = 5 \text{ length units}$$

$$\therefore CB = CA = 5 \text{ length units}$$

$$\therefore CO = \sqrt{4^2 + 3^2} = 5 \text{ length units}$$

$$\text{3 The slope of } \overline{AB} = \frac{6-0}{0-8} = -\frac{3}{4}$$

$$\therefore \text{the slope of } \overline{OC} = \frac{0-3}{0-4} = \frac{3}{4}$$

$$\therefore \text{the slope of } \overline{OA} = \text{zero}$$

$$\therefore \text{the slope of } \overline{OB} \text{ is undefined}$$

$$\text{4 The equation of } \overline{AB} \text{ is : } y = -\frac{3}{4}x + 6$$

$$\therefore \text{the equation of } \overline{CO} \text{ is : } y = \frac{3}{4}x$$

32

$$\therefore \text{The slope of } \overline{CD} = \frac{3-1}{4-3} = 2$$

$$\therefore \text{The equation of } \overline{CD} \text{ is : } y = 2x + c$$

$$\therefore C \in \overline{CD}$$

$$\therefore (3, 1) \text{ satisfies the equation}$$

$$\therefore 1 = 2 \times 3 + c \quad \therefore 1 = 6 + c$$

$$\therefore c = -5$$

$$\therefore \text{The equation of } \overline{CD} \text{ is : } y = 2x - 5$$

$$\text{Let } A(x, 0), \quad \therefore A \in \overline{CD}$$

$$\therefore (x, 0) \text{ satisfies the equation of } \overline{CD}$$

$$\therefore 0 = 2x - 5$$

$$\therefore x = \frac{5}{2}$$

$$\therefore A\left(\frac{5}{2}, 0\right)$$

$$\therefore AO = 2\frac{1}{2} \text{ length units}$$

$$\therefore \overline{CD} \text{ intercepts from the negative side of y-axis} \\ 5 \text{ length units}$$

$$\therefore OB = 5 \text{ length units}$$

33

$$\text{1 The slope of } \overline{AC} = \frac{-1}{-1} = 1$$

$$\therefore \text{from the equation of } \overline{AC} : y = x - 3$$

$$\therefore OH = 3 \text{ length unit.}$$

$$\text{2 } \therefore \text{The slope of } \overline{BC} = \sqrt{3}$$

$$\therefore \tan(\angle CBD) = \sqrt{3} \quad \therefore m(\angle CBD) = 60^\circ$$

$$\therefore \text{the slope of } \overline{AC} = 1$$

$$\therefore \tan(\angle CAD) = 1 \quad \therefore m(\angle CAD) = 45^\circ$$

- 3  $\therefore \angle CBD$  is an exterior angle of  $\triangle ABC$   
 $\therefore m(\angle CBD) = m(\angle ACB) + m(\angle CAD)$   
 $\therefore m(\angle ACB) = 60^\circ - 45^\circ = 15^\circ$

- 34  
 $\therefore C(5, 2)$   
 $\therefore OB = 5$  length units,  $BC = 2$  length units  
 $\therefore ABCD$  is a square  
 $\therefore AB = BC = AD = 2$  length units  
 $\therefore OA = OB - BA = 5 - 2 = 3$  length units  
 $\therefore$  The slope of the straight line  $L = \tan(\angle AOD)$   
 $= \frac{AD}{OA} = \frac{2}{3}$   
 $\therefore$  The equation of the straight line  $L$  is:  
 $y = \frac{2}{3}x + c$   
 $\therefore$  the straight line  $L$  passes through the origin point  
 $\therefore c = 0$   
 $\therefore$  The equation of the straight line  $L$  is:  $y = \frac{2}{3}x$

- 35  
 Let the length of  $OA = l$  length unit  
 $\therefore ABCD$  is a square  $\therefore AB = BC$   
 $\therefore OA = AB$   
 $\therefore OA = AB = BC = l$  length unit  
 In  $\triangle OBC$ :  $\therefore BC \perp BO$  (properties of the square)  
 $\therefore \tan(\angle BOC) = \frac{BC}{BO} = \frac{l}{2l} = \frac{1}{2}$   
 $\therefore$  The slope of  $\overline{OC} = \tan(\angle BOC) = \frac{1}{2}$   
 $\therefore$  The equation  $\overline{OC}$  is:  $y = \frac{1}{2}x + c$   
 $\therefore \overline{OC}$  passes through the origin point  
 $\therefore c = 0$   
 $\therefore$  The equation of  $\overline{OC}$  is:  $y = \frac{1}{2}x$

- 36  
 $\therefore$  The straight line  $L_1$  passes through the origin point  
 $\therefore$  Its equation is:  $y = mx$   
 $\therefore$  the slope of the straight line  $L_1 = \tan 45^\circ = 1$   
 $\therefore$  The equation of the straight line  $L_1$  is:  $y = x$   
 (Q.E.D.1)

- $\therefore$  the straight line  $L_1 \parallel$  the straight line  $L_2$   
 $\therefore$  The slope of the straight line  $L_1$  = the slope of the straight line  $L_2 = 1$

- $\therefore$  The equation of the straight line  $L_2$  is:  $y = x + c$   
 $\therefore$  the straight line  $L_2$  passes through the point  $A(1, 5)$   
 $\therefore 5 = 1 + c$   $\therefore c = 4$   
 $\therefore$  The equation of the straight line  $L_2$  is:  
 $y = x + 4$  (Q.E.D.2)

- Let  $B(x, y)$   
 $\therefore B$  belongs to the straight line  $L_1$   $\therefore y = x$   
 $\therefore \overline{AB} \perp L_1$   $\therefore$  The slope of  $\overline{AB} = -1$   
 $\therefore \frac{y-5}{x-1} = -1$   $\therefore y-5 = -x+1$   
 $\therefore x = y$   $\therefore y-5 = -y+1$   
 $\therefore 2y = 6$   $\therefore y = 3$   
 $\therefore x = 3$   $\therefore B(3, 3)$   
 $\therefore$  The length of  $\overline{AB} = \sqrt{(1-3)^2 + (5-3)^2}$   
 $= 2\sqrt{2}$  length unit (Q.E.D.3)

- 37  
 1 2 m.  
 2 The velocity of the particle = The slope of the straight line passing through the two points  $(0, 2)$   
 $\therefore (4, 4) = \frac{4-2}{4-0} = \frac{1}{2}$   
 $\therefore$  The velocity =  $\frac{1}{2}$  m./sec.  
 3  $d = \frac{1}{2}t + 2$   
 4 2 metre  
 5 7 seconds

- 38  
 1 90 km. 2 2.5 hours  
 3 The velocity of the car = The slope of the straight line which passes through the two points  $(0.5, 30)$   
 $\therefore (2, 120) = \frac{120-30}{2-0.5} = 60$  km./hr.  
 4  $d = 60t$

- 39  
 1 Yes, A and B start motion at the same time  
 2 3.2 minutes approximately

- 3 The velocity of A = The slope of the straight line passing through the two points  $(0, 9)$  &  $(5, 0)$

$$= \frac{0-9}{5-0} = -1.8 \text{ km/minutes}$$

- 4  $d = t$

40

$\therefore$  ABCD is a square its area equals 25 square units

$\therefore AB = BC = 5$  length units

$\therefore B(3, 0)$

$\therefore OB = 3$  length units

$\therefore$  In  $\triangle AOB$  which is right-angled at O

$\therefore AO = 4$  length units (Pythagoras)

$\therefore \triangle AOB \cong \triangle BEC$  (prove by yourself)

$\therefore EC = OB = 3$  length units

$\therefore EB = AO = 4$  length units

$\therefore OE = 7$  length units  $\therefore$  The point  $C(7, 3)$

$\therefore \overline{CO}$  passes through the origin point

$\therefore$  The equation of  $\overline{CO}$  is :

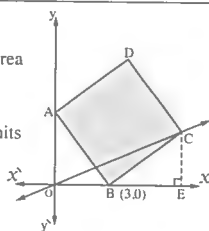
$y = mX$  (where  $m$  is the slope)

$\therefore$  the slope of  $\overline{CO} = \frac{0-3}{0-7} = \frac{3}{7}$

$\therefore$  The equation of  $\overline{CO}$  is :  $y = \frac{3}{7}X$

i.e.  $7y = 3X$

(The req.)



41

Let :  $OB = OA = X$

$\therefore \triangle AOB$  is right-angled at O

$\therefore X^2 + X^2 = (2\sqrt{2})^2$  (Pythagoras)

$\therefore 2X^2 = 8 \therefore X^2 = 4$

$\therefore X = 2 \therefore OB = OA = 2$

$\therefore$  The point  $B(2, 0)$

Let the equation of  $\overline{AB}$  by :  $y = mX + n$

$\therefore$  The slope of  $\overline{AB} = \tan(\angle BAO) = \frac{BO}{AO} = 1$

$\therefore n = 2$

$\therefore$  The equation of  $\overline{AB}$  is :  $y = X + 2$

$\therefore C(1, k) \in \overline{AB} \therefore k = 1 + 2$

$\therefore k = 3 \therefore C(1, 3)$

$\therefore \overline{CD} \perp \overline{AB}$ , the slope of  $\overline{AB} = 1$

$\therefore$  The slope of  $\overline{CD} = -1$

$\therefore$  The equation of  $\overline{CD}$  is :  $y = -X + \ell$

$\therefore (1, 3) \in \overline{CD} \therefore 3 = -1 + \ell$

$\therefore \ell = 4$

$\therefore$  The equation of  $\overline{CD}$  is :  $y = -X + 4$  (The req.)

## Answers of exams on unit five

## Model 1

1

- 1 c      2 a      3 a  
4 c      5 c      6 d

2

[a] Prove by yourself.

 [b] The equation of the straight line is :  
 $5y + 2x - 26 = 0$ 

3

[a] Prove by yourself.

 , the area of the circle =  $25\pi$  square units

 [b] 1 The point of intersection of its  
 diagonals =  $(1, 0)$ 

2 The area of the rhombus = 24 square units.

4

 [a]  $X = 8$  or  $X = 4$ 

 [b] Prove by yourself ,  $\sqrt{26}$  length units.

5

 [a]  $y = 2x + 1$ 

 [b]  $y = -x + 3$ 

## Model 2

1

- 1 c      2 a      3 d  
4 d      5 b      6 d

2

 [a] D  $(-2, 4)$  , E  $(2, 3)$ 

 [b] 1  $BC = 2\sqrt{2}$  length units.

 2 The area of the figure OABC  
 = 4 square units

 3  $m(\angle OCB) = 45^\circ$ 

3

[a] Prove by yourself.

 [b]  $X = 5$  or  $X = 1$ 

4

[a] Prove by yourself.

 , the area of  $\triangle ABC = 5$  square units

 [b] The equation of the straight line is :  
 $y = \frac{1}{3}x + \frac{5}{3}$ 

5

[a] Prove by yourself.

 [b]  $m = 10$ 

## Answers of accumulative basic skills

- |      |      |      |      |
|------|------|------|------|
| 1 d  | 2 a  | 3 c  | 4 b  |
| 5 b  | 6 a  | 7 a  | 8 a  |
| 9 d  | 10 d | 11 b | 12 a |
| 13 d | 14 a | 15 a | 16 d |
| 17 c | 18 c | 19 d | 20 c |
| 21 c | 22 c | 23 b | 24 d |
| 25 c | 26 c | 27 d | 28 c |
| 29 c | 30 d | 31 b | 32 d |
| 33 a | 34 c | 35 c | 36 b |
| 37 d | 38 b | 39 b |      |

## **Guide Answers**

# **of The Notebook**

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- **Quizzes.**
- **Final Examinations.**



Answers of the quizzes  
of algebra & statistics

## Quiz 1

1

1 c

2 d

3 c

2

[a] 1  $Y = \{2, 5, 7\}$

2  $n(Y^2) = 9$

3  $Y \times X = \{(2, 2), (5, 2), (7, 2)\}$

[b] 1  $(X \cap Y) \times Z = \{(2, 4), (6, 4)\}$

2  $(Y - Z) \times X = \{(2, 1), (2, 2), (2, 6), (5, 1), (5, 2), (5, 6), (6, 1), (6, 2), (6, 6)\}$

## Quiz 2

1

1 b

2 c

3 c

2

[a] 1  $R = \{(-2, 2), (-1, 1), (0, 0), (1, -1), (2, -2)\}$

, represent by yourself.

2 R is a function on X because each element of X has a unique image in X

[b] 1 10

2 Represent by yourself.

## Quiz 3

1

1 c

2 c

3 d

2

[a] 1  $f(1) = 1, f(-2) = -5, f(0) = -1$

2  $x = 5$

[b] 1 The domain =  $\{0, 1, 2, 3\}$   
the range =  $\{1, 2, 3, 4\}$

2  $f(x) = 4 - x$

## Quiz 4

1

1 c

2 c

3 b

2

[a]  $R = \{(1, 3), (2, 3), (2, 4), (3, 4)\}$

 R is not a function because  $2 \in X$  has more than one image in X,  $4 \in X$  has no image in X

[b] Draw by yourself

1 The maximum value = 4

 2 The equation of the axis of symmetry is :  $x = -1$ 

## Quiz 5

1

1 d

2 d

3 c

2

[a]  $R = \{(1, 3), (2, 6), (3, 9)\}$   
represent by yourself

 , R is a function,  $\sqrt{x} = 3$ 

[b] Prove by yourself

## Quiz 6

1

1 b

2 a

3 c

2

[a] Prove by yourself

[b] Represent by yourself

1 The minimum value = 0

 2 The equation of the axis of symmetry is :  $x = 2$ 

## Quiz 7

1

1 a

2 c

3 b

2

[a] Prove by yourself

 [b]  $x = 7$

### Quiz 8

1

d

2 c

3 c

2

[a] Prove by yourself

[b] 3.2 cm/sec.

### Quiz 9

1

b

2 d

3 a

2

[a] From model (A) 150 cars

→ from model (B) 50 cars

→ from model (C) 100 cars

and from model (D) 200 cars

[b] Prove by yourself.

### Quiz 10

1

d

2 a

3 d

2

The standard deviation  $\approx 1.43$  units.

Answers of model examinations of the school book of algebra & statistics

Model 1

1

- 1 b 2 c 3 b 4 a 5 c 6 b

2

[a] 1  $Y = \{2, 5, 7\}$

2  $Y \times X = \{(2, 2), (5, 2), (7, 2)\}$

[b] Let  $\frac{a}{b} = \frac{c}{d} = m$ , where  $m > 0$

$\therefore a = bm, c = dm$

$\therefore \text{L.H.S.} = \frac{a}{b-a} = \frac{bm}{bm-dm} = \frac{bm}{b(1-m)} = \frac{m}{1-m}$  (1)

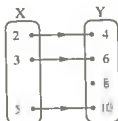
$\therefore \text{R.H.S.} = \frac{c}{d-c} = \frac{dm}{dm-bm} = \frac{dm}{d(1-m)} = \frac{m}{1-m}$  (2)

From (1), (2) :  $\therefore \frac{a}{b-a} = \frac{c}{d-c}$

3

[a] 1  $R = \{(2, 4), (3, 6), (5, 10)\}$

2 R is a function because every element of X has only one image in Y



[b] Let the number be X

$\therefore \frac{X+7}{X+11} = \frac{2}{3}$

$\therefore 3X + 21 = 2X + 22 \therefore X = 1$

$\therefore$  The required number is 1

4

[a] 1 The range =  $\{3, 1, 5\}$

2  $a + b = 8$

[b] 1  $\therefore y \propto \frac{1}{X}$

$\therefore m = 2 \times 3 = 6$

2 At  $X = 1.5$

$\therefore Xy = m$

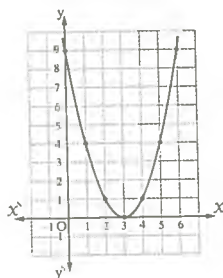
$\therefore Xy = 6$

$\therefore y = \frac{6}{1.5} = 4$

5

[a]  $f(x) = (x-3)^2$

x	0	1	2	3	4	5	6
f(x)	9	4	1	0	1	4	9



From the graph :

The vertex of the curve is  $(3, 0)$

$\therefore$  the minimum value  $= 0$

$\therefore$  the equation of the axis of symmetry is :  $X = 3$

[b] Form the table by yourself

$\therefore$  then the arithmetic mean  $= 7$

$\therefore$  the standard deviation  $\approx 1.41$

Model 2

1

- 1 a 2 c 3 d 4 b 5 c 6 a

2

[a] 1  $n(X \times Z) = 2$

2  $(Y \cap X) \times Z = \{2\} \times \{3\} = \{(2, 3)\}$

[b]  $\therefore b$  is the middle proportional between  $a$  and  $c$

$\therefore \frac{a}{b} = \frac{b}{c} = m \therefore b = cm, a = cm^2$

$\therefore \text{L.H.S.} = \frac{a-b}{a-c} = \frac{cm^2 - cm}{cm^2 - c} = \frac{cm(m-1)}{c(m^2-1)}$

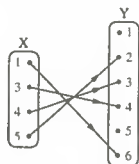
$= \frac{cm(m-1)}{c(m-1)(m+1)} = \frac{m}{m+1}$  (1)

$\therefore \text{R.H.S.} = \frac{b}{b+c} = \frac{cm}{cm+c} = \frac{cm}{c(m+1)} = \frac{m}{m+1}$  (2)

From (1), (2) :  $\therefore \frac{a-b}{a-c} = \frac{b}{b+c}$

3

[a] 1  $R = \{(1, 6), (3, 4), (4, 3), (5, 2)\}$



2  $R$  is a function because every element of  $X$  has only one image in  $Y$

[b] 1  $5a = 3b \quad \therefore \frac{a}{b} = \frac{3}{5}$

$\therefore a = 3m, b = 5m$

$\therefore \frac{7a+9b}{4a+2b} = \frac{7 \times 3m + 9 \times 5m}{4 \times 3m + 2 \times 5m} = \frac{66m}{22m} = 3$

4

[a] 1  $\therefore f(x) = 4x + b, f(3) = 15$

$\therefore 4 \times 3 + b = 15 \quad \therefore b = 3$

[b] 1  $\therefore y \propto x \quad \therefore y = mx$

$\therefore 6 = m \times 3 \quad \therefore m = 2$

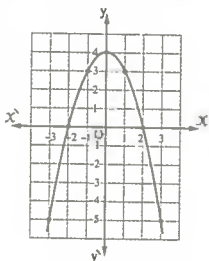
$\therefore y = 2x$

2 At  $X = 5 \quad \therefore y = 2 \times 5 = 10$

5

[a]  $f(x) = 4 - x^2$

$x$	-3	-2	-1	0	1	2	3
$f(x)$	-5	0	3	4	3	0	-5



From the graph : The vertex of the curve is  $(0, 4)$

$\therefore$  the maximum value  $= 4$

$\therefore$  the equation of the axis of symmetry is :  $x = 0$

[b] Form the table by yourself

$\therefore$  then the mean  $= 2.26$

$\therefore$  the standard deviation  $\approx 1.06$

### Answers of model for the merge students

1

1 the first

2 the third

3 30

4  $x$

5 9

6 9

2

1 a

2 a

3 d

4 b

5 c

6 c

3

1 ✓

2 ✗

3 ✗

4 ✓

5 ✓

6 ✓

4

1 1

2 6

3 8

4 10

5  $\pm 6$

6 2

Answers of governors' examinations of algebra & statistics

1 Cairo

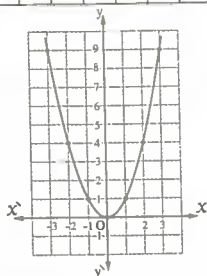
1

- 1 c 2 a 3 b 4 d 5 a 6 d

2

a)  $f(x) = x^2$

x	-3	-2	-1	0	1	2	3
f(x)	9	4	1	0	1	4	9



From the graph :

- 1 The minimum value is 0  
2 The equation of the axis of symmetry is :  $x = 0$

b) Form the table by yourself, then  $\sigma \approx 3.22$

3

a) 1  $X \times Y = \{(3, 4), (3, 5), (4, 4), (4, 5)\}$

2  $\therefore X - Y = \{3\}$

$\therefore (X - Y) \times Z = \{3\} \times \{5, 6\}$   
 $= \{(3, 5), (3, 6)\}$

b)  $\therefore \frac{x}{y} = \frac{z}{l} = m$

$\therefore x = ym, z = lm$

$\therefore \frac{y-x}{x} = \frac{y-ym}{ym} = \frac{y(1-m)}{ym} = \frac{1-m}{m}$

$\therefore \frac{l-z}{z} = \frac{l-lm}{lm} = \frac{l(1-m)}{lm} = \frac{1-m}{m}$

From (1), (2) :  $\therefore \frac{y-x}{x} = \frac{l-z}{z}$

4

a) Let the number be  $x$

$\therefore \frac{3+x}{5+x} = \frac{1}{2}$

$\therefore 6+2x = 5+x$

$\therefore x = -1$

$\therefore$  The number is -1

b) 1  $R = \{(1, 1), (2, 1), (3, 1)\}$

2  $R$  is a function, its range =  $\{1\}$

5

a) 1  $\therefore y \propto x \therefore y = mx$

$\therefore 20 = 4m \therefore m = 5 \therefore y = 5x$

2 At  $y = 40 \therefore 40 = 5x \therefore x = 8$

b)  $\therefore f(5) = 13 \therefore 13 = 2 \times 5 + k$

$\therefore 13 = 10 + k \therefore k = 3$

2

Giza

1

- 1 d 2 c 3 c 4 d 5 a 6 c

2

a) 1  $n(X \times Z) = 2$

2  $\therefore Y \cap X = \{2\}$

$\therefore (Y \cap X) \times Z = \{2\} \times \{3\} = \{(2, 3)\}$

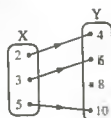
b)  $\therefore f(2) = 10 \therefore 10 = 2 \times 4 + b$

$\therefore 10 = 8 + b \therefore b = 2$

3

a)  $R = \{(2, 4), (3, 6), (5, 10)\}$

$R$  is a function because every element in  $X$  has only one image in  $Y$



b) Let the number be  $x$

$\therefore \frac{7+x}{11+x} = \frac{2}{3}$

$\therefore 21 + 3x = 22 + 2x$

$\therefore x = 1 \therefore$  The number is 1

4

a)  $\therefore 2a = 3b = 3c \therefore 2a = 3b$

$\therefore a = \frac{3}{2}b$

$\therefore 3b = 3c \therefore b = c$

$\therefore a : b : c = \frac{3}{2}b : b : b$  (multiplying by 2)

$\therefore a : b : c = 3b : 2b : 2b$

$\therefore a : b : c = 3 : 2 : 2$

$\therefore a = 3m, b = 2m, c = 2m$

$\therefore \frac{6a+b+c}{4a+6b+6c} = \frac{18m+2m+2m}{12m+12m+12m} = \frac{22m}{36m} = \frac{11}{18}$

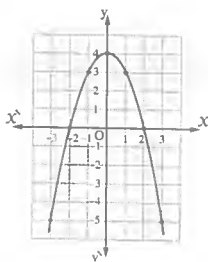
[b] Form the table by yourself, then  $\sigma \approx 1.41$

5

- [a] 1)  $\because y \propto X \quad \therefore y = mX$   
 $\therefore 6 = 3m \quad \therefore m = 2 \quad \therefore y = 2X$   
 2) At  $X = 4 \quad \therefore y = 2 \times 4 \quad \therefore y = 8$

[b]  $f(X) = 4 - X^2$

X	-3	-2	-1	0	1	2	3
f(X)	-5	0	3	4	3	0	-5



From the graph :

- \* The vertex of the curve is : (0 , 4)
- \* The equation of the symmetry axis is :  $X = 0$

### 3 Alexandria

1

- 1 c    2 a    3 b    4 c    5 d    6 c

2

[a]  $f$  is of the first degree

$$f(-2) = 3 \times -2 = -6$$

$$f(\sqrt{3}) = 3 \times \sqrt{3} = 3\sqrt{3}$$

[b]  $\because 5a = 3b \quad \therefore \frac{a}{b} = \frac{3}{5}$

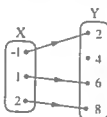
$$\therefore a = 3m \quad b = 5m$$

$$\therefore \frac{7a + 9b}{4a + 2b} = \frac{21m + 45m}{12m + 10m} = \frac{66m}{22m} = \frac{3}{2}$$

3

[a]  $R = \{(-1, 2), (1, 6), (2, 8)\}$

$R$  is a function because every element in  $X$  has only one image in  $Y$



[b]  $\because X^4 y^2 - 14 X^2 y + 49 = 0$

$$\therefore (X^2 y - 7)^2 = 0 \quad \therefore X^2 y - 7 = 0$$

$$\therefore X^2 y = 7 \quad \therefore y \propto \frac{1}{X^2}$$

4

[a]  $\because (X - 2, 3) = (5, y + 1)$

$$\therefore X - 2 = 5 \quad \therefore X = 7$$

$$\therefore y + 1 = 3 \quad \therefore y = 2$$

[b] Form the tables by yourself, then the  $\bar{X} = 2$  children,  $\sigma \approx 1$  children.

5

[a]  $\because \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$$\therefore c = dm, \quad b = dm^2, \quad a = dm^3$$

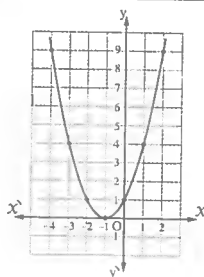
$$\therefore \frac{a}{b+d} = \frac{dm^3}{dm^2 + dm} = \frac{dm^3}{d(m^2 + 1)} = \frac{m^3}{m^2 + 1} \quad (1)$$

$$\therefore \frac{c^3}{c^2 d + d^3} = \frac{d^3 m^3}{d^3 m^2 + d^3} = \frac{d^3 m^3}{d^3(m^2 + 1)} = \frac{m^3}{m^2 + 1} \quad (2)$$

$$\text{From (1) , (2) : } \therefore \frac{a}{b+d} = \frac{c^3}{c^2 d + d^3}$$

[b]  $f(X) = X^2 + 2X + 1$

X	-4	-3	-2	-1	0	1	2
f(X)	9	4	1	0	1	4	9



From the graph :

- 1 The vertex of the curve is : (-1 , 0)
- 2 The equation of the symmetry axis is :  $X = -1$
- 3 The minimum value  $\approx 0$

### 4 El-Kalyoubia

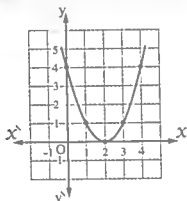
1

- 1 d    2 c    3 d    4 a    5 a    6 a



[a]  $f(x) = (x-2)^2$

$x$	0	1	2	3	4
$f(x)$	4	1	0	1	4



From the graph :

[1] The equation of the symmetry axis is :  $x = 2$

[2] The minimum value = 0

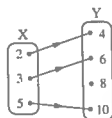
$$[b] \therefore y \propto \frac{1}{x} \quad \therefore \frac{y_1}{y_2} = \frac{x_2}{x_1}$$

$$\therefore \frac{\frac{4}{7}}{y_2} = \frac{3 \frac{1}{5}}{2 \frac{4}{5}} \quad \therefore y_2 = \frac{\frac{4}{7} \times 2 \frac{4}{5}}{3 \frac{1}{5}} = \frac{1}{2}$$



[a] [1]  $R = \{(2, 4), (3, 6), (5, 10)\}$

[2] Yes,  $R$  is a function.



$$[b] \therefore \frac{a}{b} = \frac{c}{d} = m \quad \therefore a = bm, \quad c = dm$$

$$\therefore \sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \sqrt[3]{\frac{5b^3m^3 - 3d^3m^3}{5b^3 - 3d^3}}$$

$$= \sqrt[3]{\frac{m^3(5b^3 - 3d^3)}{5b^3 - 3d^3}}$$

$$= \sqrt[3]{m^3} = m$$

$$\therefore \frac{a+c}{b+d} = \frac{bm+dm}{b+d} = \frac{m(b+d)}{b+d} = m$$

$$\text{From (1), (2)} : \therefore \sqrt[3]{\frac{5a^3 - 3c^3}{5b^3 - 3d^3}} = \frac{a+c}{b+d}$$



[a] [1]  $Z - Y = \{5, -2\}$ ,  $X \cap Y = \{4\}$   
 $\therefore (Z - Y) \times (X \cap Y) = \{5, -2\} \times \{4\}$   
 $= \{(5, 4), (-2, 4)\}$

[2]  $n(X^2) = 2 \times 2 = 4$

[b]  $\therefore f(3) = 15 \quad \therefore 15 = 4 \times 3 + b$   
 $\therefore 15 = 12 + b \quad \therefore b = 3$



[a]  $\therefore \frac{a}{2x+y} = \frac{b}{3y-x} = \frac{c}{4x+5y}$

Multiplying the terms of the 2<sup>nd</sup> ratio by 2 and adding the antecedents and consequents of the 1<sup>st</sup> and 2<sup>nd</sup> ratios

$$\therefore \frac{a+b}{2x+y+6y-2x} = \frac{a+2b}{7y}$$

= one of the given ratios (1)

Multiplying the terms of the 2<sup>nd</sup> ratio by 4

and adding the antecedents and consequents of the 2<sup>nd</sup> and 3<sup>rd</sup> ratios

$$\therefore \frac{4b+c}{12y-4x+4x+5y}$$

$$= \frac{4b+c}{17y} = \text{one of the given ratios} \quad (2)$$

From (1) and (2) :

$$\therefore \frac{a+2b}{7y} = \frac{4b+c}{17y} \quad \therefore \frac{a+2b}{7} = \frac{4b+c}{17}$$

[b] Form the tables by yourself, then  $\alpha \approx 1.43$

**5 El-Sharkia**



[1] b [2] c [3] d [4] a [5] d [6] c



[a] [1]  $X - Y = \{1, 2\}$

[2]  $\therefore Y \cap X = \{3\}$

$$\therefore (Y \cap X) \times Y = \{3\} \times \{3, 4\}$$

$$= \{(3, 3), (3, 4)\}$$

[3]  $n(Y^2) = 2 \times 2 = 4$

[b]  $\therefore \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$

$$\therefore c = dm, \quad b = dm^2, \quad a = dm^3$$

$$\therefore \frac{b+d}{c^2d+d^3} = \frac{dm^2+d}{d^3m^2+d^3} = \frac{d(m^2+1)}{d^3(m^2+1)} = \frac{1}{d^2} \quad (1)$$

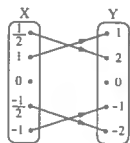
$$\therefore \frac{a}{c^3} = \frac{dm^3}{d^3m^3} = \frac{1}{d^2} \quad (2)$$

$$\text{From (1), (2)} : \therefore \frac{b+d}{c^2d+d^3} = \frac{a}{c^3}$$

1

[a]  $R = \left\{ \left( \frac{1}{2}, 2 \right), (1, 1), \left( -\frac{1}{2}, -2 \right), (-1, -1) \right\}$

R is not a function because the element  $0 \in X$  has no image in Y



[b] (1)  $\therefore y \propto \frac{1}{x^2} \quad \therefore x^2 y = m$

$\therefore \left( \frac{2}{3} \right)^2 \times 9 = m \quad \therefore \frac{4}{9} \times 9 = m$

$\therefore m = 4 \quad \therefore x^2 y = 4$

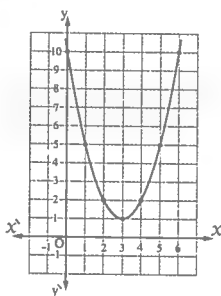
(2) At  $x = \frac{1}{2} \quad \therefore \left( \frac{1}{2} \right)^2 \times y = 4$

$\therefore \frac{1}{4} y = 4 \quad \therefore y = 16$

4

[a]  $f(x) = (x-3)^2 + 1$

x	0	1	2	3	4	5	6
f(x)	10	5	2	1	2	5	10



From the graph :

(1) The vertex of the curve is : (3, 1)

(2) The minimum value = 1

(3) The equation of the axis of symmetry is :  $x = 3$

[b]  $\therefore \frac{x}{3} = \frac{y}{2} = \frac{z}{5} = m$

$\therefore x = 3m, y = 2m, z = 5m$

$\therefore \frac{xy + yz}{x^2 + y^2} = \frac{6m^2 + 10m^2}{9m^2 + 4m^2} = \frac{16m^2}{13m^2} = \frac{16}{13}$

5

[a] Form the table by yourself, then  $\alpha \approx 3.29$

[b]  $\therefore f(a) = b \quad \therefore b = a^2 + b$

$\therefore a^2 = 0 \quad \therefore a = 0$

$\therefore a^2 b + 5 = (0) b^2 + 5 = 5$

## 6 El-Monofia

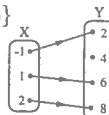
1

- (1) c (2) b (3) c (4) d (5) b (6) a

2

[a] (1)  $R = \{(-1, 2), (1, 6), (2, 8)\}$

(2) R is a function because every element in X has only one image in Y, the range =  $\{2, 6, 8\}$



[b]  $\therefore$  The straight line which represents the function f cuts the y-axis at (b, 3)

$\therefore b = 0$

$\therefore (0, 3)$  satisfies the function

$\therefore 3 = 6 \times 0 - a \quad \therefore a = -3$

$\therefore 2a - 5b = 2 \times -3 - 5 \times 0 = -6$

3

[a] (1)  $X \times Y = \{(1, 2), (1, 3)\}$

(2)  $\therefore Y - Z = \{2\}$

$\therefore X \times (Y - Z) = \{1\} \times \{2\} = \{(1, 2)\}$

(3)  $n(Z^2) = 3 \times 3 = 9$

[b]  $\therefore b$  is the middle proportional between a and c

$\therefore b^2 = ac$

$\therefore \frac{a^2 + b^2}{b^2} = \frac{a^2 + ac}{ac} = \frac{a(a+c)}{ac} = \frac{a+c}{c}$  (1)

$\therefore \frac{b^2 + c^2}{c^2} = \frac{ac + c^2}{c^2} = \frac{c(a+c)}{c^2} = \frac{a+c}{c}$  (2)

From (1), (2) :  $\therefore \frac{a^2 + b^2}{b^2} = \frac{b^2 + c^2}{c^2}$

4

[a]  $\therefore a : b : c = 2 : 3 : 5$

$\therefore a = 2m, b = 3m, c = 5m$

$\therefore a + b + c = 35$

$\therefore 2m + 3m + 5m = 35$

$\therefore 10m = 35 \quad \therefore m = 3.5$

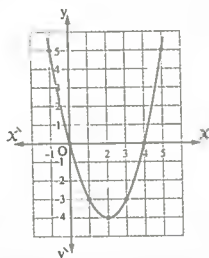
$\therefore a = 7, b = 10.5, c = 17.5$

[b] 1  $\because a \propto \frac{1}{x^2} \therefore a = \frac{m}{x^2}$   
 $\therefore 3 = \frac{m}{2^2} \therefore 3 = \frac{m}{4} \therefore m = 12$   
 $\therefore a = \frac{12}{x^2} \therefore y = \frac{12}{x^2} + 7$   
 2 At  $x = \sqrt{3} \therefore y = \frac{12}{(\sqrt{3})^2} + 7 = 4 + 7 = 11$

5

[a]  $f(x) = x^2 - 4x$

x	-1	0	1	2	3	4	5
f(x)	5	0	-3	-4	-3	0	5



From the graph :

- 1 The vertex of the curve is : (2, -4)
- 2 The equation of the line of symmetry is :  $x = 2$
- 3 The minimum value = -4

[b] Form the table by yourself, then  $\sigma \approx 9.32$

### 7 El-Gharbia

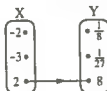
1

- 1 c    2 d    3 a    4 c    5 c    6 b

2

[a]  $R = \{(2, 8)\}$

R is not a function because the elements -2, -3 belonging to X have no images in Y



[b]  $\because x^4 y^2 - 14 x^2 y + 49 = 0$

$\therefore (x^2 y - 7)^2 = 0 \therefore x^2 y - 7 = 0$

$\therefore x^2 y = 7 \therefore y \propto \frac{1}{x^2}$

3

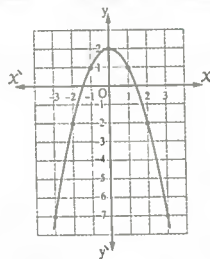
[a]  $\because \frac{a}{b} = \frac{c}{d} = m \therefore a = bm, c = dm$   
 $\therefore \frac{a+b}{b} = \frac{bm+b}{b} = \frac{b(m+1)}{b} = m+1$  (1)

$\therefore \frac{c+d}{d} = \frac{dm+d}{d} = \frac{d(m+1)}{d} = m+1$  (2)

From (1), (2) :  $\therefore \frac{a+b}{b} = \frac{c+d}{d}$

[b]  $f(x) = 2 - x^2$

x	-3	-2	-1	0	1	2	3
f(x)	-7	-2	1	2	1	-2	-7



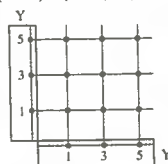
From the graph :

- \* The equation of the axis of symmetry is :  $x = 0$
- \* The maximum value = 2

4

[a]  $\because Y = \{1, 3, 5\}$

$\therefore Y^2 = \{(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)\}$



[b] Let the number be X

$\therefore \frac{5+x^2}{11+x^2} = \frac{3}{5} \therefore 25 + 5x^2 = 33 + 3x^2$

$\therefore 2x^2 = 8 \therefore x^2 = 4$

$\therefore x = 2$  or  $x = -2$  (refused)

$\therefore$  The number is 2

5

[a]  $\because$  The straight line which represents the function cuts the y-axis at (m, 3)

$$\therefore m = 0$$

$\therefore (0, 3)$  satisfies the function

$$\therefore 3 = 6 \times 0 - \ell \quad \therefore \ell = -3$$

[b] Form the table by yourself

, then  $\bar{X} = 16$ ,  $\sigma \approx 3.9$

## 8 El-Dakahlia

1

[a] 1 a

2 c

3 b

[b]  $\therefore b$  is the middle proportional between  $a$  &  $c$

$$\therefore b^2 = ac$$

$$\therefore \frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{a^2}{ac} + \frac{ac}{c^2} = \frac{a}{c} + \frac{a}{c} = \frac{2a}{c}$$

2

[a] 1 b

2 a

3 c

[b] 1  $\therefore R$  is a function

$$\therefore a = 5, b = 7 \text{ or } a = 7, b = 5$$

$$\therefore 3a + 3b = 3(a + b) = 3 \times 12 = 36$$

2 The range =  $\{5, 7\}$

3

$$[a] \therefore \frac{a}{4X + y} = \frac{b}{X - 4y}$$

Adding the antecedents and consequents of 1<sup>st</sup> and 2<sup>nd</sup> ratios

$$\therefore \frac{a + b}{5X - 3y} = \text{one of the given ratios} \quad (1)$$

Subtracting the antecedent and consequent of 2<sup>nd</sup> ratio from the antecedent and consequent of 1<sup>st</sup> ratio

$$\therefore \frac{a - b}{3X + 5y} = \text{one of the given ratios} \quad (2)$$

$$\text{From (1), (2): } \therefore \frac{a + b}{5X - 3y} = \frac{a - b}{3X + 5y}$$

[b] Form the table by yourself

, then  $\sigma \approx 3.29$

4

[a] Let  $C = (0, \ell)$

$\therefore$  the curve of the function  $f$  passes through the point  $C$

$$\therefore \ell = 0^2 - (k - 2) \times 0 - k + 4$$

$$\therefore \ell = 4 - k \quad \therefore B(4 - k, 4 - k)$$

$$\therefore 4 - k = (4 - k)^2 - (k - 2) \times (4 - k) - k + 4$$

$$\therefore (4 - k)^2 - (k - 2)(4 - k) = 0$$

$$\therefore (4 - k)(4 - k - k + 2) = 0$$

$$\therefore (4 - k)(6 - 2k) = 0$$

$$\therefore 4 - k = 0$$

$$\therefore k = 4 \text{ (refused)}$$

$$\text{or } 6 - 2k = 0$$

$$\therefore 6 = 2k$$

$$\therefore k = 3$$

$$[b] \therefore b \propto \frac{1}{X^2}$$

$$\therefore b = \frac{m}{X^2}$$

$$\therefore y = 1 + \frac{m}{X^2}$$

$$\therefore 5 = 1 + \frac{m}{1}$$

$$\therefore m = 4$$

$$\therefore y = 1 + \frac{4}{X^2}$$

$$\text{, at } X = 2$$

$$\therefore y = 1 + \frac{4}{2^2} = 1 + 1 = 2$$

5

$$[a] \therefore 3f(2) + 3\ell(X) = 6$$

$$\therefore f(2) + \ell(X) = 2$$

$$\therefore a + 2^2 + c = 2$$

$$\therefore a + 4 + c = 2$$

$$\therefore a + c = -2$$

$$\therefore 2f(0) + 2\ell(7) = 2[f(0) + \ell(7)]$$

$$= 2[a + 0^2 + c]$$

$$= 2(a + c) = 2 \times -2 = -4$$

[b] 1 The domain =  $\{3, 5, 7\}$

2 The rule of the function is :  $f(X) = 3X$

## 9 Ismailia

1

1 a

2 b

3 d

4 b

5 c

6 d

2

$$[a] 1 X \times Y = \{(2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5)\}$$

$$2 X^2 = \{(2, 2), (2, 3), (3, 2), (3, 3)\}$$

$$3 n(Y^2) = 3 \times 3 = 9$$

$$[b] \therefore \frac{a}{b} = \frac{4}{3}$$

$$\therefore a = 4 \text{ m, } b = 3 \text{ m}$$

$$\therefore \frac{2a + b}{5a - 3b} = \frac{8 \text{ m} + 3 \text{ m}}{20 \text{ m} - 9 \text{ m}} = \frac{11 \text{ m}}{11 \text{ m}} = 1$$

3

$$[a] \therefore y \propto \frac{1}{X^2} \quad \therefore \frac{y_1}{y_2} = \frac{X_2^2}{X_1^2}$$

$$\therefore \frac{5}{y_2} = \frac{2^2}{3^2}$$

$$\therefore \frac{5}{y_2} = \frac{4}{9}$$

$$\therefore y_2 = \frac{5 \times 9}{4} = \frac{45}{4}$$

[b]  $\therefore$  The straight line which represents the function cuts the  $y$ -axis at  $(b, 5)$

$$\therefore b = 0$$

$$\therefore (0, 5) \text{ satisfies the function}$$

$$\therefore 5 = 3 \times 0 - a \quad \therefore a = -5$$

1

$$[a] \therefore \frac{1+2X}{3+2X} = \frac{3+2X}{7+2X}$$

$$\therefore (1+2X)(7+2X) = (3+2X)^2$$

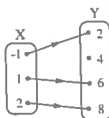
$$\therefore 7+16X+4X^2 = 9+12X+4X^2$$

$$\therefore 7+16X = 9+12X$$

$$\therefore 4X = 2 \quad \therefore X = \frac{1}{2}$$

$$[b] [1] R = \{(-1, 2), (1, 6), (2, 8)\}$$

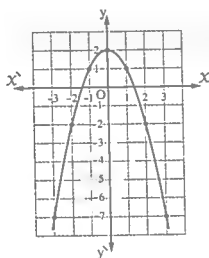
[2] R is a function because every element in X has only one image in Y



2

$$[a] f(X) = 2 - X^2$$

X	-3	-2	-1	0	1	2	3
f(X)	-7	-2	1	2	1	-2	-7



From the graph :

[1] The vertex of the curve is : (0, 2)

[2] The equation of the axis of symmetry is : X = 0

[3] The maximum value = 2

[b] Form the table by yourself, then  $\sigma \approx 3.29$

10 Suez

1

[1] a [2] c [3] d [4] b [5] c [6] b

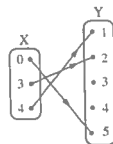
2

$$[a] \therefore \frac{a}{b} = \frac{3}{4} \quad \therefore a = 3m, b = 4m$$

$$\therefore \frac{4a+b}{2a-b} = \frac{12m+4m}{6m-4m} = \frac{16m}{2m} = 8$$

$$[b] [1] R = \{(0, 5), (3, 2), (4, 1)\}$$

2



[3] Yes, R is a function.

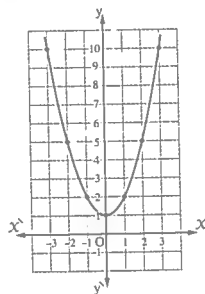
3

$$[a] [1] X = \{2, 3\}, Y = \{6, 9\}$$

$$[2] Y \times Y = \{(6, 6), (6, 9), (9, 6), (9, 9)\}$$

$$[b] f(X) = 1 + X^2$$

X	-3	-2	-1	0	1	2	3
f(X)	10	5	2	1	2	5	10



From the graph :

[1] The vertex of the curve is : (0, 1)

[2] The equation of the axis of symmetry is : X = 0

[3] The minimum value = 1

4

$$[a] \therefore \frac{X}{y} = \frac{z}{r} = m \quad \therefore X = ym, z = rm$$

$$\therefore \frac{X^2 + 2z^2}{y^2 + 2r^2} = \frac{y^2 m^2 + 2r^2 m^2}{y^2 + 2r^2} = \frac{m^2 (y^2 + 2r^2)}{y^2 + 2r^2} = m^2 \quad (1)$$

$$\therefore \frac{Xz}{yr} = \frac{ym \times rm}{yr} = m^2 \quad (2)$$

$$\text{From (1), (2) : } \therefore \frac{X^2 + 2z^2}{y^2 + 2r^2} = \frac{Xz}{yr}$$

[b] [1] The variation is inverse.

$$[2] \therefore y \propto \frac{1}{X} \quad \therefore Xy = m \quad \therefore m = 12$$

$$[3] \text{ At } X = 3 \quad \therefore 3y = 12 \quad \therefore y = 4$$

10

[a] 1  $f(2) + g(2) = 2^2 - 3 \times 2 + 2 - 3$   
 $= 4 - 6 - 1 = -3$

2  $f(3) + g(3) = 3^2 - 3 \times 3 + 3 - 3$   
 $= 9 - 9 + 3 - 3$   
 $= 0$

[b] Form the table by yourself, then  $\sigma \approx 3.29$

### Port Said

1

- 1 c   2 b   3 d   4 d   5 a   6 a

2

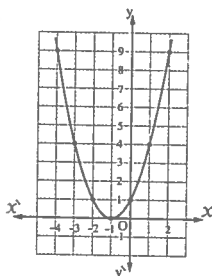
[a] 1  $n(X \times Z) = 2 \times 2 = 4$

2  $\therefore X - Y = \{1\}$

$\therefore (X - Y) \cap Z = \emptyset$

[b]  $f(x) = x^2 + 2x + 1$

x	-4	-3	-2	-1	0	1	2
f(x)	9	4	1	0	1	4	9



From the graph:

1 The vertex of the curve is:  $(-1, 0)$

2 The minimum value = 0

3

[a]  $\therefore f(3) = 15$     $\therefore 15 = 4 \times 3 + b$

$\therefore 15 = 12 + b$     $\therefore b = 3$

[b] 1  $\therefore y \propto \frac{1}{x}$     $\therefore xy = m$     $\therefore 2.5 \times 6 = m$

$\therefore m = 15$     $\therefore xy = 15$

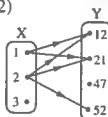
2 At  $x = 5$     $\therefore 5y = 15$     $\therefore y = 3$

4

[a] 1  $R = \{(1, 12), (1, 21), (2, 12), (2, 21), (2, 52)\}$

2  $2R21$  because  $2 \in X$

its image in  $Y$  is 21



[b]  $\therefore \frac{7}{x} = \frac{x}{\frac{1}{y}}$     $\therefore \frac{7}{x} = xy$

$\therefore x^2y = 7$     $\therefore (x^2y)^2 = 7^2$     $\therefore x^4y^2 = 49$

5

[a]  $\therefore \frac{x}{3} = \frac{y}{4} = \frac{z}{5} = m$

$\therefore x = 3m$  ,  $y = 4m$  ,  $z = 5m$

$\therefore \frac{2y - z}{3x - 2y + z} = \frac{8m - 5m}{9m - 8m + 5m} = \frac{3m}{6m} = \frac{1}{2}$

[b] Form the table by yourself

, then  $\bar{X} = 8$  ,  $\sigma \approx 4$

### Damietta

1

- 1 a   2 b   3 d   4 d   5 c   6 b

2

[a] 1  $n(X \times Y) = 2 \times 2 = 4$

2  $\therefore X - Y = \{5\}$

$\therefore (X - Y) \times Z = \{5\} \times \{3\} = \{(5, 3)\}$

3  $Y^2 = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

[b]  $\therefore \frac{a}{b} = \frac{b}{c} = m$     $\therefore b = cm$  ,  $a = cm^2$

$\therefore \frac{a-b}{a-c} = \frac{cm^2 - cm}{cm^2 - c} = \frac{cm(m-1)}{c(m^2-1)}$   
 $= \frac{m(m-1)}{(m-1)(m+1)} = \frac{m}{m+1}$  (1)

$\therefore \frac{b}{b+c} = \frac{cm}{cm+c} = \frac{cm}{c(m+1)} = \frac{m}{m+1}$  (2)

From (1), (2):  $\therefore \frac{a-b}{a-c} = \frac{b}{b+c}$

3

[a] 1  $R = \{(1, 6), (3, 4), (4, 3), (5, 2)\}$

2  $R$  is a function because every element in  $X$  has only one image in  $Y$

, the range =  $\{6, 4, 3, 2\}$

$$\begin{aligned}
 [b] \therefore \frac{21X-y}{7X-z} &= \frac{y}{z} \\
 \therefore 7Xy - yz &= 21Xz - yz \\
 \therefore 7Xy &= 21Xz \\
 \therefore y &= 3z \qquad \therefore y \propto z
 \end{aligned}$$

4

 [a] Form the table by yourself, then  $\sigma \approx 3.29$ 

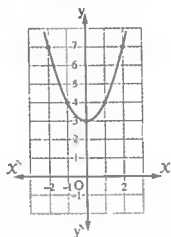
$$\begin{aligned}
 [b] [1] \therefore y &\propto X \qquad \therefore y = mX \\
 \therefore 6 &= 3m \qquad \therefore m = 2 \qquad \therefore y = 2X \\
 [2] \text{ At } X &= 5 \qquad \therefore y = 2 \times 5 = 10
 \end{aligned}$$

5

$$\begin{aligned}
 [a] \therefore \frac{X}{3} &= \frac{y}{4} = \frac{z}{5} = m \\
 \therefore X &= 3m, \quad y = 4m, \quad z = 5m \\
 \therefore \sqrt{3X^2 + 3y^2 + z^2} &= \sqrt{27m^2 + 48m^2 + 25m^2} \\
 &= \sqrt{100m^2} = 10m \quad (1) \\
 \therefore 2X + y &= 6m + 4m = 10m \quad (2) \\
 \text{From (1), (2)} \therefore \sqrt{3X^2 + 3y^2 + z^2} &= 2X + y
 \end{aligned}$$

$$[b] f(X) = X^2 + 3$$

X	-2	-1	0	1	2
f(X)	7	4	3	4	7



From the graph :

- [1] The equation of symmetry line is :  $X = 0$   
 [2] The minimum value = 3

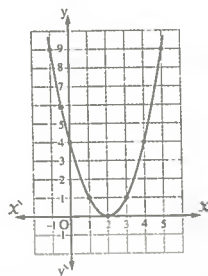
### 13 Kafr El-Sheikh

1

$$[a] [1] c \qquad [2] d \qquad [3] b$$

$$[b] f(X) = (X-2)^2$$

X	-1	0	1	2	3	4	5
f(X)	9	4	1	0	1	4	9



From the graph :

- \* The vertex of the curve is : (2, 0)
- \* The equation of the symmetry axis is :  $X = 2$
- \* The minimum value = 0

2

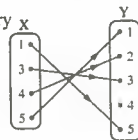
$$[a] [1] a \qquad [2] c \qquad [3] d$$

$$\begin{aligned}
 [b] \therefore \frac{X}{y} &= \frac{y}{z} = m \qquad \therefore y = zm, \quad X = zm^2 \\
 \therefore \frac{X-y}{X-z} &= \frac{zm^2 - zm}{zm^2 - z} = \frac{zm(m-1)}{z(m-1)(m+1)} \\
 &= \frac{m}{m+1} \quad (1) \\
 \therefore \frac{y}{y+z} &= \frac{zm}{zm+z} = \frac{zm}{z(m+1)} = \frac{m}{m+1} \quad (2) \\
 \text{From (1), (2)} \therefore \frac{X-y}{X-z} &= \frac{y}{y+z}
 \end{aligned}$$

3

$$[a] [1] R = \{(1, 5), (3, 3), (4, 2), (5, 1)\}$$

[2] R is a function because every  $x$  element in  $X$  has only one image in  $Y$   
 its range =  $\{1, 2, 3, 5\}$



$$\begin{aligned}
 [b] \therefore \frac{X}{y} &= \frac{2}{3} \qquad \therefore X = 2m, \quad y = 3m \\
 \therefore \frac{3X+2y}{6y-X} &= \frac{6m+6m}{18m-2m} = \frac{12m}{16m} = \frac{3}{4}
 \end{aligned}$$

4

$$[a] [1] X \times Y = \{(2, 4), (2, 0), (-1, 4), (-1, 0)\}$$

$$\begin{aligned}
 [2] \therefore Y \cap Z &= \{4\} \\
 \therefore (Y \cap Z) \times X &= \{4\} \times \{2, -1\} \\
 &= \{(4, 2), (4, -1)\}
 \end{aligned}$$

$$[3] n(Y^2) = 2 \times 2 = 4$$

[b]  $\therefore f(2) = 1 \quad \therefore 1 = 2 \times 2 + a$   
 $\therefore 1 = 4 + a \quad \therefore a = -3$

5

[a] 1  $\therefore y \propto \frac{1}{x^2} \quad \therefore x^2 y = m$   
 $\therefore (4)^2 \times 2 = m \quad \therefore m = 32$   
 $\therefore x^2 y = 32$

2 At  $X = 16 \quad \therefore (16)^2 \times y = 32 \quad \therefore y = \frac{1}{8}$

[b] Form the table by yourself  
 , then  $\bar{X} = 7, \sigma \approx 1.41$

### 14 El-Beheira

1

- 1 d 2 b 3 c 4 a 5 b 6 c

2

[a] 1  $\therefore Y \cap Z = \{2\}$   
 $\therefore X \times (Y \cap Z) = \{1\} \times \{2\} = \{(1, 2)\}$

2  $n(Z^2) = 3 \times 3 = 9$

[b] Let the number be  $X$

$\therefore \frac{5+X^2}{11+X^2} = \frac{3}{5} \quad \therefore 25 + 5X^2 = 33 + 3X^2$

$\therefore 2X^2 = 8 \quad \therefore X^2 = 4$

$\therefore X = -2$  (refused) or  $X = 2$

$\therefore$  The number is 2

3

[a]  $\therefore (a, 3)$  satisfies the function

$\therefore 3 = 4a - 5 \quad \therefore 4a = 8 \quad \therefore a = 2$

[b]  $\therefore \frac{a+b}{3} = \frac{b+c}{6} = \frac{c+a}{5}$

Adding the antecedents and consequents of the three ratios

$\therefore \frac{a+b+b+c+c+a}{3+6+5} = \frac{2a+2b+2c}{14}$   
 $= \frac{a+b+c}{7}$

= one of the given ratios (1)

Multiplying the terms of the 2<sup>nd</sup> ratio by -1 and adding the antecedents and consequents of the three ratios

$\therefore \frac{a+b-b-c+c+a}{3-6+5} = \frac{2a}{2} = a$   
 = one of the given ratios (2)

From (1), (2) :  $\therefore \frac{a+b+c}{7} = a$

$\therefore \frac{a+b+c}{a} = 7$

4

[a] 1  $R = \{(1, 5), (3, 3), (5, 1)\}$

2  $R$  is a function because every element in  $X$  has only one image in  $Y$   
 , its range =  $\{5, 3, 1\}$

[b] Form the table by yourself , then  $\sigma \approx 2.28$

5

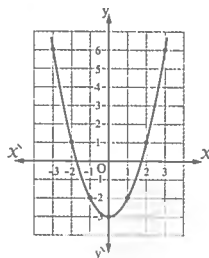
[a] 1  $\therefore y \propto X \quad \therefore y = mX$

$\therefore 6 = 3m \quad \therefore m = 2 \quad \therefore y = 2X$

2 At  $X = 5 \quad \therefore y = 2 \times 5 = 10$

[b]  $f(X) = X^2 - 3$

$X$	-3	-2	-1	0	1	2	3
$f(X)$	6	1	-2	-3	-2	1	6



From the graph :

1 The equation of the axis of symmetry is :  $X = 0$

2 The minimum value = -3

### 15 El-Fayoum

1

- 1 d 2 c 3 a 4 b 5 b 6 c

2

[a]  $\therefore \frac{a}{b} = \frac{2}{3} \quad \therefore a = 2m, b = 3m$

$\therefore \frac{3a-b}{a+2b} = \frac{6m-3m}{2m+6m} = \frac{3m}{8m} = \frac{3}{8}$

[b]  $\therefore f(-3) = 8 \quad \therefore 8 = -3a + 5$

$\therefore -3a = 3 \quad \therefore a = -1$

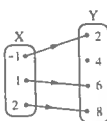
3

$$[a] \because \frac{x}{y} = \frac{y}{z} \quad \therefore y^2 = xz$$

$$\frac{x^2 + y^2}{y^2 + z^2} = \frac{x^2 + xz}{xz + z^2} = \frac{x(x+z)}{z(x+z)} = \frac{x}{z}$$

$$[b] R = \{(-1, 2), (1, 6), (2, 8)\}$$

R is a function because  
every element in X has  
only one image in Y



4

$$[a] \because y \propto X \quad \therefore y = mX \quad \therefore 20 = 7m$$

$$\therefore m = \frac{20}{7} \quad \therefore y = \frac{20}{7}X$$

$$\text{At } X = 14 \quad \therefore y = \frac{20}{7} \times 14 = 40$$

$$[b] \because (5 - 2X, y^3) = (1, 27)$$

$$\therefore 5 - 2X = 1 \quad \therefore -2X = -4 \quad \therefore X = 2$$

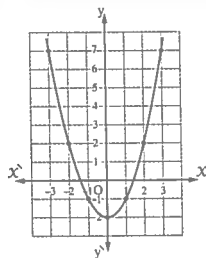
$$y^3 = 27 \quad \therefore y = 3$$

$$\therefore \sqrt[3]{3X + y} = \sqrt[3]{3 \times 2 + 3} = 3$$

5

$$[a] f(X) = X^2 - 2$$

X	-3	-2	-1	0	1	2	3
f(X)	7	2	-1	-2	-1	2	7



From the graph :

\* The vertex of the curve is : (0, -2)

\* The minimum value = -2

[b] Form the table by yourself, then  $a = 4$

16 Beni Suez

1

1 c 2 b 3 a 4 d 5 a 6 d

2

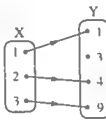
[a] Let the number be X

$$\therefore \frac{7+X}{11+X} = \frac{2}{3} \quad \therefore 21 + 3X = 22 + 2X$$

$$\therefore X = 1 \quad \therefore \text{The number is 1}$$

$$[b] R = \{(1, 1), (2, 4), (3, 9)\}$$

R is a function because  
every element in X has  
only one image in Y



3

$$[a] \because \frac{x}{2} = \frac{y}{3} = \frac{z}{4} = \frac{3X - 2y + 5z}{5k}$$

Multiplying the two terms of the 1<sup>st</sup> ratio by 3, the two terms of the 2<sup>nd</sup> ratio by -2, the two terms of the 3<sup>rd</sup> ratio by 5 and adding the antecedents and consequents of the three ratios

$$\therefore \frac{3X - 2y + 5z}{6 - 6 + 20} = \frac{3X - 2y + 5z}{20}$$

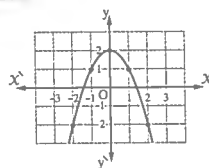
= one of the given ratios

$$\therefore \frac{3X - 2y + 5z}{5k} = \frac{3X - 2y + 5z}{20}$$

$$\therefore 5k = 20 \quad \therefore k = 4$$

$$[b] f(X) = 2 - X^2$$

X	-2	-1	0	1	2
f(X)	-2	1	2	1	-2



From the graph :

\* The vertex of the curve is : (0, 2)

\* The maximum value = 2

4

$$[a] \because y \propto X \quad \therefore y = mX$$

$$\therefore 3 = 15m \quad \therefore m = \frac{1}{5} \quad \therefore y = \frac{1}{5}X$$

$$\text{at } y = 100 \quad \therefore 100 = \frac{1}{5}X \quad \therefore X = 500$$

$$[b] 1) X \times Y = \{(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)\}$$

$$2) Y \times X = \{(3, 1), (3, 2), (4, 1), (4, 2), (5, 1), (5, 2)\}$$

$$3) X^2 = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$$

5

[a]  $\because f(3) + g(5) = 15 \quad \therefore 3 \times 3 + k + k = 15$   
 $\therefore 9 + 2k = 15 \quad \therefore 2k = 6 \quad \therefore k = 3$

[b] Form the table by yourself, then  $\sigma \approx 3.29$

### 17 El-Menia

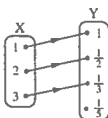
1

1 d    2 c    3 b    4 a    5 d    6 a

2

[a]  $R = \left\{ (1, 1), \left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right) \right\}$

R is a function because every element in X has only one image in Y



[b]  $\because \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, a = cm^2$

$$\therefore \frac{a+b}{a-c} = \frac{cm^2 + cm}{cm^2 - c} = \frac{cm(m+1)}{c(m^2-1)} = \frac{m(m+1)}{(m+1)(m-1)} = \frac{m}{m-1} \quad (1)$$

$$\therefore \frac{b}{b-c} = \frac{cm}{cm-c} = \frac{cm}{c(m-1)} = \frac{m}{m-1} \quad (2)$$

From (1) and (2) :  $\therefore \frac{a+b}{a-c} = \frac{b}{b-c}$

3

[a]  $\because \frac{x}{y} = \frac{2}{3} \quad \therefore x = 2m, y = 3m$

$$\therefore \frac{3x+2y}{6y-x} = \frac{6m+6m}{18m-2m} = \frac{12m}{16m} = \frac{3}{4}$$

[b] 1  $\because Y \cap Z = \{5\}$

$$\therefore X \times (Y \cap Z) = \{3, 4\} \times \{5\} = \{(3, 5), (4, 5)\}$$

2  $\because X - Y = \{3\}$

$$\therefore (X - Y) \times Z = \{3\} \times \{6, 5\} = \{(3, 6), (3, 5)\}$$

4

[a] 1  $\because y \propto \frac{1}{x} \quad \therefore xy = m$

$$\therefore 2 \times 3 = m \quad \therefore m = 6 \quad \therefore xy = 6$$

2 At  $y = 4 \quad \therefore 4x = 6 \quad \therefore x = \frac{3}{2}$

[b] Form the table by yourself, then  $\sigma \approx 3.29$

5

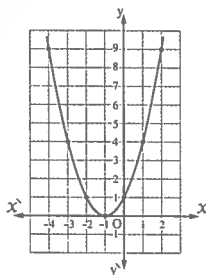
[a] The function  $f$  is of the 3<sup>rd</sup> degree

$$f(0) = 3 - 2 \times (0)^3 = 3$$

$$f(-2) = 3 - 2(-2)^3 = 19$$

[b]  $f(x) = x^2 + 2x + 1$

x	-4	-3	-2	-1	0	1	2
f(x)	9	4	1	0	1	4	9



From the graph :

1 The equation of the symmetry axis is :  $x = -1$

2 The minimum value is 0

### 18 Assiut

1

1 b    2 d    3 a    4 c    5 a    6 b

2

[a] 1  $\because X \cap Y = \{7\}$

$$\therefore (X \cap Y) \times X = \{7\} \times \{6, 7\} = \{(7, 6), (7, 7)\}$$

2  $n(Y^2) = 2 \times 2 = 4$

[b]  $\because \frac{a}{2} = \frac{b}{3} = \frac{c}{4} = m$

$$\therefore a = 2m, b = 3m, c = 4m$$

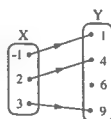
$$\therefore \frac{3c-b}{a+b} = \frac{12m-3m}{2m+3m} = \frac{9m}{5m} = \frac{9}{5}$$

3

[a]  $R = \{(-1, 1), (2, 4), (3, 9)\}$

R is a function because every element in X has only one image in Y

, the range =  $\{1, 4, 9\}$



- [b] ①  $\because y \propto \frac{1}{x} \therefore xy = m$   
 $\therefore 4 \times 3 = m \therefore m = 12 \therefore xy = 12$   
 ② At  $x = \frac{3}{4} \therefore \frac{3}{4}y = 12 \therefore y = 16$

4

[a] Let the number be  $x$

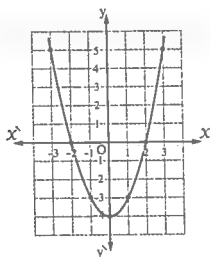
$$\therefore \frac{7+x^2}{11+x^2} = \frac{2}{3} \therefore 21 + 3x^2 = 22 + 2x^2$$

$$\therefore x^2 = 1 \therefore x = -1 \text{ (refused) or } x = 1$$

$$\therefore \text{The number is } 1$$

[b]  $f(x) = x^2 - 4$

$x$	-3	-2	-1	0	1	2	3
$f(x)$	5	0	-3	-4	-3	0	5



From the graph :

- \* The vertex of the curve is :  $(0, -4)$
- \* The minimum value  $= -4$
- \* The equation of the axis of symmetry is :  $x = 0$

5

[a]  $f(\sqrt{2}) + g(5) = (\sqrt{2})^2 - 2 + 3$   
 $= 2 - 2 + 3 = 3$

[b] Form the table by yourself  
 , then  $\bar{X} = 15, \sigma \approx 3.29$

19 Souhag

1

- ① c    ② a    ③ d    ④ b    ⑤ b    ⑥ c

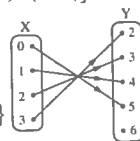
2

- [a] ①  $X = \{1\}, Y = \{1, 3, 5\}$   
 ②  $Y \times X = \{(1, 1), (3, 1), (5, 1)\}$   
 [b]  $\because \frac{x}{y} = \frac{2}{3} \therefore x = 2m, y = 3m$   
 $\therefore \frac{3x+2y}{6y-x} = \frac{6m+6m}{18m-2m} = \frac{12m}{16m} = \frac{3}{4}$

3

[a] ①  $R = \{(0, 5), (1, 4), (2, 3), (3, 2)\}$

- ②  $R$  is a function because  
 every element in  $X$  has  
 only one image in  $Y$   
 , the range  $= \{5, 4, 3, 2\}$



[b] Let the number be  $x$

$$\therefore \frac{7+x}{11+x} = \frac{2}{3} \therefore 21 + 3x = 22 + 2x$$

$$\therefore x = 1 \therefore \text{The number is } 1$$

4

[a]  $\because (a, 3)$  satisfies the relation

$$\therefore 3 = 4a - 5 \therefore 4a = 8 \therefore a = 2$$

[b] ①  $\because y \propto x \therefore y = mx$

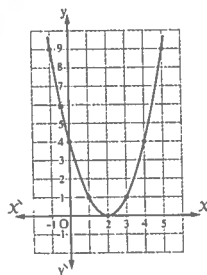
$$\therefore 6 = 3m \therefore m = 2 \therefore y = 2x$$

② At  $x = 5 \therefore y = 2 \times 5 = 10$

5

[a]  $f(x) = x^2 - 4x + 4$

$x$	-1	0	1	2	3	4	5
$f(x)$	9	4	1	0	1	4	9



From the graph :

- ① The vertex of the curve is :  $(2, 0)$   
 ② The equation of the axis of symmetry is :  $x = 2$

[b] Form the table by yourself  
 , then  $\bar{X} = 16, \sigma \approx 3.29$

20 Qena

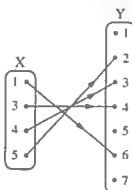
1

- ① a    ② c    ③ c    ④ b    ⑤ b    ⑥ d



$$[a] R = \{(1, 6), (3, 4), (4, 3), (5, 2)\}$$

R is a function because every element in X has only one image in Y  
 its range =  $\{2, 3, 4, 6\}$



[b]  $\therefore b$  is the middle proportional between  $a$  and  $c$

$$\therefore b^2 = ac$$

$$\therefore \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + ac}{ac + c^2} = \frac{a(a+c)}{c(a+c)} = \frac{a}{c}$$



$$[a] \begin{aligned} [1] f(\sqrt{2}) + 3g(\sqrt{2}) &= (\sqrt{2})^2 - 3\sqrt{2} + 3(\sqrt{2} - 3) \\ &= 2 - 3\sqrt{2} + 3\sqrt{2} - 9 = -7 \end{aligned}$$

$$[2] \therefore f(3) = 3^2 - 3 \times 3 = 0$$

$$g(3) = 3 - 3 = 0$$

$$\therefore f(3) = g(3)$$

[b] Let the number be  $X$

$$\therefore \frac{7+X}{11+X} = \frac{2}{3} \quad \therefore 21 + 3X = 22 + 2X$$

$$\therefore X = 1 \quad \therefore \text{The number is } 1$$



$$[a] \therefore \frac{a}{b} = \frac{3}{5} \quad \therefore a = 3m, b = 5m$$

$$\therefore \frac{7a+9b}{4a+2b} = \frac{21m+45m}{12m+10m} = \frac{66m}{22m} = 3$$

[b] Form the tables by yourself, then  $\sigma \approx 1.73$

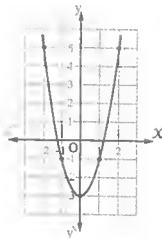


$$[a] \therefore y \propto x \quad \therefore \frac{y_1}{y_2} = \frac{x_1}{x_2}$$

$$\therefore \frac{40}{80} = \frac{14}{x_2} \quad \therefore x_2 = \frac{80 \times 14}{40} = 28$$

$$[b] f(x) = 2x^2 - 3$$

$x$	-2	-1	0	1	2
$f(x)$	5	-1	-3	-1	5



From the graph :

[1] The vertex of the curve is :  $(0, -3)$

[2] The equation of the axis of symmetry is :  $x = 0$

[3] The minimum value =  $-3$

21

Luxor



[1] d [2] c [3] b [4] a [5] b [6] d



[a] [1] The domain =  $\{1, 2, 3, 4, 5\}$

[2] The range =  $\{3, 5, 7, 9, 11\}$

$$[3] f(x) = 2x + 1$$

[b] Let the two numbers be  $2x, 3x$

$$\therefore \frac{2x-7}{3x-7} = \frac{1}{2} \quad \therefore 4x - 14 = 3x - 7$$

$$\therefore x = 7$$

$\therefore$  The two numbers are : 14, 21



[a] [1]  $\therefore R$  is a function from  $X$  to  $Y$

$\therefore$  Each element in  $X$  has only one image in  $Y$

$\therefore$  the image of  $-2 = (-2)^2 - 1 = 3 \in Y$

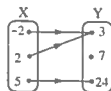
the image of  $2 = (2)^2 - 1 = 3 \in Y$

the image of  $5 = (5)^2 - 1 = 24 \in Y$

$$\therefore l = 24$$

$$[2] R = \{(-2, 3), (2, 3), (5, 24)\}$$

[3]



$$[b] \therefore y = a - 9, y \propto \frac{1}{x^2} \therefore y = \frac{m}{x^2}$$

$$\therefore \frac{m}{x^2} = a - 9$$

$$\therefore m = x^2(a - 9)$$

$$\therefore m = \left(\frac{2}{3}\right)^2 (18 - 9) = 4$$

$$\therefore y = \frac{4}{x^2}$$

$$\text{at } X = 1 \quad \therefore y = 4$$

**4**

[a] Let  $A(X, 0)$

$\therefore A(X, 0)$  belongs to the straight line of the function  $f$

$$\therefore 4 - 2X = 0 \quad \therefore -2X = -4$$

$$\therefore X = 2 \quad \therefore A(2, 0)$$

let  $B(0, y)$

$\therefore B(0, y)$  belongs to the straight line of the function  $f$

$$\therefore 4 - 2 \times 0 = y \quad \therefore y = 4$$

$$\therefore B(0, 4)$$

$\therefore$  the area of  $\triangle AOB = \frac{1}{2} \times 2 \times 4 = 4$  square unit.

[b]  $\therefore \frac{X}{7} = \frac{y}{3} \quad \therefore X = 7m, y = 3m$

$$\therefore \frac{2X - 3y}{X + 2y} = \frac{14m - 9m}{7m + 6m} = \frac{5m}{13m} = \frac{5}{13} \quad (1)$$

$$\therefore \frac{10}{26} = \frac{5}{13} \quad (2)$$

$$\text{From (1), (2): } \therefore \frac{2X - 3y}{X + 2y} = \frac{10}{26}$$

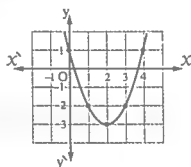
$\therefore (2X - 3y), (X + 2y), 10, 26$  are proportional.

**5**

[a] Form the table by yourself, then  $\sigma \approx 7.07$

[b]  $f(X) = 1 - 4X + X^2$

$X$	0	1	2	3	4
$f(X)$	1	-2	-3	-2	1



From the graph :

1 The vertex of the curve is :  $(2, -3)$

2 The equation of the axis of symmetry is :  $X = 2$

3 The minimum value is  $-3$

## 22 Aswan

**1**

1 c 2 d 3 a 4 c 5 b 6 c

**2**

[a] 1  $Y = \{2, 5, 7\}$

2  $Y \times X = \{(2, 2), (5, 2), (7, 2)\}$

[b]  $\therefore \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, a = cm^2$

$$\therefore \frac{a-b}{a-c} = \frac{cm^2 - cm}{cm^2 - c} = \frac{cm(m-1)}{c(m^2-1)} = \frac{cm(m-1)}{c(m-1)(m+1)} = \frac{m}{m+1} \quad (1)$$

$$\therefore \frac{b}{b+c} = \frac{cm}{cm+c} = \frac{cm}{c(m+1)} = \frac{m}{m+1} \quad (2)$$

From (1) and (2) :  $\therefore \frac{a-b}{a-c} = \frac{b}{b+c}$

**3**

[a] 1  $R = \{(2, 4), (3, 6), (5, 10)\}$

2 Yes,  $R$  is a function.

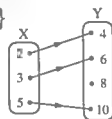
[b]  $\therefore y \propto \frac{1}{X} \quad \therefore XY = m$

$$\therefore 4 \times 2 = m$$

$$\therefore m = 8 \quad \therefore XY = 8$$

$$\text{at } X = 16 \quad \therefore 16Y = 8$$

$$\therefore Y = \frac{1}{2}$$


**4**

[a]  $\therefore (a, 3)$  satisfies the function.

$$\therefore 3 = 4a - 5 \quad \therefore 4a = 8 \quad \therefore a = 2$$

[b]  $\therefore \frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a-b+5c}{3X}$

Multiplying the two terms of the 1<sup>st</sup> ratio by 2 and the 2<sup>nd</sup> by -1 and the 3<sup>rd</sup> by 5 and adding the antecedents and consequents of the three ratios

$$\therefore \frac{2a-b+5c}{4-3+20} = \text{one of the given ratios}$$

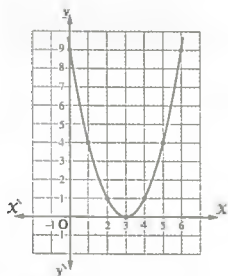
$$\therefore \frac{2a-b+5c}{21} = \frac{2a-b+5c}{3X}$$

$$\therefore 3X = 21 \quad \therefore X = 7$$

**5**

[a]  $f(X) = (X-3)^2$

$X$	0	1	2	3	4	5	6
$f(X)$	9	4	1	0	1	4	9



**From the graph :**

- \* The vertex of the curve is : (3 , 0)
- \* The minimum value = 0
- \* The equation of the axis of symmetry is :  $X = 3$

[b] Form the tables by yourself, then  $\bar{X} = 2$  ,  $\sigma \approx 0.96$

### 23 New Valley

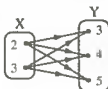
1

- 1 d    2 b    3 b    4 d    5 a    6 a

2

- [a] 1  $X \times Y = \{(2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5)\}$

2  $n(X \times Y) = 6$



[b]  $\therefore X^2 y^2 - 14 X y + 49 = 0$

$\therefore (Xy - 7)^2 = 0 \quad \therefore Xy - 7 = 0$

$\therefore Xy = 7 \quad \therefore y \propto \frac{1}{X}$

3

[a] Let the number be  $X$

$\therefore \frac{7+X^2}{11+X^2} = \frac{4}{5}$

$\therefore 35 + 5X^2 = 44 + 4X^2 \quad \therefore X^2 = 9$

$\therefore X = 3$  (refused) or  $X = -3$

$\therefore$  The number is  $-3$

[b]  $R = \{(4, 2), (8, 4)\}$

$R$  is not a function because  $2 \in X$  has no image in  $X$

4

[a] 1  $\therefore \frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a-5b+3c}{X}$

Multiplying the terms of the 1<sup>st</sup> ratio by 2 and the 2<sup>nd</sup> ratio by  $-5$  and the 3<sup>rd</sup> ratio by 3 and adding the antecedents and consequents of the three ratios

$\therefore \frac{2a-5b+3c}{4-15+12} = \text{one of the given ratios}$

$\therefore \frac{2a-5b+3c}{1} = \frac{2a-5b+3c}{X} \quad \therefore X = 1$

2 Adding the antecedents and consequents of the three ratios

$\therefore \frac{a+b+c}{2+3+4} = \text{one of the given ratios}$

$\therefore \frac{a+b+c}{9} = \frac{b}{3} \quad \therefore \frac{a+b+c}{b} = \frac{9}{3} = 3$

[b] 1  $\therefore f(k) = 5 \quad \therefore 5 = 2k - 3$

$\therefore 2k = 8 \quad \therefore k = 4$

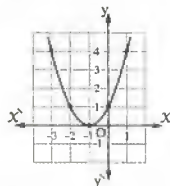
2  $\therefore (2, k) \in f \quad \therefore k = 2 \times 2 - 3 = 1$

5

[a] Form the tables by yourself, then  $\bar{X} = 7$  ,  $\sigma \approx 2$

[b]  $f(X) = (X+1)^2$

$X$	-3	-2	-1	0	1
$f(X)$	4	1	0	1	4



**From the graph :**

1 The vertex of the curve is :  $(-1, 0)$

2 The equation of the symmetry axis is :  $X = -1$

3 The minimum value = 0

### 24 South Sinai

1

- 1 a    2 b    3 c    4 d    5 a    6 d

2

$R = \{(2, 4), (3, 6), (4, 8)\}$

Yes,  $R$  is a function, its range =  $\{4, 6, 8\}$

3

[a] Let the number be  $X$

$$\therefore \frac{7+X}{11+X} = \frac{2}{3} \quad \therefore 21 + 3X = 22 + 2X$$

$$\therefore X = 1 \quad \therefore \text{The number is } 1$$

[b]  $\therefore y \propto X \quad \therefore y = mX$

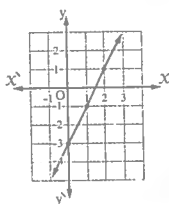
$$\therefore 14 = 42m \quad \therefore m = \frac{1}{3} \quad \therefore y = \frac{1}{3}X$$

$$\text{at } X = 60 \quad \therefore y = \frac{1}{3} \times 60 = 20$$

4

[a]  $f(X) = 2X - 3$

$X$	0	1	2
$f(X)$	-3	-1	1



[b]  $\therefore b$  is the middle proportional between  $a$  and  $c$

$$\therefore b^2 = ac$$

$$\therefore \frac{a+b^2}{b^2+c^2} = \frac{a^2+ac}{ac+c^2} = \frac{a(a+c)}{c(a+c)} = \frac{a}{c}$$

5

[a]  $\therefore (X^3, y+1) = (27, \sqrt[3]{125})$

$$\therefore X^3 = 27 \quad \therefore X = 3$$

$$\therefore y+1 = \sqrt[3]{125} \quad \therefore y+1 = 5 \quad \therefore y = 4$$

[b] Form the table by yourself, then  $\bar{X} = 20$

$$\sigma \approx 2.28$$

25 North Sinai

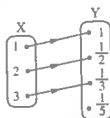
1

1 d 2 c 3 c 4 d 5 b 6 a

2

[a]  $R = \left\{ (1, 1), \left(2, \frac{1}{2}\right), \left(3, \frac{1}{3}\right) \right\}$

Yes,  $R$  is a function



[b] 1  $\therefore y \propto \frac{1}{X} \quad \therefore Xy = m$

$$\therefore 2 \times 3 = m \quad \therefore m = 6 \quad \therefore Xy = 6$$

2 At  $X = 1.5 \quad \therefore 1.5y = 6 \quad \therefore y = 4$

3

[a]  $\therefore (3, b)$  satisfies the function.

$$\therefore b = 5 \times 3 + 4 = 19$$

[b]  $\therefore \frac{X}{y} = \frac{3}{4} \quad \therefore X = 3m, y = 4m$

$$\therefore \frac{3X+y}{X+5y} = \frac{9m+4m}{3m+20m} = \frac{13m}{23m} = \frac{13}{23}$$

4

[a]  $X = \{1, 4, 5\}, Y = \{2\}$

$$\therefore Y^2 = \{(2, 2)\}$$

[b]  $\therefore b$  is the middle proportional between  $a$  and  $c$

$$\therefore b^2 = ac$$

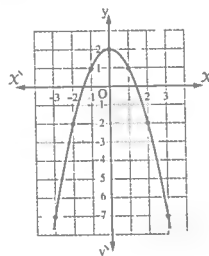
$$\therefore \frac{5c^2-2b^2}{5b^2-2a^2} = \frac{5c^2-2ac}{5ac-2a^2} = \frac{c(5c-2a)}{a(5c-2a)} = \frac{c}{a}$$

5

[a] Form the table by yourself, then  $\sigma \approx 3.29$

[b]  $f(X) = 2 - X^2$

$X$	-3	-2	-1	0	1	2	3
$f(X)$	-7	-2	1	2	1	-2	-7



From the graph :

- 1 The vertex of the curve is :  $(0, 2)$
- 2 The equation of the axis of symmetry is :  $X = 0$
- 3 The maximum value = 2

26 Red Sea

1

1 b 2 a 3 c 4 d 5 c 6 a

2

- [a] ①  $X = \{1\}$       ②  $n(Y) = 3$   
 ③  $Y \times X = \{(1, 1), (5, 1), (7, 1)\}$

[b]  $\therefore b$  is the middle proportional between  $a$  &  $c$

$$\therefore b^2 = ac$$

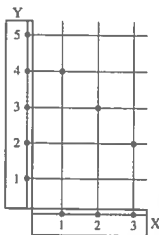
$$\therefore \frac{a^2 + b^2}{b^2 + c^2} = \frac{a^2 + ac}{ac + c^2} = \frac{a(a+c)}{c(a+c)} = \frac{a}{c}$$

3

- [a]  $\therefore f(2) = 15$        $\therefore 15 = 4 \times 2 + a$   
 $\therefore 15 = 8 + a$        $\therefore a = 7$

- [b] ①  $R = \{(1, 4), (2, 3), (3, 2)\}$

② Yes,  $R$  is a function.



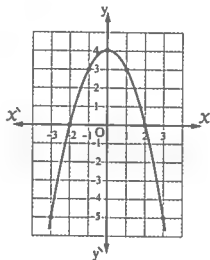
4

- [a]  $\therefore \frac{x}{y} = \frac{2}{3}$        $\therefore x = 2m$  ,  $y = 3m$   
 $\therefore \frac{3x+2y}{6y-x} = \frac{6m+6m}{18m-2m} = \frac{12m}{16m} = \frac{3}{4}$   
 [b] ①  $\therefore y \propto x$        $\therefore y = mx$        $\therefore 2 = 6m$   
 $\therefore m = \frac{1}{3}$        $\therefore y = \frac{1}{3}x$   
 ② At  $x = 15$        $\therefore y = \frac{1}{3} \times 15 = 5$

5

[a]  $f(x) = 4 - x^2$

$x$	-3	-2	-1	0	1	2	3
$f(x)$	-5	0	3	4	3	0	-5



From the graph :

- ① The vertex of the curve is :  $(0, 4)$   
 ② The equation of the axis of symmetry is :  $x = 0$

[b] Form the table by yourself, then  $\sigma \approx 4.98$

## 27 Matrouh

1

- ① a      ② b      ③ d      ④ a      ⑤ d      ⑥ a

2

- [a]  $R = \{(1, 3), (2, 6), (3, 9)\}$

$R$  is a function because every element in  $X$  has only one image in  $Y$

, its range =  $\{3, 6, 9\}$

- [b]  $\therefore \frac{a}{b} = \frac{2}{5}$        $\therefore a = 2m$  ,  $b = 5m$   
 $\therefore \frac{2a-2b}{3a+2b} = \frac{4m-10m}{6m+10m} = \frac{-6m}{16m} = \frac{-3}{8}$

3

- [a] ①  $X = \{1\}$  ,  $Y = \{1, 3, 5\}$   
 ②  $Y^2 = \{(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)\}$

- [b]  $\therefore \frac{x}{2a+b} = \frac{y}{2b-c} = \frac{z}{2c-a}$

Multiplying the two terms of the 1<sup>st</sup> ratio by 2 and adding the antecedents and consequents of the 1<sup>st</sup> and the 2<sup>nd</sup> ratios

$$\therefore \frac{2x+y}{4a+2b+2b-c} = \frac{2x+y}{4a+4b-c}$$

= one of the given ratios. (1)

Multiplying the terms of the 1<sup>st</sup> ratio by 2 and the 2<sup>nd</sup> by 2 and adding the antecedents and consequents of the three ratios

$$\therefore \frac{2x+2y+z}{4a+2b+4b-2c+2c-a} = \frac{2x+2y+z}{3a+6b}$$

= one of the given ratios (2)

From (1) and (2) :

$$\therefore \frac{2x+y}{4a+4b-c} = \frac{2x+2y+z}{3a+6b}$$

4

- [a]  $\therefore (a, 3)$  satisfies the function

$$\therefore 3 = 4a - 5 \quad \therefore 4a = 8 \quad \therefore a = 2$$

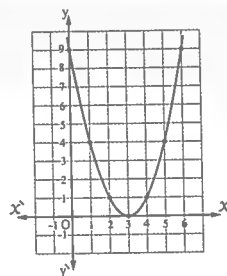
[b] Form the tables by yourself, then  $\bar{X} = 2$   
 $\sigma \approx 0.96$

5

[a]  $\because y \propto \frac{1}{x} \quad \therefore xy = m$   
 $\therefore 3 \times 10 = m \quad \therefore m = 30 \quad \therefore xy = 30$   
 at  $x = 5 \quad \therefore 5y = 30 \quad \therefore y = 6$

[b]  $f(x) = (x-3)^2$

$x$	0	1	2	3	4	5	6
$f(x)$	9	4	1	0	1	4	9



From the graph :

- \* The vertex of the curve is : (3, 0)
- \* The minimum value = 0

Answers of quizzes of  
trigonometry & geometry

Quiz 1

1

1 b

2 a

3 a

2

Prove by yourself

Quiz 2

1

1 c

2 b

3 c

2

[a]  $30^\circ$

[b]  $\frac{7}{2}$

Quiz 3

1

1 d

2 b

3 d

2

[a] 314 square units

[b] 0

Quiz 4

1

1 b

2 c

3 b

2

[a] Prove by yourself

→ the area of  $\triangle ABC = 10$  square units.

[b]  $30^\circ$

Quiz 5

1

1 a

2 c

3 b

2

[a] Prove by yourself

[b] Prove by yourself

→ the area = 8 square units.

Quiz 6

1

1 a

2 b

3 c

2

1 Prove by yourself

2 Prove by yourself

3  $y = \frac{4}{3}x - \frac{4}{3}$

## Answers of model examinations of the school book of trigonometry &amp; geometry

## Model 1

1

1 a

2 c

3 b

4 a

5 b

6 a

2

$$[a] \therefore \sin 60^\circ = \frac{\sqrt{3}}{2} \quad (1)$$

$$2 \sin 30^\circ \cos 30^\circ = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} \quad (2)$$

$$\text{From (1), (2)}: \therefore \sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ$$

$$[b] \therefore \text{The slope of } \overline{AB} = \frac{5+1}{6+3} = \frac{2}{3}$$

$$\therefore \text{the slope of } \overline{BC} = \frac{3-5}{3-6} = \frac{2}{3}$$

$$\therefore \text{The slope of } \overline{AB} = \text{the slope of } \overline{BC}$$

$$\therefore \overline{AB} \parallel \overline{BC}$$

$\therefore$  B is a common point between the two straight lines.

$\therefore$  The points A, B and C are collinear.

3

$$[a] \therefore 4 \cos 60^\circ \sin 30^\circ = 4 \times \frac{1}{2} \times \frac{1}{2} = 1$$

$$\therefore \tan X = 1 \quad \therefore X = 45^\circ$$

[b] Let B (X, y)

$$\therefore (6, -4) = \left( \frac{X+5}{2}, \frac{y-3}{2} \right)$$

$$\therefore \frac{X+5}{2} = 6 \quad \therefore X+5 = 12 \quad \therefore X = 7$$

$$\therefore \frac{y-3}{2} = -4 \quad \therefore y-3 = -8 \quad \therefore y = -5$$

$$\therefore B(7, -5)$$

4

$$[a] \therefore m_1 = \frac{k-1}{2-3} = 1-k$$

$$\therefore m_2 = \tan 45^\circ = 1$$

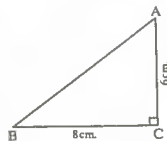
$$\therefore L_1 \parallel L_2 \quad \therefore m_1 = m_2$$

$$\therefore 1-k = 1 \quad \therefore k = 0$$

$$[b] \therefore m(\angle C) = 90^\circ$$

$$\therefore (AB)^2 = (6)^2 + (8)^2 \\ = 100$$

$$\therefore AB = 10 \text{ cm.}$$



$$[1] \cos A \cos B - \sin A \sin B$$

$$= \frac{6}{10} \times \frac{8}{10} - \frac{8}{10} \times \frac{6}{10} = 0$$

$$[2] \therefore \cos B = \frac{8}{10} \quad \therefore m(\angle B) \approx 36^\circ 52' 12''$$

5

[a]  $\therefore$  The slope of the straight line = 2

$\therefore$  The equation of the straight line is:

$$y = 2x + c$$

$\therefore (1, 0)$  satisfies the equation.

$$\therefore 0 = 2 \times 1 + c \quad \therefore c = -2$$

$\therefore$  The equation of the straight line is:  $y = 2x - 2$

$$[b] \therefore MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9} = \sqrt{25} \\ = 5 \text{ length units}$$

$$MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16} \\ = \sqrt{25} = 5 \text{ length units}$$

$$MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16} \\ = \sqrt{25} = 5 \text{ length units}$$

$$\therefore MA = MB = MC$$

$\therefore$  A, B and C are located on the circle M

$$\therefore \text{the circumference} = 2\pi r = 2 \times \pi \times 5 \\ = 10\pi \text{ length units}$$

## Model 2

1

1 a

2 d

3 b

4 c

5 b

6 b

2

$$[a] \therefore \cos E \tan 30^\circ = \cos^2 45^\circ$$

$$\therefore \cos E \times \frac{1}{\sqrt{3}} = \left( \frac{1}{\sqrt{2}} \right)^2$$

$$\therefore \cos E = \frac{\sqrt{3}}{2} \quad \therefore m(\angle E) = 30^\circ$$

$$\begin{aligned}
 [b] \therefore AB &= \sqrt{(3-1)^2 + (3-5)^2} = \sqrt{4+4} \\
 &= 2\sqrt{2} \text{ length units} \\
 BC &= \sqrt{(1-1)^2 + (5-3)^2} = \sqrt{4} = 2 \text{ length units} \\
 AC &= \sqrt{(3-1)^2 + (3-3)^2} = \sqrt{4} = 2 \text{ length units} \\
 \therefore BC &= AC \qquad \therefore \Delta ABC \text{ is isosceles.}
 \end{aligned}$$

13

$$\begin{aligned}
 [a] \therefore \text{The slope of the straight line} &= \frac{-3-3}{-1-1} = 3 \\
 \therefore \text{The equation of the straight line is: } &y = 3x + c \\
 \therefore (1, 3) \text{ satisfies the equation} & \\
 \therefore 3 &= 3 \times 1 + c \qquad \therefore c = 0 \\
 \therefore \text{The equation of the straight line is: } &y = 3x \\
 \therefore c &= 0 \\
 \therefore \text{The straight line passes through the} &\text{origin point.}
 \end{aligned}$$

$$\begin{aligned}
 [b] \therefore (3, 1) &= \left( \frac{1+x}{2}, \frac{y+3}{2} \right) \\
 \therefore \frac{1+x}{2} &= 3 \qquad \therefore 1+x = 6 \\
 \therefore x &= 5 \qquad \therefore \frac{y+3}{2} = 1 \\
 \therefore y+3 &= 2 \qquad \therefore y = -1 \\
 \therefore (x, y) &= (5, -1)
 \end{aligned}$$

4

$$\begin{aligned}
 [a] \therefore \text{The straight line passes through the two points} & \\
 (1, 0) \text{ and } (0, 4) & \\
 \therefore \text{The slope} &= \frac{4-0}{0-1} = -4 \\
 \therefore \text{The equation of the straight line is:} & \\
 y &= -4x + c \\
 \therefore \text{the intercepted part from y-axis} &= 4 \\
 \therefore \text{The equation of the straight line} & \\
 \text{is: } y &= -4x + 4
 \end{aligned}$$

$$\begin{aligned}
 [b] \therefore m(\angle B) &= 90^\circ \\
 \therefore (AB)^2 &= (10)^2 - (8)^2 = 36 \\
 \therefore AB &= 6 \text{ cm.} \\
 \therefore \sin^2 A + 1 &= \left( \frac{8}{10} \right)^2 + 1 = \frac{41}{25} \quad (1) \\
 \therefore 2 \cos^2 C + \cos^2 A &= 2 \times \left( \frac{8}{10} \right)^2 + \left( \frac{6}{10} \right)^2 = \frac{41}{25} \quad (2) \\
 \text{From (1), (2):} & \\
 \therefore \sin^2 A + 1 &= 2 \cos^2 C + \cos^2 A
 \end{aligned}$$

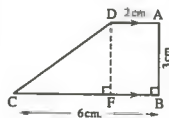


6

$$\begin{aligned}
 [a] \therefore m_1 &= \frac{4-3}{2+1} = \frac{1}{3} \\
 m_2 &= \frac{1}{3} \qquad \therefore m_1 = m_2 \qquad \therefore L_1 \parallel L_2
 \end{aligned}$$

$$\begin{aligned}
 [b] \text{ Draw } \overline{DF} \perp \overline{BC} \\
 \therefore \overline{AD} \parallel \overline{BC}, \overline{AB} \perp \overline{BC} \\
 \therefore \overline{DF} \perp \overline{BC}
 \end{aligned}$$

$$\begin{aligned}
 \therefore ABFD \text{ is a rectangle} \\
 \therefore BF &= AD = 2 \text{ cm.} \\
 \therefore AB &= DF = 3 \text{ cm.} \\
 \therefore FC &= 6 - 2 = 4 \text{ cm.}
 \end{aligned}$$


 From  $\Delta DFC$  which is right-angled at F

$$\begin{aligned}
 \therefore (DC)^2 &= (3)^2 + (4)^2 = 25 \\
 \therefore DC &= 5 \text{ cm.} \\
 \therefore \cos(\angle BCD) &= \frac{4}{5}
 \end{aligned}$$

## Answers of model for the merge students

1

$$\begin{array}{lll}
 1 \checkmark & 2 \checkmark & 3 \times \\
 4 \times & 5 \times & 6 \checkmark
 \end{array}$$

2

$$\begin{array}{lll}
 1 b & 2 c & 3 d \\
 4 c & 5 a & 6 c
 \end{array}$$

3

$$\begin{array}{lll}
 1 0 & 2 1 & 3 10 \\
 4 2 & 5 -3 & 6 \frac{\sqrt{3}}{2}
 \end{array}$$

4

$$\begin{array}{lll}
 1 \frac{1}{2} & 2 \frac{3}{5} & 3 3 \\
 4 2 & 5 5 \text{ length units} & 6 (-5, 2)
 \end{array}$$

Answers of governorates' examinations  
of trigonometry & geometry

1 Cairo

1

- 1 d 2 a 3 c 4 b 5 d 6 c

2

[a]  $\therefore X \sin 45^\circ \cos 45^\circ = \sin 30^\circ$

$\therefore X \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2} \therefore \frac{1}{2} X = \frac{1}{2}$

$\therefore X = 1$

[b]  $\therefore$  The slope of the straight line = 2

$\therefore$  Its equation is :  $y = 2X + c$

$\therefore (1, 0)$  satisfies the equation.

$\therefore 0 = 2 \times 1 + c \therefore c = -2$

$\therefore$  The equation is :  $y = 2X - 2$

3

[a]  $\therefore m(\angle Y) = 90^\circ$

$\therefore (XZ)^2 = (6)^2 + (8)^2 = 100$

$\therefore XZ = 10$  cm.

$\therefore \cos X \cos Z - \sin X \sin Z$   
 $= \frac{6}{10} \times \frac{8}{10} - \frac{8}{10} \times \frac{6}{10} = 0$

[b]  $\therefore AB = \sqrt{(-3-2)^2 + (0-4)^2} = \sqrt{25+16}$   
 $= \sqrt{41}$  length units

$\therefore BC = \sqrt{(-7+3)^2 + (5-0)^2} = \sqrt{16+25}$   
 $= \sqrt{41}$  length units

$\therefore CD = \sqrt{(-2+7)^2 + (9-5)^2} = \sqrt{25+16}$   
 $= \sqrt{41}$  length units

$\therefore AD = \sqrt{(-2-2)^2 + (9-4)^2} = \sqrt{16+25}$   
 $= \sqrt{41}$  length units

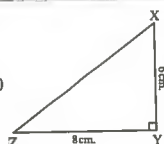
$\therefore AB = BC = CD = AD \therefore ABCD$  is a rhombus

$\therefore AC = \sqrt{(-7-2)^2 + (5-4)^2} = \sqrt{81+1}$   
 $= \sqrt{82}$  length units

$\therefore BD = \sqrt{(-2+3)^2 + (9-0)^2} = \sqrt{1+81}$   
 $= \sqrt{82}$  length units

$\therefore AC = BD$

$\therefore ABCD$  is a square.



4

[a] 1 In  $\triangle ABC$  :

$\therefore m(\angle B) = 90^\circ$

$\therefore (BC)^2 = (25)^2 - (15)^2 = 400$

$\therefore BC = 20$  cm.

2  $\therefore \sin(\angle ACB) = \frac{15}{25}$

$\therefore m(\angle ACB) \approx 36^\circ 52' 12''$

3 The area =  $20 \times 15 = 300$  cm<sup>2</sup>

[b] Let  $B(X, y)$

$\therefore (6, -4) = \left(\frac{5+X}{2}, \frac{-3+y}{2}\right)$

$\therefore \frac{5+X}{2} = 6 \therefore 5+X = 12 \therefore X = 7$

$\therefore \frac{-3+y}{2} = -4 \therefore -3+y = -8 \therefore y = -5$

$\therefore B(7, -5)$

5

[a]  $\therefore$  The two straight lines are parallel

$\therefore m_1 = m_2 \therefore \frac{-a}{2} = \tan 45^\circ$

$\therefore \frac{-a}{2} = 1 \therefore a = -2$

[b]  $\therefore$  The slope of the straight line =  $\frac{-1-2}{-2-4} = \frac{1}{2}$

$\therefore$  Its equation is :  $y = \frac{1}{2}X + c$

$\therefore (4, 2)$  satisfies the equation.

$\therefore 2 = \frac{1}{2} \times 4 + c \therefore c = 0$

$\therefore$  The equation is :  $y = \frac{1}{2}X$

$\therefore c = 0$

$\therefore$  The straight line passes through the origin point.

2 Giza

1

- 1 d 2 d 3 a 4 b 5 c 6 c

2

[a]  $\therefore$  The slope = 2

$\therefore$  The equation is :  $y = 2X + c$

$\therefore (1, -1)$  satisfies the equation.

$\therefore -1 = 2 \times 1 + c \therefore c = -3$

$\therefore$  The equation is :  $y = 2X - 3$

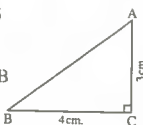
[b] 1  $\therefore m(\angle C) = 90^\circ$

$\therefore (AB)^2 = (3)^2 + (4)^2 = 25$

$\therefore AB = 5 \text{ cm.}$

$\therefore \cos A \cos B - \sin A \sin B$

$= \frac{3}{5} \times \frac{4}{5} - \frac{4}{5} \times \frac{3}{5} = 0$



2  $\therefore \tan B = \frac{3}{4} \quad \therefore m(\angle B) \approx 36^\circ 52' 12''$

3

[a]  $\therefore \sin 60^\circ = \frac{\sqrt{3}}{2}$  (1)

$\therefore 2 \sin 30^\circ \cos 30^\circ = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$  (2)

From (1), (2)  $\therefore \sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ$

[b]  $\therefore L_1 \perp L_2 \quad \therefore m_1 \times m_2 = -1$

$\therefore \frac{k-1}{2-3} \times \tan 45^\circ = -1$

$\therefore (1-k) \times 1 = -1 \quad \therefore k = 2$

4

[a]  $\therefore \cos E \tan 30^\circ = \cos^2 45^\circ$

$\therefore \cos E \times \frac{1}{\sqrt{3}} = \left(\frac{1}{\sqrt{2}}\right)^2$

$\therefore \cos E = \frac{\sqrt{3}}{2} \quad \therefore m(\angle E) = 30^\circ$

[b]  $\therefore AB = \sqrt{(1-3)^2 + (5-3)^2} = \sqrt{4+4}$   
 $= 2\sqrt{2} \text{ length units}$

$\therefore BC = \sqrt{(1-1)^2 + (3-5)^2} = \sqrt{0+4}$   
 $= 2 \text{ length units}$

$\therefore AC = \sqrt{(1-3)^2 + (3-3)^2} = \sqrt{4+0}$   
 $= 2 \text{ length units}$

$\therefore BC = AC$

$\therefore \triangle ABC \text{ is isosceles.}$

5

[a]  $m = \frac{-5}{4}$

The intercepted part is  $\frac{5}{2}$  from the negative part of the y-axis.

[b]  $\therefore MA = \sqrt{(3+1)^2 + (-1-2)^2} = \sqrt{16+9}$   
 $= 5 \text{ length units}$

$\therefore MB = \sqrt{(-4+1)^2 + (6-2)^2} = \sqrt{9+16}$   
 $= 5 \text{ length units}$

$\therefore MC = \sqrt{(2+1)^2 + (-2-2)^2} = \sqrt{9+16}$   
 $= 5 \text{ length units}$

$\therefore MA = MB = MC$

$\therefore A, B, C \text{ belong to the circle } M$

$\therefore \text{the area} = 3.14 \times 5^2 = 78.5 \text{ square units.}$

### 3 Alexandria

1

1 b

2 c

3 a

4 d

5 d

6 a

2

[a]  $\therefore X \sin 30^\circ \cos^2 45^\circ = \sin^3 60^\circ$

$\therefore X \times \frac{1}{2} \times \left(\frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^3$

$\therefore \frac{1}{4} X = \frac{3}{4} \quad \therefore X = 3$

[b]  $\therefore$  The two diagonals of the parallelogram bisect each other

Let M be the intersection point of the diagonals

$\therefore M = \left(\frac{3+0}{2}, \frac{2-3}{2}\right) = \left(\frac{3}{2}, -\frac{1}{2}\right)$

Let D(x, y)

$\therefore \left(\frac{3}{2}, -\frac{1}{2}\right) = \left(\frac{4+x}{2}, \frac{-5+y}{2}\right)$

$\therefore \frac{4+x}{2} = \frac{3}{2} \quad \therefore 4+x=3 \quad \therefore x=-1$

$\therefore \frac{-5+y}{2} = -\frac{1}{2} \quad \therefore -5+y=-1 \quad \therefore y=4$

$\therefore D(-1, 4)$

3

[a]  $\therefore MA = \sqrt{(3+1)^2 + (-1-2)^2} = \sqrt{16+9}$   
 $= 5 \text{ length units}$

$\therefore MB = \sqrt{(-4+1)^2 + (6-2)^2} = \sqrt{9+16}$   
 $= 5 \text{ length units}$

$\therefore MC = \sqrt{(2+1)^2 + (-2-2)^2} = \sqrt{9+16}$   
 $= 5 \text{ length units}$

$\therefore MA = MB = MC$

$\therefore A, B, C \text{ are located on the circle } M$

$\therefore \text{the circumference} = 2 \times 3.14 \times 5$   
 $= 31.4 \text{ length units.}$

- [b]  $\therefore$  The slope of the given straight line =  $-\frac{1}{2}$   
 $\therefore$  The slope of the required straight line = 2  
 $\therefore$  Its equation is :  $y = 2X + c$   
 $\therefore$  it intercepts a part of 7 units from the positive part of the y-axis  
 $\therefore$  Its equation is :  $y = 2X + 7$

4

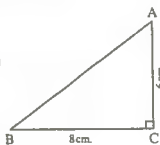
- [a]  $\therefore m_1 = \frac{5+7}{4+3} = 1$  ,  $m_2 = \tan 45^\circ = 1$   
 $\therefore m_1 = m_2$   
 $\therefore$  The two straight lines are parallel.

- [b]  $\therefore m(\angle C) = 90^\circ$

$$\therefore (AB)^2 = (6)^2 + (8)^2 = 100$$

$$\therefore AB = 10 \text{ cm.}$$

$$\therefore \cos A \cos B - \sin A \sin B = \frac{6}{10} \times \frac{8}{10} - \frac{8}{10} \times \frac{6}{10} = 0$$



5

- [a] Let D be the midpoint of  $\overline{BC}$

$$\therefore D = \left( \frac{3+1}{2}, \frac{7-3}{2} \right) = (2, 2)$$

$$\therefore \text{The slope of } \overline{AD} = \frac{2+6}{2-4} = -4$$

$$\therefore \text{Its equation is : } y = -4X + c$$

$$\therefore (4, -6) \text{ satisfies the equation.}$$

$$\therefore -6 = -4 \times 4 + c \quad \therefore c = 10$$

$$\therefore \text{The equation is : } y = -4X + 10$$

- [b] 1 In  $\triangle ABC$  :  $\therefore m(\angle B) = 90^\circ$

$$\therefore \sin(\angle ACB) = \frac{15}{25}$$

$$\therefore m(\angle ACB) \approx 36^\circ 52' 12''$$

$$2 \therefore (BC)^2 = (25)^2 - (15)^2 = 400$$

$$\therefore BC = 20 \text{ cm.}$$

$$\therefore \text{The area} = 20 \times 15 = 300 \text{ cm}^2$$

### 4 El-Kalyoubia

1

- 1 d    2 b    3 c    4 a    5 c    6 c

2

$$[a] \therefore \sqrt{(x-6)^2 + (5-1)^2} = 2\sqrt{5} \text{ (Squaring both sides)}$$

$$\therefore (x-6)^2 + 16 = 20$$

$$\therefore x^2 - 12x + 36 + 16 - 20 = 0$$

$$\therefore x^2 - 12x + 32 = 0$$

$$\therefore (x-8)(x-4) = 0$$

$$\therefore x = 8 \text{ or } x = 4$$

$$[b] \sin 45^\circ \cos 45^\circ + \sin 30^\circ \cos 60^\circ - \cos^2 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{2} - \left( \frac{\sqrt{3}}{2} \right)^2$$

$$= \frac{1}{2} + \frac{1}{4} - \frac{3}{4} = 0$$

3

- [a]  $\therefore$  The two diagonals of the parallelogram bisect each other

Let M be the intersection point of the diagonals

$$\therefore M = \left( \frac{3+0}{2}, \frac{2-3}{2} \right) = \left( \frac{3}{2}, -\frac{1}{2} \right)$$

Let D (X, y)

$$\therefore \left( \frac{3}{2}, -\frac{1}{2} \right) = \left( \frac{4+X}{2}, \frac{-5+y}{2} \right)$$

$$\therefore \frac{4+X}{2} = \frac{3}{2} \quad \therefore 4+X = 3 \quad \therefore X = -1$$

$$\therefore \frac{-5+y}{2} = \frac{-1}{2} \quad \therefore -5+y = -1 \quad \therefore y = 4$$

$$\therefore D(-1, 4)$$

- [b]  $\therefore m(\angle B) = 90^\circ$

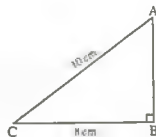
$$\therefore (AB)^2 = (10)^2 - (8)^2 = 36$$

$$\therefore AB = 6 \text{ cm.}$$

$$\therefore \sin^2 A + 1 = \left( \frac{8}{10} \right)^2 + 1 = \frac{41}{25} \quad (1)$$

$$\therefore 2 \cos^2 C + \cos^2 A = 2 \times \left( \frac{8}{10} \right)^2 + \left( \frac{6}{10} \right)^2 = \frac{41}{25} \quad (2)$$

$$\text{From (1), (2) : } \therefore \sin^2 A + 1 = 2 \cos^2 C + \cos^2 A$$



4

$$[a] \therefore L_1 \parallel L_2 \quad \therefore m_1 = m_2$$

$$\therefore \frac{k-1}{2-3} = \tan 45^\circ$$

$$\therefore -k+1 = 1$$

$$\therefore k = 0$$

[b]  $\therefore$  The slope of the given straight line =  $-\frac{1}{3}$

$\therefore$  The slope of the required straight line = 3

$\therefore$  Its equation is :  $y = 3x + c$

$\therefore (1, 2)$  satisfies the equation.

$\therefore 2 = 3 \times 1 + c \quad \therefore c = -1$

$\therefore$  The equation is :  $y = 3x - 1$

5

[a] [1] In  $\triangle ABC$  :  $\therefore m(\angle B) = 90^\circ$

$\therefore \sin(\angle ACB) = \frac{15}{25}$

$\therefore m(\angle ACB) = 36^\circ 52' 12''$

[2]  $\therefore (BC)^2 = (25)^2 - (15)^2 = 400$

$\therefore BC = 20$  cm.

$\therefore$  The area =  $20 \times 15 = 300$  cm<sup>2</sup>.

[b]  $\therefore$  The straight line passes through the two points  
(4, 0), (0, 9)

$\therefore$  The slope of the straight line =  $\frac{9-0}{0-4} = -\frac{9}{4}$

and the intercepted part = 9 units from the positive part of y-axis

$\therefore$  The equation of the straight line is :

$y = -\frac{9}{4}x + 9$

5

El-Sharkia

1

[1] b [2] b [3] d [4] a [5] c [6] c

2

[a]  $\therefore \frac{\sin 30^\circ \sin 60^\circ}{\sin 45^\circ \cos 45^\circ} = \frac{\frac{1}{2} \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}} = \frac{\sqrt{3}}{2}$  (1)

$\therefore \cos 30^\circ = \frac{\sqrt{3}}{2}$  (2)

From (1), (2) :  $\therefore \frac{\sin 30^\circ \sin 60^\circ}{\sin 45^\circ \cos 45^\circ} = \cos 30^\circ$

[b]  $\therefore MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9}$

= 5 length units

$\therefore MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16}$

= 5 length units

and  $MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16}$

= 5 length units

$\therefore MA = MB = MC$

$\therefore A, B$  and  $C$  lie on the circle  $M$

$\therefore$  the circumference =  $2 \times 3.14 \times 5$

= 31.4 length units.

3

[a] The slope of  $\overline{BC} = \frac{3+7}{1-3} = -5$

$\therefore$  The slope of the required straight line = -5

$\therefore$  Its equation is :  $y = -5x + c$

$\therefore A(5, 1)$  satisfies the equation.

$\therefore 1 = -5 \times 5 + c \quad \therefore c = 26$

$\therefore$  The equation is :  $y = -5x + 26$

[b] Draw  $\overline{AD} \perp \overline{BC}$

[1]  $\therefore \overline{AD} \perp \overline{BC} \therefore AC = AB$

$\therefore BD = CD = 6$  cm.

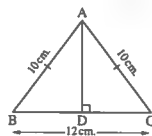
In  $\triangle ADB$  :

$\therefore m(\angle ADB) = 90^\circ$

$\therefore (AD)^2 = (10)^2 - (6)^2 = 64$

$\therefore AD = 8$  cm.  $\therefore \sin B = \frac{8}{10}$

[2] The area of  $\triangle ABC = \frac{1}{2} \times 12 \times 8 = 48$  cm<sup>2</sup>.



4

[a] [1]  $\therefore$  The midpoint of  $\overline{AC} = \left(\frac{3+5}{2}, \frac{3-1}{2}\right)$   
= (4, 1)

$\therefore$  The point of intersection of the two diagonals is : (4, 1)

[2] Let  $D(x, y)$

$\therefore (4, 1) = \left(\frac{2+x}{2}, \frac{-2+y}{2}\right)$

$\therefore \frac{2+x}{2} = 4 \quad \therefore x = 6$

$\therefore \frac{-2+y}{2} = 1 \quad \therefore y = 4 \quad \therefore D = (6, 4)$

[b]  $\therefore$  The slope of the straight line  $= \frac{3-5}{0-4} = \frac{1}{2}$

$\therefore$  Its equation is :  $y = \frac{1}{2}x + c$

$\therefore (0, 3)$  satisfies the equation.

$\therefore 3 = \frac{1}{2} \times 0 + c \quad \therefore c = 3$

$\therefore$  The equation is :  $y = \frac{1}{2}x + 3$

at  $y = 0 \quad \therefore 0 = \frac{1}{2}x + 3 \quad \therefore x = -6$

$\therefore$  The intersection point of the straight line with the  $x$ -axis is :  $(-6, 0)$

5

[a] 1  $\therefore \cos X = \sin 30^\circ \cos 60^\circ$

$\therefore \cos X = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$\therefore X = 75^\circ 31' 21''$

2  $\tan 75^\circ 31' 21'' \approx 3.873$

[b]  $\therefore$  The slope of the given straight line  $= \frac{-3}{2}$

$\therefore$  The slope of the required straight line  $= \frac{2}{3}$

$\therefore$  the required straight line cuts 3 units of the positive part of  $y$ -axis

$\therefore$  Its equation is :  $y = \frac{2}{3}x + 3$

## El-Monofia

1

1 a    2 d    3 d    4 b    5 b    6 c

2

[a]  $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ - \tan^2 45^\circ$

$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - (1)^2$

$= \frac{1}{4} + \frac{3}{4} - 1 = 0$

[b] 1  $\therefore AB = \sqrt{(5-7)^2 + (1+3)^2} = \sqrt{4+16}$

$= 2\sqrt{5}$  length units

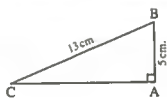
$\therefore$  The area  $= 3.14 \times (\sqrt{5})^2 = 15.7 \text{ cm}^2$

2  $M = \left(\frac{7+5}{2}, \frac{-3+1}{2}\right) = (6, -1)$

3

[a]  $\therefore m(\angle A) = 90^\circ$

$\therefore (AC)^2 = (13)^2 - (5)^2$   
 $= 144$



$\therefore AC = 12 \text{ cm.}$

$\therefore \sin C \cos B + \cos C \sin B$

$= \frac{5}{13} \times \frac{5}{13} + \frac{12}{13} \times \frac{12}{13} = 1$

[b]  $\therefore$  The slope of the given straight line  $= \frac{1-0}{2-5} = \frac{-1}{3}$

$\therefore$  The slope of the required straight line  $= 3$

$\therefore$  Its equation is :  $y = 3x + c$

$\therefore (1, 3)$  satisfies the equation.

$\therefore 3 = 3 \times 1 + c \quad \therefore c = 0$

$\therefore$  The equation is :  $y = 3x$

a

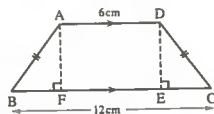
[a] Draw  $\overline{AF} \perp \overline{BC}$

$\therefore \overline{DE} \perp \overline{BC}$

$\therefore \overline{AD} \parallel \overline{BC}$

$\therefore \overline{AF} \perp \overline{BC}$

$\therefore \overline{DE} \perp \overline{BC}$



$\therefore ADEF$  is a rectangle  $\therefore EF = AD = 6 \text{ cm.}$

$\therefore BF + EC = 6 \text{ cm.}$

$\therefore BF = EC = 3 \text{ cm. } (\Delta ABF \cong \Delta DCE)$

$\therefore$  The area of the trapezium  $= \frac{1}{2} (AD + BC) \times AF$

$\therefore 36 = \frac{1}{2} (6 + 12) \times AF$

$\therefore AF = 4 \text{ cm.}$

$\therefore DE = AF = 4 \text{ cm.}$

In  $\Delta ABF$  :  $\therefore m(\angle AFB) = 90^\circ$

$\therefore (AB)^2 = (3)^2 + (4)^2 = 25 \quad \therefore AB = 5 \text{ cm.}$

$\therefore DC = AB = 5 \text{ cm.}$

$\therefore \sin B + \cos C = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}$

[b]  $\therefore AB = \sqrt{(5+1)^2 + (1-3)^2} = \sqrt{36+4}$

$= \sqrt{40}$  length units

$\therefore BC = \sqrt{(6-5)^2 + (4-1)^2} = \sqrt{1+9}$

$= \sqrt{10}$  length units

$\therefore AC = \sqrt{(6+1)^2 + (4-3)^2} = \sqrt{49+1}$

$= \sqrt{50}$  length units

$\therefore (AC)^2 = 50$

$\therefore (AB)^2 + (BC)^2 = 40 + 10 = 50$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore \Delta ABC$  is a right-angled triangle at B

6

[a] The slope =  $-\frac{4}{5}$  and the intercepted part = 2 units from the positive part of the y-axis.

[b] [1]  $\therefore$  The slope of  $\overline{CD} = \frac{6-2}{-3-3} = \frac{-2}{3}$

$\therefore$  The equation of  $\overline{CD}$  is :  $y = \frac{-2}{3}x + c$

$\therefore A(3, 2)$  satisfies the equation.

$\therefore 2 = \frac{-2}{3} \times 3 + c \quad \therefore c = 4$

$\therefore$  The equation is :  $y = \frac{-2}{3}x + 4$

[2] At  $X = 0 \quad \therefore y = \frac{-2}{3} \times 0 + 4 \quad \therefore y = 4$

$\therefore OD = 4$  units

$\therefore$  at  $y = 0 \quad \therefore 0 = \frac{-2}{3}x + 4 \quad \therefore x = 6$

$\therefore OC = 6$  units

$\therefore$  The area of  $\Delta DOC = \frac{1}{2} \times 4 \times 6$   
 $= 12$  square units.

## 7 El-Gharbia

1

- [1] c [2] c [3] b [4] b [5] a [6] d

2

[a]  $\therefore \tan X = 4 \cos 60^\circ \sin 30^\circ$

$\therefore \tan X = 4 \times \frac{1}{2} \times \frac{1}{2} = 1 \quad \therefore X = 45^\circ$

[b] [1]  $\therefore \overline{XY} \perp \overline{YZ}$

$\therefore$  The slope of  $\overline{XY} \times$  the slope of  $\overline{YZ} = -1$

$\therefore \frac{2-5}{4-3} \times \frac{a-2}{-5-4} = -1$

$\therefore -3 \times \frac{a-2}{-9} = -1 \quad \therefore a-2 = -3$

$\therefore a = -1$

[2]  $\therefore XY = \sqrt{(4-3)^2 + (2-5)^2} = \sqrt{1+9}$   
 $= \sqrt{10}$  length units

$\therefore YZ = \sqrt{(-5-4)^2 + (-1-2)^2} = \sqrt{81+9}$   
 $= \sqrt{90}$  length units

$\therefore$  The area of  $\Delta XYZ = \frac{1}{2} \times \sqrt{10} \times \sqrt{90}$   
 $= 15$  square units.

3

[a] Let the measures be  $3X, 5X$

$\therefore 3X + 5X = 180^\circ \quad \therefore 8X = 180^\circ$

$\therefore X = 22^\circ 30'$

$\therefore$  The measures are :  $67^\circ 30', 112^\circ 30'$

[b]  $\therefore$  The slope of the given straight line =  $-1$

$\therefore$  The slope of the required straight line =  $1$

$\therefore$  Its equation is :  $y = x + c$

$\therefore (-1, 2)$  satisfies the equation.

$\therefore 2 = -1 + c \quad \therefore c = 3$

$\therefore$  The equation is :  $y = x + 3$

4

[a]  $\therefore MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9}$   
 $= 5$  length units

$\therefore MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16}$   
 $= 5$  length units

and  $MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16}$   
 $= 5$  length units

$\therefore MA = MB = MC$

$\therefore A, B$  and  $C$  lie on the circle  $M$

$\therefore$  the circumference =  $2 \times 5 \times \pi$   
 $= 10\pi$  length units.

[b] Draw  $\overline{DF} \perp \overline{BC}$

$\therefore \overline{AD} \parallel \overline{BC}, \overline{AB} \perp \overline{BC}$

$\therefore \overline{DF} \perp \overline{BC}$

$\therefore ABFD$  is a rectangle

$\therefore BF = AD = 6$  cm.

$\therefore FC = 4$  cm,  $DF = AB = 3$  cm.

$\therefore$  From  $\Delta DFC$  which is right-angled at  $F$

$(DC)^2 = 3^2 + 4^2 = 25 \quad \therefore DC = 5$  cm.

$\therefore \cos(\angle DCB) = \tan(\angle ACB) = \frac{4}{5} - \frac{3}{10} = \frac{1}{2}$

5

[a] [1]  $\therefore$  The midpoint of  $\overline{AC} = \left(\frac{3+0}{2}, \frac{2-3}{2}\right)$   
 $= \left(1\frac{1}{2}, -\frac{1}{2}\right)$

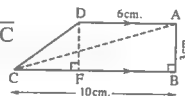
$\therefore$  The intersection point of the two diagonals is  $\left(1\frac{1}{2}, -\frac{1}{2}\right)$

[2] Let  $D(x, y)$

$\therefore$  The midpoint of  $\overline{AC}$  = the midpoint of  $\overline{BD}$

$\therefore \left(1\frac{1}{2}, -\frac{1}{2}\right) = \left(\frac{x+4}{2}, \frac{y-5}{2}\right)$

$\therefore \frac{x+4}{2} = 1\frac{1}{2}$



$$\therefore x + 4 = 3$$

$$\therefore x = -1$$

$$\therefore \frac{y-5}{2} = -\frac{1}{2} \quad \therefore y-5 = -1 \quad \therefore y = 4$$

$$\therefore D(-1, 4)$$

[b] [1] Let A(x, 0), B(0, y)

$$\therefore (3, 4) = \left(\frac{x+0}{2}, \frac{0+y}{2}\right)$$

$$\therefore \frac{x}{2} = 3 \quad \therefore x = 6$$

$$\therefore \frac{y}{2} = 4 \quad \therefore y = 8$$

$$\therefore A(6, 0), B(0, 8)$$

[2]  $\therefore$  The slope of  $\overline{AB} = \frac{8-0}{0-6} = -\frac{4}{3}$

$$\therefore \text{The equation of } \overline{AB} \text{ is } y = -\frac{4}{3}x + c$$

$$\therefore (0, 8) \text{ satisfies the equation.}$$

$$\therefore 8 = -\frac{4}{3} \times 0 + c \quad \therefore c = 8$$

$$\therefore \text{The equation is } y = -\frac{4}{3}x + 8$$

## 8 El-Dakahlia

[1]

[a] [1] c

[2] b

[3] b

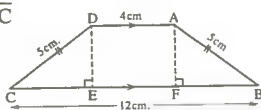
[b] Draw  $\overline{AF} \perp \overline{BC}$

$$\therefore \overline{DE} \perp \overline{BC}$$

$$\therefore \overline{AD} \parallel \overline{BC}$$

$$\therefore \overline{AF} \perp \overline{BC}$$

$$\therefore \overline{DE} \perp \overline{BC}$$



$$\therefore \text{AFED is a rectangle} \quad \therefore FE = AD = 4 \text{ cm.}$$

$$\therefore BF + EC = 8 \text{ cm.}$$

$$\therefore BF = EC = 4 \text{ cm. } (\triangle ABF \cong \triangle DCE)$$

$$\therefore \text{From } \triangle ABF \text{ which is right-angled at F}$$

$$(AF)^2 = (5)^2 - (4)^2 = 9$$

$$\therefore AF = 3 \text{ cm.}$$

$$\therefore DE = AF = 3 \text{ cm. (AFED is a rectangle)}$$

$$\therefore \frac{\tan B \cos C}{\sin^2 C + \cos^2 B} = \frac{\frac{3}{4} \times \frac{4}{5}}{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \frac{3}{5}$$

[2]

[a] [1] b

[2] b

[3] d

[b] [1]  $\therefore MB = \sqrt{(8-5)^2 + (11-7)^2} = \sqrt{9+16}$   
 $= 5 \text{ length units}$

$$\therefore \text{The circumference} = 2 \times 5 \times 3.14$$

$$= 31.4 \text{ length units.}$$

[2] Let A(x, y)

$$\therefore (5, 7) = \left(\frac{x+8}{2}, \frac{y+11}{2}\right)$$

$$\therefore \frac{x+8}{2} = 5 \quad \therefore x+8 = 10 \quad \therefore x = 2$$

$$\therefore \frac{y+11}{2} = 7 \quad \therefore y+11 = 14 \quad \therefore y = 3$$

$$\therefore A(2, 3)$$

$$\therefore \therefore \text{the slope of } \overline{AB} = \frac{11-3}{8-2} = \frac{4}{3}$$

$$\therefore \text{The slope of the required straight}$$

$$\text{line} = -\frac{3}{4}$$

$$\therefore \text{Its equation is } y = -\frac{3}{4}x + c$$

$$\therefore A(2, 3) \text{ satisfies the equation.}$$

$$\therefore 3 = -\frac{3}{4} \times 2 + c \quad \therefore c = \frac{9}{2}$$

$$\therefore \text{The equation is } y = -\frac{3}{4}x + \frac{9}{2}$$

[3]

[a]  $\therefore$  The midpoint of  $\overline{AC} = \left(\frac{-1+7}{2}, \frac{3+4}{2}\right)$   
 $= \left(3, \frac{7}{2}\right)$

$$\therefore \text{the midpoint of } \overline{BD} = \left(\frac{5+1}{2}, \frac{1+6}{2}\right)$$

$$= \left(3, \frac{7}{2}\right)$$

$$\therefore \text{The midpoint of } \overline{AC} = \text{the midpoint of } \overline{BD}$$

$$\therefore \text{The two diagonals bisect each other.}$$

$$\therefore ABCD \text{ is a parallelogram.}$$

[b] [1] Let A(0, n), B(n, 0)

$$\therefore \text{The slope} = \frac{0-n}{n-0} = -1 \quad \therefore k = -1$$

$$\therefore (2, 3) \text{ satisfies the equation.}$$

$$\therefore 3 = -1 \times 2 + c \quad \therefore c = 5$$

[2] A(0, n) satisfies the equation.

$$\therefore n = -1 \times 0 + 5 \quad \therefore n = 5$$

$$\therefore A(0, 5), B(5, 0)$$

$$\therefore \text{The area of } \triangle ABO = \frac{1}{2} \times 5 \times 5$$

$$= \frac{25}{2} \text{ square units.}$$

4

[a] [1]  $\therefore$  The intercepted part of the y-axis by  $\overline{BC}$  is 3 units

$$\therefore C = (0, 3)$$

$$\therefore BC = \sqrt{(0-2)^2 + (3-1)^2} = \sqrt{4+4} = 2\sqrt{2} \text{ length units.}$$

[2]  $\therefore B(2, 1) \therefore OA = 2$  length units

$\therefore AB = 1$  length unit

$\therefore \overline{AB} \parallel \overline{OC}, AB \neq OC$

$\therefore OABC$  is a trapezium

$$\therefore \text{The area of } OABC = \frac{1}{2}(1+3) \times 2 = 4 \text{ square units.}$$

[3] Draw  $\overline{BE} \perp \overline{OC}$

$\therefore \overline{BE} \perp \overline{OC}, \overline{AO} \perp \overline{OC}$

$\therefore \overline{AB} \parallel \overline{OC}$

$\therefore ABEO$  is a rectangle

$$\therefore OE = AB = 1 \text{ length unit}$$

$$BE = OA = 2 \text{ length units}$$

$\therefore CE = 3 - 1 = 2$  length units.

$$\text{In } \triangle BEC : \therefore \tan(\angle BCE) = \frac{2}{2} = 1$$

$$\therefore m(\angle OCB) = 45^\circ$$

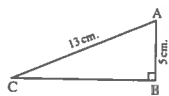
[b] [1]  $\therefore m(\angle B) = 90^\circ$

$$\therefore \sin^2 A + \cos^2 A$$

$$= \frac{(BC)^2}{(AC)^2} + \frac{(AB)^2}{(AC)^2}$$

$$= \frac{(BC)^2 + (AB)^2}{(AC)^2} = \frac{(AC)^2}{(AC)^2} = 1$$

$$[2] \therefore \sin C = \frac{5}{13} \therefore m(\angle C) \approx 22^\circ 37'$$



5

[a]  $\therefore$  The slope =  $\tan 135^\circ = -1$

$\therefore$  The equation is :  $y = -X + c$

$\therefore (3, 4)$  satisfies the equation.

$$\therefore 4 = -3 + c \therefore c = 7$$

$\therefore$  The equation is :  $y = -X + 7$

$$[b] \therefore \tan^2 60^\circ - \tan^2 45^\circ = (\sqrt{3})^2 - (1)^2 = 3 - 1 = 2$$

$$\therefore \sin^2 60^\circ + \cos^2 60^\circ + 2 \sin 30^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + 2 \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} + 1 = 2 \quad (2)$$

From (1), (2):

$$\therefore \tan^2 60^\circ - \tan^2 45^\circ = \sin^2 60^\circ + \cos^2 60^\circ + 2 \sin 30^\circ$$

9

Ismailia

1

1 a

2 c

3 b

4 a

5 c

6 d

2

$$[a] \therefore X \cos^2 30^\circ = \tan^2 60^\circ \cos^2 45^\circ$$

$$\therefore X \times \left(\frac{\sqrt{3}}{2}\right)^2 = (\sqrt{3})^2 \times \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\therefore \frac{3}{4} X = 3 \times \frac{1}{2} \therefore X = 2$$

$$[b] \therefore \text{The midpoint of } \overline{BC} = \left(\frac{3+1}{2}, \frac{7-3}{2}\right) = (2, 2)$$

$$\therefore \text{The slope of the required straight line} = \frac{-1-2}{5-2} = -1$$

$\therefore$  Its equation is :  $y = -X + c$

$\therefore A(5, -1)$  satisfies the equation.

$$\therefore -1 = -5 + c \therefore c = 4$$

$\therefore$  The equation is :  $y = -X + 4$

3

$$[a] \therefore AB = \sqrt{(-4-1)^2 + (2+2)^2} = \sqrt{25+16} = \sqrt{41} \text{ length units}$$

$$\therefore BC = \sqrt{(1+4)^2 + (6-2)^2} = \sqrt{25+16} = \sqrt{41} \text{ length units}$$

$$\therefore AC = \sqrt{(1-1)^2 + (6+2)^2} = \sqrt{0+64} = 8 \text{ length units}$$

$$\therefore AB = BC$$

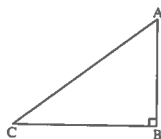
$\therefore \triangle ABC$  is an isosceles triangle.

[b]  $\therefore m(\angle B) = 90^\circ$

$$\therefore \frac{\sin A}{\cos C} = \frac{\frac{BC}{AC}}{\frac{BC}{AC}} = 1$$

$$\therefore \tan D = \frac{\sin A}{\cos C} = 1$$

$$\therefore m(\angle D) = 45^\circ$$



4

$$[a] \because L_1 \parallel L_2 \quad \therefore m_1 = m_2$$

$$\therefore \frac{1-k}{k-2} = \tan 45^\circ \quad \therefore \frac{-3}{k-2} = 1$$

$$\therefore k-2 = -3 \quad \therefore k = -1$$

[b] In  $\triangle BED$ :

$$\therefore m(\angle BED) = 90^\circ, m(\angle B) = 60^\circ$$

$$\therefore m(\angle BDE) = 30^\circ, BE = \frac{1}{2} BD = 2 \text{ cm.}$$

$$\therefore \sin 60^\circ = \frac{DE}{BD} \quad \therefore \frac{\sqrt{3}}{2} = \frac{DE}{4}$$

$$\therefore DE = 2\sqrt{3} \text{ cm.}$$

In  $\triangle CDE$ :

$$\therefore m(\angle CED) = 90^\circ, CE = 5 - 2 = 3 \text{ cm.}$$

$$\therefore \tan(\angle DCE) = \frac{2\sqrt{3}}{3}$$

5

[a] [1] The midpoint of  $\overline{AC} = \left(\frac{3-3}{2}, \frac{3-3}{2}\right) = (0, 0)$

$\therefore$  The intersection point of the diagonals is:  
(0, 0)

[2]  $\therefore$  The slope of  $\overline{AC} = \frac{-3-3}{-3-3} = 1$

$$\therefore \overline{AC} \perp \overline{BD}$$

$$\therefore \text{The slope of } \overline{BD} = -1$$

$$\therefore \overline{BD} \text{ passes through } (0, 0)$$

$$\therefore \text{The equation of } \overline{BD} \text{ is: } y = -x$$

[b]  $\therefore A(0, 2), B(4, 0), C(-1, 0)$

$$\therefore \text{The slope of } \overline{AB} = m_1 = \frac{2-0}{0-4} = -\frac{1}{2}$$

$$\therefore \text{the slope of } \overline{AC} = m_2 = \frac{0-2}{-1-0} = 2$$

$$\therefore m_1 \times m_2 = -\frac{1}{2} \times 2 = -1$$

$$\therefore \overline{AB} \perp \overline{AC}$$

$\therefore \triangle ABC$  is a right-angled triangle at A

$$\therefore \text{its area} = \frac{1}{2} \times 2 \times 5 = 5 \text{ square units.}$$

10 Suez

1

[1] d

[2] a

[3] c

[4] d

[5] b

[8] a

2

[a]  $\therefore 2 \sin 30^\circ + 4 \cos 60^\circ = 2 \times \frac{1}{2} + 4 \times \frac{1}{2} = 3$  (1)  
 $\therefore \tan^2 60^\circ = (\sqrt{3})^2 = 3$  (2)

From (1), (2):

$$\therefore 2 \sin 30^\circ + 4 \cos 60^\circ = \tan^2 60^\circ$$

[b]  $\therefore$  The midpoint of  $\overline{AC} = \left(\frac{-1+6}{2}, \frac{-1+0}{2}\right)$   
 $= \left(\frac{5}{2}, -\frac{1}{2}\right)$

$$\therefore \text{the midpoint of } \overline{BD} = \left(\frac{2+3}{2}, \frac{3-4}{2}\right)$$

$$= \left(\frac{5}{2}, -\frac{1}{2}\right)$$

$\therefore$  The midpoint of  $\overline{AC}$  = the midpoint of  $\overline{BD}$

$\therefore \overline{AC}$  and  $\overline{BD}$  bisect each other.

3

[a]  $\therefore \cos 3X = \frac{\sin 60^\circ \sin 30^\circ}{\tan 45^\circ \sin^2 45^\circ}$

$$= \frac{\frac{\sqrt{3}}{2} \times \frac{1}{2}}{1 \times \left(\frac{1}{\sqrt{2}}\right)^2} = \frac{\frac{\sqrt{3}}{4}}{\frac{1}{2}} = \frac{\sqrt{3}}{2}$$

$$\therefore 3X = 30^\circ \quad \therefore X = 10^\circ$$

[b]  $\therefore$  The slope of  $\overline{AB} = \frac{-4+3}{5-2} = -\frac{1}{3}$

$\therefore$  The slope of the required straight line = 3

$\therefore$  Its equation is:  $y = 3X + c$

$\therefore (1, 2)$  satisfies the equation.

$$\therefore 2 = 3 \times 1 + c \quad \therefore c = -1$$

$$\therefore \text{The equation is: } y = 3X - 1$$

4

[a]  $\therefore m(\angle C) = 90^\circ$

$$\therefore (AC)^2 = (5)^2 - (4)^2 = 9$$

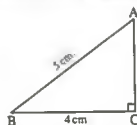
$$\therefore AC = 3 \text{ cm.}$$

$$\therefore \sin A \cos B + \cos A \sin B = \frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5}$$

$$= 1$$

[b]  $\therefore \frac{y-1}{x} = \frac{1}{3} \quad \therefore y = \frac{1}{3}x + 1$

$\therefore$  The slope of the given straight line =  $\frac{1}{3}$



$\therefore$  The slope of the required straight line =  $\frac{1}{3}$

$\therefore$  it intersects a part from the negative direction of the y-axis of length 3 units

$\therefore$  The equation is :  $y = \frac{1}{3}x - 3$

5

$$\begin{aligned} \text{[a]} \therefore AB &= \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{9+16} \\ &= 5 \text{ length units} \end{aligned}$$

$$\begin{aligned} \therefore BC &= \sqrt{(-4-3)^2 + (3-4)^2} = \sqrt{49+1} \\ &= 5\sqrt{2} \text{ length units} \end{aligned}$$

$$\begin{aligned} \therefore AC &= \sqrt{(-4-0)^2 + (3-0)^2} = \sqrt{16+9} \\ &= 5 \text{ length units} \end{aligned}$$

$$\begin{aligned} \therefore \text{The perimeter of } \triangle ABC &= 5 + 5\sqrt{2} + 5 \\ &= 10 + 5\sqrt{2} \text{ length units.} \end{aligned}$$

$$\text{[b]} \therefore \overline{AB} \parallel \overline{CD}$$

$$\therefore \frac{2+2}{3-9} = \frac{-3+x}{4+x}$$

$$\therefore -9+3x = -8-2x$$

$$\therefore x = \frac{1}{5}$$

$$\therefore m_1 = m_2$$

$$\therefore \frac{-2}{3} = \frac{-3+x}{4+x}$$

$$\therefore 5x = 1$$

$$\therefore C\left(-\frac{1}{5}, -\frac{1}{5}\right)$$

## 11 Port Said

1

1 a    2 b    3 c    4 b    5 d    6 b

2

$$\text{[a]} \therefore m_1 = \frac{4-3}{2+1} = \frac{1}{3}$$

$$\therefore m_2 = \frac{1}{3}$$

$$\therefore m_1 = m_2$$

$\therefore$  The two straight lines are parallel.

$$\text{[b]} \therefore \sin 90^\circ = 1$$

$$\therefore \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = 1$$

From (1), (2):

$$\therefore \sin 90^\circ = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

3

$$\text{[a]} \therefore \cos E = \frac{\cos^2 45^\circ}{\tan 30^\circ} = \frac{\left(\frac{1}{\sqrt{2}}\right)^2}{\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}}{2}$$

$$\therefore m(\angle E) = 30^\circ$$

$$\begin{aligned} \text{[b]} \therefore AB &= \sqrt{(3+3)^2 + (4-0)^2} = \sqrt{36+16} \\ &= 2\sqrt{13} \text{ length units} \end{aligned}$$

$$\begin{aligned} \therefore BC &= \sqrt{(1-3)^2 + (-6-4)^2} = \sqrt{4+100} \\ &= 2\sqrt{26} \text{ length units} \end{aligned}$$

$$\begin{aligned} \therefore AC &= \sqrt{(1+3)^2 + (-6-0)^2} = \sqrt{16+36} \\ &= 2\sqrt{13} \text{ length units} \end{aligned}$$

$\therefore AB = AC \quad \therefore \triangle ABC$  is an isosceles triangle.

4

$$\text{[a]} \therefore \frac{y-1}{x} = \frac{1}{3}$$

$$\therefore y = \frac{1}{3}x + 1$$

$\therefore$  The slope of the given straight line =  $\frac{1}{3}$

$\therefore$  The slope of the required straight line =  $\frac{1}{3}$

$\therefore$  it intercepts a part from the negative direction of the y-axis of length 3 units

The equation is :  $y = \frac{1}{3}x - 3$

$$\text{[b]} \therefore \text{The slope of } \overline{AD} = \frac{1-3}{-2-2} = \frac{1}{2}$$

$$\therefore \text{the slope of } \overline{BC} = \frac{2+2}{6+2} = \frac{1}{2}$$

$\therefore$  The slope of  $\overline{AD}$  = the slope of  $\overline{BC}$

$$\therefore \overline{AD} \parallel \overline{BC}$$

$$\therefore \therefore \text{the slope of } \overline{AB} = \frac{2-3}{6-2} = \frac{-1}{4}$$

$$\therefore \text{the slope of } \overline{CD} = \frac{1+2}{-2+2} \text{ is undefined}$$

$\therefore$  The slope of  $\overline{AB} \neq$  the slope of  $\overline{CD}$

$\therefore \overline{AB}$  is not parallel to  $\overline{CD}$

From (1), (2):

$\therefore ABCD$  is a trapezoid.

5

$$\text{[a]} \therefore \text{The midpoint of } \overline{BC} = \left(\frac{3+1}{2}, \frac{7-3}{2}\right) = (2, 2)$$

$$\therefore \text{The slope of the straight line} = \frac{2+6}{2-5} = \frac{-8}{3}$$

$$\therefore \text{Its equation is : } y = \frac{-8}{3}x + c$$

$\therefore (5, -6)$  satisfies the equation.

$$\therefore -6 = \frac{-8}{3} \times 5 + c \quad \therefore c = \frac{22}{3}$$

$$\therefore \text{The equation is : } y = \frac{-8}{3}x + \frac{22}{3}$$

[b]  $\therefore m(\angle Y) = 90^\circ$

$\therefore (YZ)^2 = (13)^2 - (5)^2 = 144$

$\therefore YZ = 12 \text{ cm.}$

$\therefore \sin X \cos Z + \cos X \sin Z$

$= \frac{12}{13} \times \frac{12}{13} + \frac{5}{13} \times \frac{5}{13} = 1$



## 12 Damietta

1

- 1 a    2 d    3 d    4 c    5 b    6 d

2

[a]  $\therefore$  The slope of the straight line  $= \frac{5-0}{0-5} = -1$

$\therefore$  Its equation is :  $y = -x + c$

$\therefore (0, 5)$  satisfies the equation.

$\therefore 5 = 0 + c \quad \therefore c = 5$

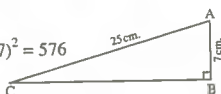
$\therefore$  The equation is :  $y = -x + 5$

[b]  $\therefore m(\angle B) = 90^\circ$

$\therefore (BC)^2 = (25)^2 - (7)^2 = 576$

$\therefore BC = 24 \text{ cm.}$

$\therefore \sin^2 A + \sin^2 C = \left(\frac{24}{25}\right)^2 + \left(\frac{7}{25}\right)^2 = 1$



3

[a]  $\therefore$  The points are located on one straight line

$\therefore \frac{3-1}{a-0} = \frac{5-1}{2-0} \quad \therefore \frac{2}{a} = 2 \quad \therefore a = 1$

[b]  $\therefore$  The slope of the given straight line  $= \frac{-1}{3}$

$\therefore$  The slope of the required straight line  $= \frac{-1}{3}$

$\therefore$  Its equation is :  $y = \frac{-1}{3}x + c$

$\therefore (3, 7)$  satisfies the equation.

$\therefore 7 = \frac{-1}{3} \times 3 + c \quad \therefore c = 8$

$\therefore$  The equation is :  $y = \frac{-1}{3}x + 8$

4

[a]  $\therefore 2 \sin X = \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ$

$\therefore 2 \sin X = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$\therefore 2 \sin X = 1 \quad \therefore \sin X = \frac{1}{2}$

$\therefore X = 30^\circ$

[b]  $\therefore$  The slope of the straight line  $= 2$  and it intersects from the positive part of y-axis 7 units.

$\therefore$  Its equation is :  $y = 2x + 7$

5

[a]  $\therefore \tan 60^\circ = \sqrt{3} \quad 2 \times \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}} \quad (1)$

$\therefore \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{\frac{2}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \sqrt{3} \quad (2)$

From (1) , (2) :

$\therefore \tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$

[b]  $\therefore AB = \sqrt{(-2-3)^2 + (4+1)^2} = \sqrt{25 + 25} = 5\sqrt{2} \text{ length units}$

$\therefore BC = \sqrt{(3-4)^2 + (-1-5)^2} = \sqrt{1 + 36} = \sqrt{37} \text{ length units}$

$\therefore AC = \sqrt{(-2-4)^2 + (4-5)^2} = \sqrt{36 + 1} = \sqrt{37} \text{ length units}$

$\therefore BC = AC$

$\therefore \triangle ABC$  is an isosceles triangle.

## 13 Kafr El-Sheikh

1

- 1 c    2 a    3 b    4 d    5 c    6 d

2

[a]  $\therefore AB = \sqrt{(3-1)^2 + (0-4)^2} = \sqrt{4 + 16} = 2\sqrt{5} \text{ length units}$

$\therefore BC = \sqrt{(1+1)^2 + (4-2)^2} = \sqrt{4 + 4} = 2\sqrt{2} \text{ length units}$

$\therefore AC = \sqrt{(3+1)^2 + (0-2)^2} = \sqrt{16 + 4} = 2\sqrt{5} \text{ length units}$

$\therefore AB = AC$

$\therefore \triangle ABC$  is an isosceles triangle.

[b]  $\sin^2 45^\circ \cos 60^\circ + \frac{1}{2} \tan 60^\circ \sin 60^\circ$   
 $= \left(\frac{1}{\sqrt{2}}\right)^2 \times \frac{1}{2} + \frac{1}{2} \times \sqrt{3} \times \frac{\sqrt{3}}{2}$   
 $= \frac{1}{4} + \frac{3}{4} = 1$

3

[a]  $\because L_1 \parallel L_2 \quad \therefore m_1 = m_2$   
 $\therefore 2 - k = \tan 45^\circ \quad \therefore 2 - k = 1$

$\therefore k = 1$

[b]  $\because \sqrt{3} \tan X = 4 \sin 60^\circ \cos 30^\circ$

$\therefore \sqrt{3} \tan X = 4 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$\therefore \sqrt{3} \tan X = 3 \quad \therefore \tan X = \sqrt{3}$

$\therefore X = 60^\circ$

4

[a]  $\because \sqrt{(2-X)^2 + (5-3)^2} = 2\sqrt{2}$  (Squaring both sides)

$\therefore (2-X)^2 + (2)^2 = 8$

$\therefore X^2 - 4X + 4 + 4 = 8 \quad \therefore X^2 - 4X = 0$

$\therefore X(X-4) = 0 \quad \therefore X = 0 \text{ or } X = 4$

[b]  $\therefore$  The slope = 3

$\therefore$  The equation is:  $y = 3X + c$

$\because (5, -2)$  satisfies the equation.

$\therefore -2 = 3 \times 5 + c \quad \therefore c = -17$

$\therefore$  The equation is:  $y = 3X - 17$

5

[a] Let B ( $X, y$ )

$\therefore (2, 3) = \left( \frac{X-1}{2}, \frac{y+3}{2} \right)$

$\therefore \frac{X-1}{2} = 2 \quad \therefore X-1 = 4 \quad \therefore X = 5$

$\therefore \frac{y+3}{2} = 3 \quad \therefore y+3 = 6 \quad \therefore y = 3$

$\therefore B(5, 3)$

[b]  $\therefore \angle A, \angle C$  are complementary angles

$\therefore \sin A = \cos C$

$\therefore \sin A + \cos C = \sin A + \sin A = 1$

$\therefore \sin A = \frac{1}{2} \quad \therefore m(\angle A) = 30^\circ$

14 El-Beheira

1

[1] c [2] b [3] b [4] b [5] b [6] c

2

[a]  $\because m_1 = \frac{4-3}{2+1} = \frac{1}{3}, m_2 = \frac{1}{3} \quad \therefore m_1 = m_2$

$\therefore$  The two straight lines are parallel.

[b] Draw  $\overline{DE} \perp \overline{BC}$

$\because \overline{AD} \parallel \overline{BC}, \overline{AB} \perp \overline{BC}$   
 $\therefore \overline{DE} \perp \overline{BC}$

$\therefore ABED$  is a rectangle

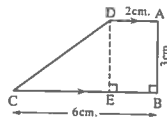
$\therefore DE = AB = 3 \text{ cm.}$

$\therefore BE = AD = 2 \text{ cm.} \quad \therefore CE = 6 - 2 = 4 \text{ cm.}$

In  $\triangle DEC$ :  $\because m(\angle DEC) = 90^\circ$

$\therefore (DC)^2 = (3)^2 + (4)^2 = 25 \quad \therefore DC = 5 \text{ cm.}$

$\therefore \cos(\angle BCD) = \frac{4}{5}$



3

[a]  $\therefore$  The slope = 3

$\therefore$  The equation is:  $y = 3X + c$

$\because (1, 2)$  satisfies the equation.

$\therefore 2 = 3 \times 1 + c \quad \therefore c = -1$

$\therefore$  The equation is:  $y = 3X - 1$

[b]  $\because 2 \sin X = \tan^2 60^\circ - 2 \tan 45^\circ$

$\therefore 2 \sin X = (\sqrt{3})^2 - 2 \times 1 \quad \therefore 2 \sin X = 1$

$\therefore \sin X = \frac{1}{2} \quad \therefore X = 30^\circ$

4

[a]  $\because L_1 \perp L_2$

$\therefore \frac{k-1}{2-3} \times \tan 45^\circ = -1 \quad \therefore m_1 \times m_2 = -1$

$\therefore 1 - k = -1 \quad \therefore (1-k) \times 1 = -1$

$\therefore k = 2$

[b]  $\because \sqrt{2} AB = AC$

$\therefore \frac{AB}{AC} = \frac{1}{\sqrt{2}}$

Let  $AB = 1$  length unit

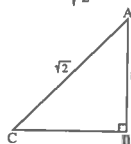
$\therefore AC = \sqrt{2}$  length unit

$\because m(\angle B) = 90^\circ$

$\therefore (BC)^2 = (\sqrt{2})^2 - (1)^2 = 1$

$\therefore BC = 1$  length unit

$\therefore \sin C = \frac{1}{\sqrt{2}}, \cos C = \frac{1}{\sqrt{2}}, \tan C = 1$



5

[a]  $\because AB = BC$

$\therefore \sqrt{(X-3)^2 + (3-2)^2} = \sqrt{(3-5)^2 + (2-1)^2}$   
 (Squaring both sides)

$\therefore (X-3)^2 + 1 = 4 + 1$

$\therefore X^2 - 6X + 9 + 1 - 4 - 1 = 0$

$\therefore X^2 - 6X + 5 = 0 \quad \therefore (X-5)(X-1) = 0$

$\therefore X = 5 \text{ or } X = 1$  (refused because  $B \notin \overline{AC}$ )

$$\begin{aligned}
 [b] \therefore AB &= \sqrt{(2-6)^2 + (-4-0)^2} \\
 &= \sqrt{16+16} = \sqrt{32} = 4\sqrt{2} \text{ length unit} \\
 , BC &= \sqrt{(-4-2)^2 + (2+4)^2} = \sqrt{36+36} = \sqrt{72} \\
 &= 6\sqrt{2} \text{ length unit} \\
 , CA &= \sqrt{(6+4)^2 + (0-2)^2} = \sqrt{100+4} = \sqrt{104} \\
 &= 2\sqrt{26} \text{ length unit}
 \end{aligned}$$

$$\therefore (AB)^2 + (BC)^2 = 32 + 72 = 104 = (CA)^2$$

$\therefore \Delta ABC$  is right-angled at B

Let E be the midpoint of  $\overline{AC}$

$$\therefore E = \left( \frac{6-4}{2}, \frac{0+2}{2} \right) = (1, 1)$$

$\therefore$  In the rectangle the two diagonals bisect each other

$\therefore$  E is the midpoint of  $\overline{BD}$

Let D (X, y)

$$\therefore (1, 1) = \left( \frac{X+2}{2}, \frac{y-4}{2} \right) \therefore \frac{X+2}{2} = 1$$

$$\therefore X+2=2 \therefore X=0$$

$$\therefore \frac{y-4}{2} = 1 \therefore y-4=2$$

$$\therefore y=6 \therefore D(0, 6)$$

### 15 El-Fayoum

1

1 b 2 d 3 c 4 c 5 c 6 c

2

$$\begin{aligned}
 [a] \therefore MA &= \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9} \\
 &= 5 \text{ length units} \\
 , MB &= \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16} \\
 &= 5 \text{ length units} \\
 and MC &= \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16} \\
 &= 5 \text{ length units} \\
 \therefore MA &= MB = MC \\
 \therefore A, B \text{ and } C &\text{ lie on the circle M} \\
 , the circumference &= 2 \times 3.14 \times 5 \\
 &= 31.4 \text{ length units.}
 \end{aligned}$$

$$\begin{aligned}
 [b] \therefore \tan^2 60^\circ - \tan^2 45^\circ &= \left( \sqrt{3} \right)^2 - (1)^2 = 2 \quad (1) \\
 , \sin^2 60^\circ + \cos^2 60^\circ + 2 \sin 30^\circ &
 \end{aligned}$$

$$= \left( \frac{\sqrt{3}}{2} \right)^2 + \left( \frac{1}{2} \right)^2 + 2 \times \frac{1}{2} = 2 \quad (2)$$

From (1), (2):

$$\therefore \tan^2 60^\circ - \tan^2 45^\circ = \sin^2 60^\circ + \cos^2 60^\circ + 2 \sin 30^\circ$$

$$[a] \therefore \text{The slope of } \overline{AB} = \frac{5-3}{3-1} = 1$$

$\therefore$  The slope of the required straight line = -1

$\therefore$  Its equation is:  $y = -X + c$

$\therefore$  the midpoint of AB

$$= \left( \frac{1+3}{2}, \frac{3+5}{2} \right) = (2, 4)$$

$\therefore$  the required straight line passes through the midpoint of AB

$$\therefore 4 = -2 + c \therefore c = 6$$

$\therefore$  The equation of the required straight line is:

$$y = -X + 6$$

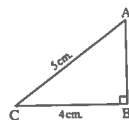
$$[b] \therefore m(\angle B) = 90^\circ$$

$$\therefore (AB)^2 = (5)^2 - (4)^2 = 9$$

$$\therefore AB = 3 \text{ cm.}$$

$$\therefore 2 \cos^2 C + \sin^2 A$$

$$= 2 \left( \frac{4}{5} \right)^2 + \left( \frac{4}{5} \right)^2 = \frac{48}{25}$$



1

$$[a] \therefore \text{The midpoint of } \overline{AC} = \left( \frac{3}{2}, \frac{-2-7}{2} \right)$$

$$= \left( \frac{3}{2}, \frac{9}{2} \right)$$

$$, \text{ the midpoint of } \overline{BD} = \left( \frac{-5+5}{2}, \frac{-9}{2} \right)$$

$$= \left( \frac{3}{2}, \frac{-9}{2} \right)$$

$\therefore$  The midpoint of  $\overline{AC}$  = the midpoint of  $\overline{BD}$

$\therefore \overline{AC}$  and  $\overline{BD}$  bisect each other

$\therefore$  The points A, B, C and D are the vertices of a parallelogram.

$$[b] \therefore 4X = \cos^2 30^\circ \tan^2 30^\circ \tan^2 45^\circ$$

$$\therefore 4X = \left( \frac{\sqrt{3}}{2} \right)^2 \times \left( \frac{1}{\sqrt{3}} \right)^2 \times (1)^2$$

$$\therefore 4X = \frac{3}{4} \times \frac{1}{3} \times 1 \therefore 4X = \frac{1}{4} \therefore X = \frac{1}{16}$$

5

[a]  $\therefore$  The two straight lines are perpendicular

$$\therefore m_1 \times m_2 = -1 \quad \therefore \frac{3}{4} \times \frac{-4}{k} = -1$$

$$\therefore \frac{3}{k} = 1 \quad \therefore k = 3$$

[b]  $\therefore$  The straight line passes through (1, 0), (0, 4)

$$\therefore \text{Its slope} = \frac{4-0}{0-1} = -4$$

$$\therefore \text{Its equation is : } y = -4x + c$$

$$\therefore (0, 4) \text{ satisfies the equation.}$$

$$\therefore 4 = -4 \times 0 + c \quad \therefore c = 4$$

$$\therefore \text{The equation is : } y = -4x + 4$$

## 16 Beni Suef

1

1 b 2 c 3 d 4 a 5 d 6 b

2

[a] Let B (X, y)

$$\therefore (6, -4) = \left( \frac{5+X}{2}, \frac{-3+Y}{2} \right)$$

$$\therefore \frac{5+X}{2} = 6 \quad \therefore 5+X = 12 \quad \therefore X = 7$$

$$\therefore \frac{-3+Y}{2} = -4 \quad \therefore -3+Y = -8 \quad \therefore Y = -5$$

$$\therefore B(7, -5)$$

[b] Draw  $\overline{DE} \perp \overline{BC}$

$$\therefore \overline{AD} \parallel \overline{BC}, \overline{AB} \perp \overline{BC}$$

$$\therefore \overline{DE} \perp \overline{BC}$$

$$\therefore ABED \text{ is a rectangle}$$

$$\therefore DE = AB = 12 \text{ cm.}$$

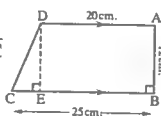
$$\therefore BE = AD = 20 \text{ cm.} \quad \therefore CE = 25 - 20 = 5 \text{ cm.}$$

$$\text{In } \triangle DEC : \therefore m(\angle DEC) = 90^\circ$$

$$\therefore (DC)^2 = (12)^2 + (5)^2 = 169$$

$$\therefore DC = 13 \text{ cm.} \quad \therefore \tan C = \frac{12}{5}$$

$$\therefore m(\angle C) \approx 67^\circ 22' 48''$$



3

$$[a] \therefore \frac{1}{2} \sin 60^\circ = \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} \quad (1)$$

$$\therefore \sin 30^\circ \cos 30^\circ = \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} \quad (2)$$

From (1), (2):

$$\therefore \frac{1}{2} \sin 60^\circ = \sin 30^\circ \cos 30^\circ$$

[b]  $\therefore$  The slope = 2

$$\therefore \text{The equation of the straight line is : } y = 2x + c$$

$$\therefore (2, 3) \text{ satisfies the equation.}$$

$$\therefore 3 = 2 \times 2 + c \quad \therefore c = -1$$

$$\therefore \text{The equation is : } y = 2x - 1$$

4

[a]  $\therefore \cos E \tan 30^\circ = \sin^2 45^\circ$

$$\therefore \cos E \times \frac{1}{\sqrt{3}} = \left( \frac{1}{\sqrt{2}} \right)^2$$

$$\therefore \cos E = \frac{\sqrt{3}}{2} \quad \therefore m(\angle E) = 30^\circ$$

[b]  $\therefore m_1 = \frac{3+1}{6-2} = 1, \quad m_2 = \tan 45^\circ = 1$

$$\therefore m_1 = m_2$$

$\therefore$  The two straight lines are parallel.

5

$$[a] \therefore MA = \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9}$$

$$= 5 \text{ length units}$$

$$\therefore MB = \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16}$$

$$= 5 \text{ length units}$$

$$\text{and } MC = \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16}$$

$$= 5 \text{ length units}$$

$$\therefore MA = MB = MC$$

$\therefore A, B \text{ and } C \text{ are located on the circle } M$

[b] The slope =  $\frac{2}{3}$

and the intersected part =  $\frac{5}{3}$  units

from the negative direction of the y-axis.

## 17 El-Menia

1

1 b 2 b 3 c 4 d 5 b 6 d

2

[a]  $\cos 60^\circ \sin 30^\circ - \sin 60^\circ \tan 60^\circ + \cos^2 30^\circ$

$$= \frac{1}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \sqrt{3} + \left( \frac{\sqrt{3}}{2} \right)^2$$

$$= \frac{1}{4} - \frac{3}{2} + \frac{3}{4} = -\frac{1}{2}$$

[b]  $\therefore$  The slope of the given straight line

$$= \frac{-4+3}{5-2} = \frac{-1}{3}$$

∴ The slope of the required straight line = 3

∴ Its equation is :  $y = 3x + c$

∵ (1, 2) satisfies the equation.

$$\therefore 2 = 3 \times 1 + c \quad \therefore c = -1$$

∴ The equation is :  $y = 3x - 1$

3

$$[a] \therefore 2 \sin X = \tan^2 60^\circ - 2 \tan 45^\circ$$

$$\therefore 2 \sin X = (\sqrt{3})^2 - 2 \times 1 \quad \therefore 2 \sin X = 1$$

$$\therefore \sin X = \frac{1}{2} \quad \therefore X = 30^\circ$$

$$[b] \therefore m(\angle A) = 90^\circ$$

$$\therefore (AB)^2 = (25)^2 - (15)^2 = 400$$

$$\therefore AB = 20 \text{ cm.}$$

$$\therefore \cos C \cos B - \sin C \sin B = \frac{15}{25} \times \frac{20}{25} - \frac{20}{25} \times \frac{15}{25} = 0$$

4

$$[a] \therefore \text{The slope of } \overline{AB} = m_1 = \frac{0+4}{1+1} = 2$$

$$\therefore \text{the slope of } \overline{BC} = m_2 = \frac{2-0}{2-1} = 2$$

$$\therefore m_1 = m_2 \quad \therefore \overline{AB} \parallel \overline{BC}$$

∵ B is a common point

∴ A, B, C are collinear.

$$[b] \text{ Let } B(x, y) \quad \therefore (6, -4) = \left( \frac{5+x}{2}, \frac{-3+y}{2} \right)$$

$$\therefore \frac{5+x}{2} = 6 \quad \therefore 5+x = 12 \quad \therefore x = 7$$

$$\therefore \frac{-3+y}{2} = -4 \quad \therefore -3+y = -8 \quad \therefore y = -5$$

$$\therefore B(7, -5)$$

5

$$[a] \therefore m_1 = \tan 45^\circ = 1, \quad m_2 = 1$$

$$\therefore m_1 = m_2$$

∴ The two straight lines are parallel.

$$[b] \therefore \sqrt{(a+2)^2 + (7-3)^2} = 5 \quad (\text{Squaring both sides})$$

$$\therefore (a+2)^2 + (4)^2 = 25$$

$$\therefore a^2 + 4a + 4 + 16 - 25 = 0$$

$$\therefore a^2 + 4a - 5 = 0 \quad \therefore (a-1)(a+5) = 0$$

$$\therefore a = 1 \text{ or } a = -5$$

18

Assiut

1

$$[1] \text{ c} \quad [2] \text{ d} \quad [3] \text{ c} \quad [4] \text{ a} \quad [5] \text{ c} \quad [6] \text{ b}$$

2

$$[a] \therefore m(\angle C) = 90^\circ$$

$$\therefore (AC)^2 = (13)^2 - (12)^2 = 25$$

$$\therefore AC = 5 \text{ cm.}$$

$$\therefore \sin A \cos B + \cos A \sin B = \frac{12}{13} \times \frac{12}{13} + \frac{5}{13} \times \frac{5}{13} = 1$$

$$[b] \therefore AB = \sqrt{(5-1)^2 + (1-1)^2} = \sqrt{16} = 4 \text{ length units}$$

$$\therefore BC = \sqrt{(3-5)^2 + (4-1)^2} = \sqrt{4+9}$$

$$= \sqrt{13} \text{ length units}$$

$$\therefore AC = \sqrt{(3-1)^2 + (4-1)^2} = \sqrt{4+9} = \sqrt{13} \text{ length units.}$$

$$\therefore BC = AC \quad \therefore \triangle ABC \text{ is isosceles.}$$

3

$$[a] \therefore 2 \sin X = \tan^2 60^\circ - 4 \sin 30^\circ$$

$$\therefore 2 \sin X = (\sqrt{3})^2 - 4 \times \frac{1}{2} \quad \therefore 2 \sin X = 1$$

$$\therefore \sin X = \frac{1}{2} \quad \therefore X = 30^\circ$$

$$[b] \therefore \text{The midpoint of } \overline{AC} = \left( \frac{3+1}{2}, \frac{2+4}{2} \right) = (2, 3)$$

∴ The point of intersection of the diagonals is :  
(2, 3)

Let D(x, y)

$$\therefore (2, 3) = \left( \frac{4+x}{2}, \frac{-5+y}{2} \right)$$

$$\therefore \frac{4+x}{2} = 2 \quad \therefore 4+x = 4 \quad \therefore x = 0$$

$$\therefore \frac{-5+y}{2} = 3$$

$$\therefore -5+y = 6 \quad \therefore y = 11 \quad \therefore D(0, 11)$$

4

$$[a] \cos 60^\circ + \cos^2 30^\circ + \tan^2 45^\circ$$

$$= \frac{1}{2} + \left( \frac{\sqrt{3}}{2} \right)^2 + (1)^2 = \frac{1}{2} + \frac{3}{4} + 1 = \frac{9}{4}$$

$$[b] \therefore m_1 = \frac{4-3}{\sqrt{3}-2\sqrt{3}} = \frac{-1}{\sqrt{3}}, \quad m_2 = \tan 60^\circ = \sqrt{3}$$

$$\therefore m_1 \times m_2 = \frac{-1}{\sqrt{3}} \times \sqrt{3} = -1$$

∴ The two straight lines are perpendicular.

5

- [a]  $\therefore$  The slope of the given straight line  $= \frac{-1}{3}$   
 $\therefore$  The slope of the required straight line  $= \frac{-1}{3}$   
 $\therefore$  Its equation is  $y = \frac{-1}{3}x + c$   
 $\therefore (3, -5)$  satisfies the equation.  
 $\therefore -5 = \frac{-1}{3} \times 3 + c \quad \therefore c = -4$   
 $\therefore$  The equation is  $y = \frac{-1}{3}x - 4$
- [b]  $\therefore \frac{y-1}{x} = \frac{1}{2}$   
 $\therefore y = \frac{1}{2}x + 1$   
 $\therefore$  The slope  $= \frac{1}{2}$  and the intercepted part equals 1 unit from the positive direction of the y-axis.

19 Souhag

1

- 1 c    2 b    3 d    4 a    5 c    6 d

2

- [a]  $\therefore \cos X = 2 \cos^2 30^\circ - 1$   
 $\therefore \cos X = 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1 \quad \therefore \cos X = 2 \times \frac{3}{4} - 1$   
 $\therefore \cos X = \frac{1}{2} \quad \therefore X = 60^\circ$
- [b]  $\therefore$  The slope of  $\overline{AB} = m_1 = \frac{-2-4}{-1-1} = 3$   
 $\therefore$  the slope of  $\overline{BC} = m_2 = \frac{-3+2}{2+1} = \frac{-1}{3}$   
 $\therefore m_1 \times m_2 = 3 \times \frac{-1}{3} = -1 \quad \therefore \overline{AB} \perp \overline{BC}$   
 $\therefore \Delta ABC$  is right-angled at B

3

- [a] 1  $\therefore m(\angle C) = 90^\circ$   
 $\therefore (AC)^2 = (13)^2 - (12)^2 = 25 \quad \therefore AC = 5 \text{ cm.}$
- 2  $\sin A \cos B + \cos A \sin B$   
 $= \frac{12}{13} \times \frac{12}{13} + \frac{5}{13} \times \frac{5}{13} = 1$
- [b]  $\therefore$  The slope  $= 2$   
 $\therefore$  The equation of the straight line is  $y = 2x + c$   
 $\therefore (1, 0)$  satisfies the equation.  
 $\therefore 0 = 2 \times 1 + c \quad \therefore c = -2$   
 $\therefore$  The equation is  $y = 2x - 2$

4

- [a]  $\therefore 2 \sin 30^\circ = 2 \times \frac{1}{2} = 1$  (1)  
 $\therefore \tan^2 60^\circ - 2 \tan 45^\circ = (\sqrt{3})^2 - 2 \times 1 = 1$  (2)  
 From (1), (2):  $\therefore 2 \sin 30^\circ = \tan^2 60^\circ - 2 \tan 45^\circ$
- [b]  $\therefore$  The slope of the straight line  $= \frac{-3-3}{-1-1} = 3$   
 $\therefore$  Its equation is  $y = 3x + c$   
 $\therefore (1, 3)$  satisfies the equation.  
 $\therefore 3 = 3 \times 1 + c \quad \therefore c = 0$   
 $\therefore$  The equation is  $y = 3x$   
 $\therefore c = 0$   
 $\therefore$  The straight line passes through the origin point.

5

- [a]  $\therefore$  The slope of  $\overline{AB} = m_1 = \frac{5+1}{6+3} = \frac{2}{3}$   
 $\therefore$  the slope of  $\overline{BC} = m_2 = \frac{3-5}{3-6} = \frac{2}{3}$   
 $\therefore m_1 = m_2 \quad \therefore \overline{AB} \parallel \overline{BC}$   
 $\therefore B$  is a common point  
 $\therefore A, B, C$  are collinear.
- [b]  $\therefore m_1 = \frac{5+2}{4+3} = 1, m_2 = \tan 45^\circ = 1$   
 $\therefore m_1 = m_2$   
 $\therefore$  The two straight lines are parallel.

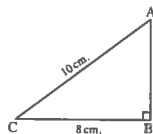
20 Qena

1

- 1 b    2 c    3 a    4 b    5 c    6 b

2

- [a]  $\therefore m(\angle B) = 90^\circ$   
 $\therefore (AB)^2 = (10)^2 - (8)^2 = 36$   
 $\therefore AB = 6 \text{ cm.}$   
 $\therefore \sin^2 A + 1$   
 $= \left(\frac{8}{10}\right)^2 + 1 = \frac{41}{25}$  (1)  
 $\therefore 2 \cos^2 C + \cos^2 A = 2 \times \left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2 = \frac{41}{25}$  (2)  
 From (1), (2):  $\therefore \sin^2 A + 1 = 2 \cos^2 C + \cos^2 A$
- [b]  $\therefore$  The slope of  $\overline{AB} = m_1 = \frac{-1-1}{0-1} = 2$   
 $\therefore$  the slope of  $\overline{BC} = m_2 = \frac{3+1}{2-0} = 2$   
 $\therefore m_1 = m_2 \quad \therefore \overline{AB} \parallel \overline{BC}$



$\therefore B$  is a common point

$\therefore A, B, C$  are collinear.

13

[a]  $\therefore \sin X \tan 30^\circ = \sin^2 45^\circ$

$$\therefore \sin X \times \frac{1}{\sqrt{3}} = \left(\frac{1}{\sqrt{2}}\right)^2 \quad \therefore \sin X = \frac{\sqrt{3}}{2}$$

$$\therefore X = 60^\circ$$

[b]  $\therefore m_1 = \frac{4-3}{2+1} = \frac{1}{3}, \quad m_2 = \frac{1}{3}$

$$\therefore m_1 = m_2$$

$\therefore$  The two straight lines are parallel.

14

[a]  $\therefore \sin 60^\circ = \frac{\sqrt{3}}{2}$  (1)

$$2 \sin 30^\circ \cos 30^\circ = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} \quad (2)$$

$$\text{From (1), (2): } \therefore \sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ$$

[b]  $\therefore AB = \sqrt{(6-5)^2 + (-2-3)^2} = \sqrt{1+25}$   
 $= \sqrt{26} \text{ length units}$

$$BC = \sqrt{(1-6)^2 + (-1+2)^2} = \sqrt{25+1}$$
  
 $= \sqrt{26} \text{ length units}$

$$CD = \sqrt{(0-1)^2 + (4+1)^2} = \sqrt{1+25}$$
  
 $= \sqrt{26} \text{ length units}$

$$DA = \sqrt{(5-0)^2 + (3-4)^2} = \sqrt{25+1}$$
  
 $= \sqrt{26} \text{ length units}$

$$\therefore AB = BC = CD = DA$$

$\therefore ABCD$  is a rhombus

$$\therefore AC = \sqrt{(1-5)^2 + (-1-3)^2} = \sqrt{16+16}$$
  
 $= 4\sqrt{2} \text{ length units}$

$$BD = \sqrt{(0-6)^2 + (4+2)^2} = \sqrt{36+36}$$
  
 $= 6\sqrt{2} \text{ length units}$

$$\therefore \text{The area of the rhombus} = \frac{1}{2} AC \times BD$$
  
 $= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$   
 $= 24 \text{ square units.}$

15

[a]  $\therefore AB = \sqrt{(3+3)^2 + (4-0)^2} = \sqrt{36+16}$   
 $= 2\sqrt{13} \text{ length units}$

$$BC = \sqrt{(1-3)^2 + (-6-4)^2} = \sqrt{4+100}$$
  
 $= 2\sqrt{26} \text{ length units}$

$$CA = \sqrt{(-3-1)^2 + (0+6)^2} = \sqrt{16+36}$$
  
 $= 2\sqrt{13} \text{ length units}$

$$\therefore AB = AC$$

$\therefore \triangle ABC$  is an isosceles triangle and its vertex is  $A$

Let  $D$  be the midpoint of  $\overline{BC}$  (The base of  $\triangle ABC$ )

$$\therefore D = \left(\frac{3+1}{2}, \frac{4-6}{2}\right) = (2, -1)$$

$$\therefore AD = \sqrt{(2+3)^2 + (-1-0)^2} = \sqrt{25+1}$$
  
 $= \sqrt{26} \text{ length units}$

$\therefore$  The length of the segment perpendicular to  $\overline{BC}$  from  $A = \sqrt{26} \text{ length units.}$

[b]  $\therefore$  The midpoint of  $\overline{AC} = \left(\frac{3+0}{2}, \frac{2-3}{2}\right)$   
 $= \left(1\frac{1}{2}, -\frac{1}{2}\right)$

$\therefore$  The point of intersection of the two diagonals is  $\left(1\frac{1}{2}, -\frac{1}{2}\right)$

and let  $D(X, y)$

$\therefore$  The midpoint of  $\overline{AC}$  = the midpoint of  $\overline{BD}$

$$\therefore \left(1\frac{1}{2}, -\frac{1}{2}\right) = \left(\frac{X+4}{2}, \frac{y-5}{2}\right)$$

$$\therefore \frac{X+4}{2} = 1\frac{1}{2} \quad \therefore X+4 = 3 \quad \therefore X = -1$$

$$\therefore \frac{y-5}{2} = -\frac{1}{2} \quad \therefore y-5 = -1 \quad \therefore y = 4$$

$$\therefore D(-1, 4)$$

21

Luxor

1

1) b    2) c    3) c    4) b    5) d    6) c

2

[a]  $\therefore \sqrt{(3a-1-a)^2 + (1-5)^2} = 5$  (Squaring both sides)

$$\therefore (2a-1)^2 + (-4)^2 = 25$$

$$\therefore 4a^2 - 4a + 1 + 16 - 25 = 0$$

$$\therefore 4a^2 - 4a - 8 = 0 \quad \therefore a^2 - a - 2 = 0$$

$$\therefore (a-2)(a+1) = 0 \quad \therefore a = 2 \text{ or } a = -1$$

[b]  $\therefore 3 \tan X - 4 \sin^2 30^\circ = 8 \cos^2 60^\circ$

$$\therefore 3 \tan X - 4 \times \left(\frac{1}{2}\right)^2 = 8 \times \left(\frac{1}{2}\right)^2$$

$$\therefore 3 \tan X = 2 + 1 \quad \therefore \tan X = 1 \quad \therefore X = 45^\circ$$

3

[a]  $\therefore$  The slope of the given straight line =  $-\frac{2}{3}$

$\therefore$  The slope of the required straight line =  $-\frac{2}{3}$

$\therefore$  Its equation is :  $y = -\frac{2}{3}x + c$

$\therefore (1, 2)$  satisfies the equation.

$\therefore 2 = -\frac{2}{3} \times 1 + c \quad \therefore c = \frac{8}{3}$

$\therefore$  The equation is :  $y = -\frac{2}{3}x + \frac{8}{3}$

[b]  $\therefore m = \frac{4\sqrt{3}-\sqrt{3}}{1+2} = \sqrt{3} \quad \therefore \tan \theta = \sqrt{3} \quad \therefore \theta = 60^\circ$

4

[a]  $\therefore AB = \sqrt{(-2-4)^2 + (7+1)^2} = \sqrt{36+64}$   
 $= 10$  length units

$\therefore r = \frac{1}{2} AB = 5$  length units

$\therefore$  The area =  $3.14 \times (5)^2 = 78.5$  square units.

[b] (1)  $\therefore AB = AC, \overline{AD} \perp \overline{BC}$

$\therefore BD = CD = 6$  cm.

In  $\triangle ADC$  :

$\therefore m(\angle ADC) = 90^\circ$

$\therefore (AD)^2 = (10)^2 - (6)^2 = 64$

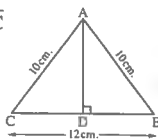
$\therefore AD = 8$  cm.

$\therefore \sin^2 C + \cos^2 C = \left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2 = 1$

(2)  $\therefore m(\angle B) = m(\angle C) \quad \therefore \sin B = \sin C$

$\therefore \sin B + \cos C = \sin C + \cos C$

$= \frac{8}{10} + \frac{6}{10} = \frac{14}{10} > 1$



5

[a]  $\therefore \overline{AB} \parallel y\text{-axis} \quad \therefore 3 - x = 0 \quad \therefore x = 3$

[b] (1) In  $\triangle AMB$  :  $\therefore m(\angle AMB) = 90^\circ$

$\therefore \cos(\angle BAM) = \frac{4}{5}$

$\therefore m(\angle BAC) \approx 36^\circ 52' 12''$

$\therefore m(\angle BAD) = 73^\circ 44' 24''$

(2)  $\therefore (BM)^2 = (5)^2 - (4)^2 = 9 \quad \therefore BM = 3$  cm.

$\therefore$  The area =  $\frac{1}{2} \times 8 \times 6 = 24$  cm<sup>2</sup>

22

Aswan

1

1 c    2 b    3 d    4 c    5 c    6 b

2

[a]  $\therefore 2 \sin X = \tan^2 60^\circ - 2 \tan^2 45^\circ$

$\therefore 2 \sin X = (\sqrt{3})^2 - 2 \times (1)^2 \quad \therefore 2 \sin X = 1$

$\therefore \sin X = \frac{1}{2} \quad \therefore X = 30^\circ$

[b]  $\therefore$  The slope of  $\overline{AB} = \frac{5-3}{3-1} = 1$

$\therefore$  The slope of the required straight line =  $-1$

$\therefore$  Its equation is :  $y = -x + c$

$\therefore$  the midpoint of  $\overline{AB} = \left(\frac{1+3}{2}, \frac{3+5}{2}\right)$   
 $= (2, 4)$

$\therefore$  the required straight line passes through the midpoint of  $\overline{AB}$

$\therefore 4 = -2 + c \quad \therefore c = 6$

$\therefore$  The equation of the required straight line is :  
 $y = -x + 6$

3

[a]  $\therefore (4, 2) = \left(\frac{2+y}{2}, \frac{4+y}{2}\right)$

$\therefore \frac{4+y}{2} = 2 \quad \therefore 4+y = 4 \quad \therefore y = 0$

[b]  $\therefore$  The slope of  $\overline{AB} = m_1 = \frac{3+1}{2+1} = \frac{4}{3}$

$\therefore$  the slope of  $\overline{BC} = m_2 = \frac{0-3}{6-2} = -\frac{3}{4}$

$\therefore m_1 \times m_2 = \frac{4}{3} \times -\frac{3}{4} = -1 \quad \therefore \overline{AB} \perp \overline{BC}$

$\therefore \triangle ABC$  is right-angled at B

4

[a]  $\therefore m(\angle Y) = 90^\circ$

$\therefore (YZ)^2 = (13)^2 - (5)^2 = 144$

$\therefore YZ = 12$  cm.

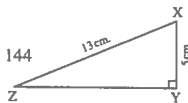
(1)  $\tan X \tan Z = \frac{12}{5} \times \frac{5}{12} = 1$

(2)  $\cos X \cos Z - \sin X \sin Z$

$= \frac{5}{13} \times \frac{12}{13} - \frac{12}{13} \times \frac{5}{13} = 0$

[b]  $\therefore$  The straight line passes through the points  
 $(1, 0), (0, 4)$

$\therefore$  Its slope =  $\frac{4-0}{0-1} = -4$



- $\therefore$  Its equation is :  $y = -4X + c$   
 $\therefore$  the straight line intercepts 4 units from the positive part of y-axis  
 $\therefore$  Its equation is :  $y = -4X + 4$

5

[a]  $\therefore m_1 = \frac{3-4}{-1-2} = \frac{1}{3}$ ,  $m_2 = \frac{1}{3}$   $\therefore m_1 = m_2$

$\therefore$  The two straight lines are parallel.

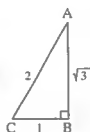
[b]  $\therefore 2AB = \sqrt{3}AC$   $\therefore \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

Let  $AB = \sqrt{3}$  length units

$\therefore AC = 2$  length units

$\therefore BC = 1$  length units

$\therefore \sin C = \frac{\sqrt{3}}{2}$ ,  $\cos C = \frac{1}{2}$ ,  $\tan C = \sqrt{3}$



## 23 New Valley

1

- [1] d [2] b [3] c [4] a [5] d [6] c

2

[a]  $\therefore m(\angle Z) = 90^\circ$

$\therefore (XY)^2 = (3)^2 + (4)^2 = 25$

$\therefore XY = 5$  cm.

[1]  $\tan X \tan Y = \frac{4}{3} \times \frac{3}{4} = 1$

[2]  $\sin^2 X + \cos^2 X = \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = 1$

[b]  $\therefore AB = \sqrt{(1-3)^2 + (5-3)^2} = \sqrt{4+4}$   
 $= 2\sqrt{2}$  length units

$\therefore BC = \sqrt{(1-1)^2 + (3-5)^2} = \sqrt{0+4}$   
 $= 2$  length units

$\therefore AC = \sqrt{(1-3)^2 + (3-3)^2} = \sqrt{4+0}$   
 $= 2$  length units

$\therefore BC = AC$

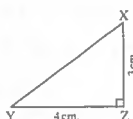
$\therefore \triangle ABC$  is an isosceles triangle

$\therefore (AB)^2 = (2\sqrt{2})^2 = 8$

$\therefore (BC)^2 + (AC)^2 = (2)^2 + (2)^2 = 8$

$\therefore (AB)^2 = (BC)^2 + (AC)^2$

$\therefore \triangle ABC$  is a right-angled triangle at C



3

[a] [1]  $\therefore \tan X = 4 \sin 30^\circ \cos 60^\circ$

$\therefore \tan X = 4 \times \frac{1}{2} \times \frac{1}{2} = 1$   $\therefore X = 45^\circ$

[2]  $\sin 45^\circ = \frac{1}{\sqrt{2}}$

[b]  $\therefore$  The slope of the straight line = 2

$\therefore$  Its equation is :  $y = 2X + c$

$\therefore (1, 0)$  satisfies the equation.

$\therefore 0 = 2 \times 1 + c$   $\therefore c = -2$

$\therefore$  The equation is :  $y = 2X - 2$

4

[a] [1]  $\therefore AB = AC$ ,  $\overline{AD} \perp \overline{BC}$

$\therefore BD = CD = 6$  cm.

In  $\triangle ADB$   $\therefore m(\angle ADB) = 90^\circ$

$\therefore \cos B = \frac{6}{10} = \frac{3}{5}$

[2]  $m(\angle B) \approx 53^\circ 7'$

[3]  $\therefore \sin(90^\circ - B) = \cos B$

$\therefore \sin(90^\circ - B) = \frac{3}{5}$

[b] [1]  $\therefore$  The midpoint of  $\overline{AC} = \left(\frac{-2+4}{2}, \frac{3-3}{2}\right)$   
 $= (1, 0)$

$\therefore$  The point of intersection of the diagonals  
 $= (1, 0)$

[2] Let  $D(X, y)$

$\therefore (1, 0) = \left(\frac{-1+X}{2}, \frac{-2+y}{2}\right)$

$\therefore \frac{-1+X}{2} = 1$   $\therefore -1+X = 2$   $\therefore X = 3$

$\therefore \frac{-2+y}{2} = 0$   $\therefore -2+y = 0$   $\therefore y = 2$

$\therefore D(3, 2)$

5

[a]  $\therefore L_1 \parallel L_2$   $\therefore m_1 = m_2$   $\therefore \frac{k-1}{3-2} = \tan 45^\circ$

$\therefore k-1 = 1$   $\therefore k = 2$

[b]  $\therefore$  The straight line passes through  $(2, 0)$ ,  $(0, 4)$

$\therefore$  Its slope =  $\frac{4-0}{0-2} = -2$

$\therefore$  Its equation is :  $y = -2X + c$

$\therefore$  the straight line intercepts 4 units from the positive part of y-axis

$\therefore$  Its equation is :  $y = -2X + 4$

## 24 South Sinai

1

1 a    2 b    3 c    4 d    5 a    6 c

2

$$[a] \therefore \cos 60^\circ = \frac{1}{2} \quad (1)$$

$$\therefore \cos^2 30^\circ - \sin^2 30^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{1}{2} \quad (2)$$

$$\text{From (1) \& (2) : } \therefore \cos 60^\circ = \cos^2 30^\circ - \sin^2 30^\circ$$

[b] Let B (X, y)

$$\therefore (1, -3) = \left(\frac{4+X}{2}, \frac{-3+y}{2}\right)$$

$$\therefore \frac{4+X}{2} = 1 \quad \therefore 4+X = 2 \quad \therefore X = -2$$

$$\therefore \frac{-3+y}{2} = -3 \quad \therefore -3+y = -6 \quad \therefore y = -3$$

$$\therefore B(-2, -3)$$

3

$$[a] \therefore \text{The slope of the straight line} = \frac{-3-3}{-1-1} = 3$$

$$\therefore \text{Its equation is : } y = 3X + c$$

$$\therefore (1, 3) \text{ satisfies the equation.}$$

$$\therefore 3 = 3 \times 1 + c \quad \therefore c = 0$$

$$\therefore \text{The equation is : } y = 3X$$

$$[b] \therefore AB = \sqrt{(1-3)^2 + (5-3)^2} = \sqrt{4+4}$$

$$= 2\sqrt{2} \text{ length units}$$

$$\therefore BC = \sqrt{(1-1)^2 + (3-5)^2} = \sqrt{4}$$

$$= 2 \text{ length units}$$

$$\therefore AC = \sqrt{(1-3)^2 + (3-3)^2} = \sqrt{4} = 2 \text{ length units}$$

$$\therefore BC = AC$$

$$\therefore \triangle ABC \text{ is an isosceles triangle.}$$

4

$$[a] \therefore \text{The slope of the straight line} = \tan 45^\circ = 1$$

$$\therefore \text{Its equation is : } y = X + c$$

$$\therefore (-2, 3) \text{ satisfies the equation.}$$

$$\therefore 3 = -2 + c \quad \therefore c = 5$$

$$\therefore \text{The equation is : } y = X + 5$$

$$[b] \frac{2 \tan 45^\circ}{1 + \tan^2 45^\circ} = \frac{2 \times 1}{1 + (1)^2} = 1$$

5

 [a]  $\therefore$  The slope of the straight line is 2 and it intersects 5 units from the positive part of the y-axis

$$\therefore \text{Its equation is : } y = 2X + 5$$

$$[b] [1] \therefore m(\angle B) = 90^\circ \quad \therefore \sin C = \frac{5}{10} = \frac{1}{2}$$

$$\therefore m(\angle C) = 30^\circ$$

$$[2] \sin^2 C + \cos^2 C = \sin^2 30^\circ + \cos^2 30^\circ$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$$

## 25 North Sinai

1

1 d    2 c    3 a    4 c    5 d    6 c

2

$$[a] \therefore \cos 60^\circ = \frac{1}{2} \quad (1)$$

$$\therefore 2 \cos^2 30^\circ - 1 = 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1$$

$$= 2 \times \frac{3}{4} - 1 = \frac{1}{2} \quad (2)$$

$$\text{From (1) \& (2) : } \therefore \cos 60^\circ = 2 \cos^2 30^\circ - 1$$

$$[b] \therefore AB = \sqrt{(-4-1)^2 + (2+2)^2} = \sqrt{25+16}$$

$$= \sqrt{41} \text{ length units}$$

$$\therefore BC = \sqrt{(1+4)^2 + (6-2)^2} = \sqrt{25+16}$$

$$= \sqrt{41} \text{ length units}$$

$$\therefore AC = \sqrt{(1-1)^2 + (6+2)^2} = \sqrt{64}$$

$$= 8 \text{ length units}$$

$$\therefore AB = BC$$

$$\therefore \triangle ABC \text{ is an isosceles triangle.}$$

3

 [a]  $\therefore$  The slope of the straight line = 2 and it cuts 7 units from the positive part of the y-axis

$$\therefore \text{Its equation is : } y = 2X + 7$$

$$[b] [1] \therefore m(\angle B) = 90^\circ$$

$$\therefore (AB)^2 = (10)^2 - (8)^2 = 36$$

$$\therefore AB = 6 \text{ cm.}$$

$$[2] \sin^2 A + \cos^2 A = \left(\frac{8}{10}\right)^2 + \left(\frac{6}{10}\right)^2$$

$$= \frac{64}{100} + \frac{36}{100} = 1$$

$$\begin{aligned}
 [4] \quad [a] \quad \therefore \cos X &= \frac{\sin 60^\circ \sin 30^\circ}{\sin^2 45^\circ} \quad \therefore \cos X = \frac{\frac{\sqrt{3}}{2} \times \frac{1}{2}}{\left(\frac{1}{\sqrt{2}}\right)^2} \\
 \therefore \cos X &= \frac{\sqrt{3}}{2} \quad \therefore X = 30^\circ \\
 [b] \quad \therefore \text{The slope of the given straight line} &= \frac{-4+3}{5-2} \\
 &= \frac{-1}{3}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{The slope of the required straight line} &= 3 \\
 \therefore \text{Its equation is : } y &= 3X + c \\
 \therefore (1, 2) \text{ satisfies the equation.} \\
 \therefore 2 &= 3 \times 1 + c \quad \therefore c = -1 \\
 \therefore \text{The equation is : } y &= 3X - 1
 \end{aligned}$$

$$\begin{aligned}
 [5] \quad [1] \quad \therefore MA &= \sqrt{(-1-3)^2 + (2+1)^2} = \sqrt{16+9} \\
 &= 5 \text{ length units} \\
 \therefore MB &= \sqrt{(-1+4)^2 + (2-6)^2} = \sqrt{9+16} \\
 &= 5 \text{ length units} \\
 \text{and } MC &= \sqrt{(-1-2)^2 + (2+2)^2} = \sqrt{9+16} \\
 &= 5 \text{ length units} \\
 \therefore MA &= MB = MC
 \end{aligned}$$

$\therefore A, B$  and  $C$  lie on the circle  $M$

$$\begin{aligned}
 [2] \quad \text{The circumference of the circle} &= 2 \times 3.14 \times 5 \\
 &= 31.4 \text{ length units.}
 \end{aligned}$$

## 26 Red Sea

$$\begin{aligned}
 [1] \quad [1] \quad b \quad [2] \quad c \quad [3] \quad a \quad [4] \quad c \quad [5] \quad b \quad [6] \quad d
 \end{aligned}$$

$$\begin{aligned}
 [2] \quad [a] \quad \therefore \sin 60^\circ &= \frac{\sqrt{3}}{2} \\
 \therefore 2 \sin 30^\circ \cos 30^\circ \tan 45^\circ &= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} \times 1 \\
 &= \frac{\sqrt{3}}{2} \quad (2)
 \end{aligned}$$

From (1), (2):

$$\therefore \sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ \tan 45^\circ$$

$$\begin{aligned}
 [b] \quad \therefore \text{The slope of the straight line} &= \frac{-1-2}{-2-4} = \frac{1}{2} \\
 \therefore \text{Its equation is : } y &= \frac{1}{2}X + c \\
 \therefore (4, 2) \text{ satisfies the equation.} \\
 \therefore 2 &= \frac{1}{2} \times 4 + c \quad \therefore c = 0 \\
 \therefore \text{The equation is : } y &= \frac{1}{2}X
 \end{aligned}$$

$$\begin{aligned}
 [3] \quad [a] \quad \therefore \tan X &= 4 \cos 60^\circ \sin 30^\circ \\
 \therefore \tan X &= 4 \times \frac{1}{2} \times \frac{1}{2} \\
 \therefore \tan X &= 1 \quad \therefore X = 45^\circ \\
 [b] \quad \therefore AB &= \sqrt{(-3-2)^2 + (0-4)^2} = \sqrt{25+16} \\
 &= \sqrt{41} \text{ length units} \\
 \therefore BC &= \sqrt{(-7+3)^2 + (5-0)^2} = \sqrt{16+25} \\
 &= \sqrt{41} \text{ length units} \\
 \therefore AC &= \sqrt{(-7-2)^2 + (5-4)^2} = \sqrt{81+1} \\
 &= \sqrt{82} \text{ length units} \\
 \therefore (AC)^2 &= (AB)^2 + (BC)^2 \\
 \therefore \triangle ABC &\text{ is a right-angled triangle at } B \\
 \therefore \text{its area} &= \frac{1}{2} \times \sqrt{41} \times \sqrt{41} = 20 \frac{1}{2} \text{ square units.}
 \end{aligned}$$

$$\begin{aligned}
 [4] \quad [a] \quad \therefore \text{The slope of the straight line} &= 2 \text{ and it intercepts} \\
 &7 \text{ units from the positive part of the } y\text{-axis} \\
 \therefore \text{Its equation is : } y &= 2X + 7 \\
 [b] \quad \therefore m(\angle B) &= 90^\circ \\
 \therefore (AB)^2 &= (13)^2 - (5)^2 = 144 \\
 \therefore AB &= 12 \text{ cm.} \\
 \therefore \sin A \cos C + \cos A \sin C \\
 &= \frac{5}{13} \times \frac{5}{13} + \frac{12}{13} \times \frac{12}{13} = 1
 \end{aligned}$$

$$\begin{aligned}
 [5] \quad [a] \quad \therefore \sqrt{(X+2)^2 + (7-3)^2} &= 5 \text{ (Squaring both sides)} \\
 \therefore (X+2)^2 + (7-3)^2 &= 25 \\
 \therefore X^2 + 4X + 4 + 16 - 25 &= 0 \\
 \therefore X^2 + 4X - 5 &= 0 \quad \therefore (X+5)(X-1) = 0 \\
 \therefore X &= -5 \text{ or } X = 1
 \end{aligned}$$

$$\begin{aligned} \text{[b]} \because L_1 \parallel L_2 & \therefore m_1 = m_2 \\ \therefore \frac{k-1}{2-3} = \tan 45^\circ & \therefore -k+1 = 1 \\ \therefore k = 0 \end{aligned}$$

### Matrouh

- 1  
1 b    2 a    3 a    4 c    5 a    6 d

$$\begin{aligned} \text{[a]} \because \tan 60^\circ &= \sqrt{3} \\ \therefore \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} &= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}} = \sqrt{3} \end{aligned} \quad (1)$$

From (1) , (2) :

$$\therefore \tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

$$\text{[b]} \because \text{The slope of } \overline{AB} = m_1 = \frac{-4-0}{2-6} = 1$$

$$\therefore \text{the slope of } \overline{BC} = m_2 = \frac{2+4}{-4-2} = -1$$

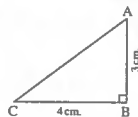
$$\therefore m_1 \times m_2 = 1 \times -1 = -1$$

$$\therefore \overline{AB} \perp \overline{BC}$$

$\therefore \Delta ABC$  is a right-angled triangle at B

$$\begin{aligned} \text{[a]} \because \sqrt{(a+2)^2 + (7-3)^2} &= 5 \text{ (Squaring both sides)} \\ \therefore (a+2)^2 + (7-3)^2 &= 25 \\ \therefore a^2 + 4a + 4 + 16 - 25 &= 0 \\ \therefore a^2 + 4a - 5 &= 0 & \therefore (a+5)(a-1) = 0 \\ \therefore a = -5 \text{ or } a = 1 \end{aligned}$$

$$\begin{aligned} \text{[b]} \because m(\angle B) &= 90^\circ \\ \therefore (AC)^2 &= (3)^2 + (4)^2 = 25 \\ \therefore AC &= 5 \text{ cm.} \\ \therefore \sin A \cos C + \cos A \sin C &= \frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5} = 1 \end{aligned}$$



$$\begin{aligned} \text{[a]} \text{ Let } A = X^\circ, B = 2X^\circ \\ \therefore X + 2X &= 90^\circ & \therefore 3X = 90^\circ \\ \therefore X &= 30^\circ & \therefore A = 30^\circ, B = 60^\circ \\ \therefore \sin A + \cos B &= \sin 30^\circ + \cos 60^\circ \\ &= \frac{1}{2} + \frac{1}{2} = 1 \end{aligned}$$

$$\begin{aligned} \text{[b]} \because \frac{X}{2} + \frac{y}{2} &= 1 \text{ (Multiplying by 2)} \\ \therefore X + y &= 2 \\ \therefore \text{The slope} &= -1 \\ \therefore \text{the intercepted part} &= 2 \text{ units from the positive part of y-axis.} \end{aligned}$$

$$\begin{aligned} \text{[a]} \because (-3, y) &= \left( \frac{X+9}{2}, \frac{-6-12}{2} \right) \\ \therefore y &= -9 \\ \therefore \frac{X+9}{2} &= -3 & \therefore X+9 = -6 \\ \therefore X &= -15 \end{aligned}$$

$$\begin{aligned} \text{[b]} \because \text{The slope of the given straight line} &= \frac{-1}{2} \\ \therefore \text{The slope of the required straight line} &= \frac{-1}{2} \\ \therefore \text{Its equation is : } y &= \frac{-1}{2}X + c \\ \therefore (3, -5) &\text{ satisfies the equation.} \\ \therefore -5 &= \frac{-1}{2} \times 3 + c & \therefore c = \frac{-7}{2} \\ \therefore \text{The equation is : } y &= \frac{-1}{2}X - \frac{7}{2} \end{aligned}$$